POPULATION DYNAMICS AND ITS CONSEQUENCES ON HEALTH RISK MODELLING FOLLOWING INHALATION EXPOSURE

Improving toxicological risk assessment

A crucial part of risk assessment following airborne spread of toxic chemicals is the translation from concentration field to injury probabilities. The impact of population movement remains an understudied and poorly understood factor in such an enterprise. An initiative to target this knowledge gap using an elemental approach, with emphasis on a mathematical method to calculate exposure probability distributions, is presented here.

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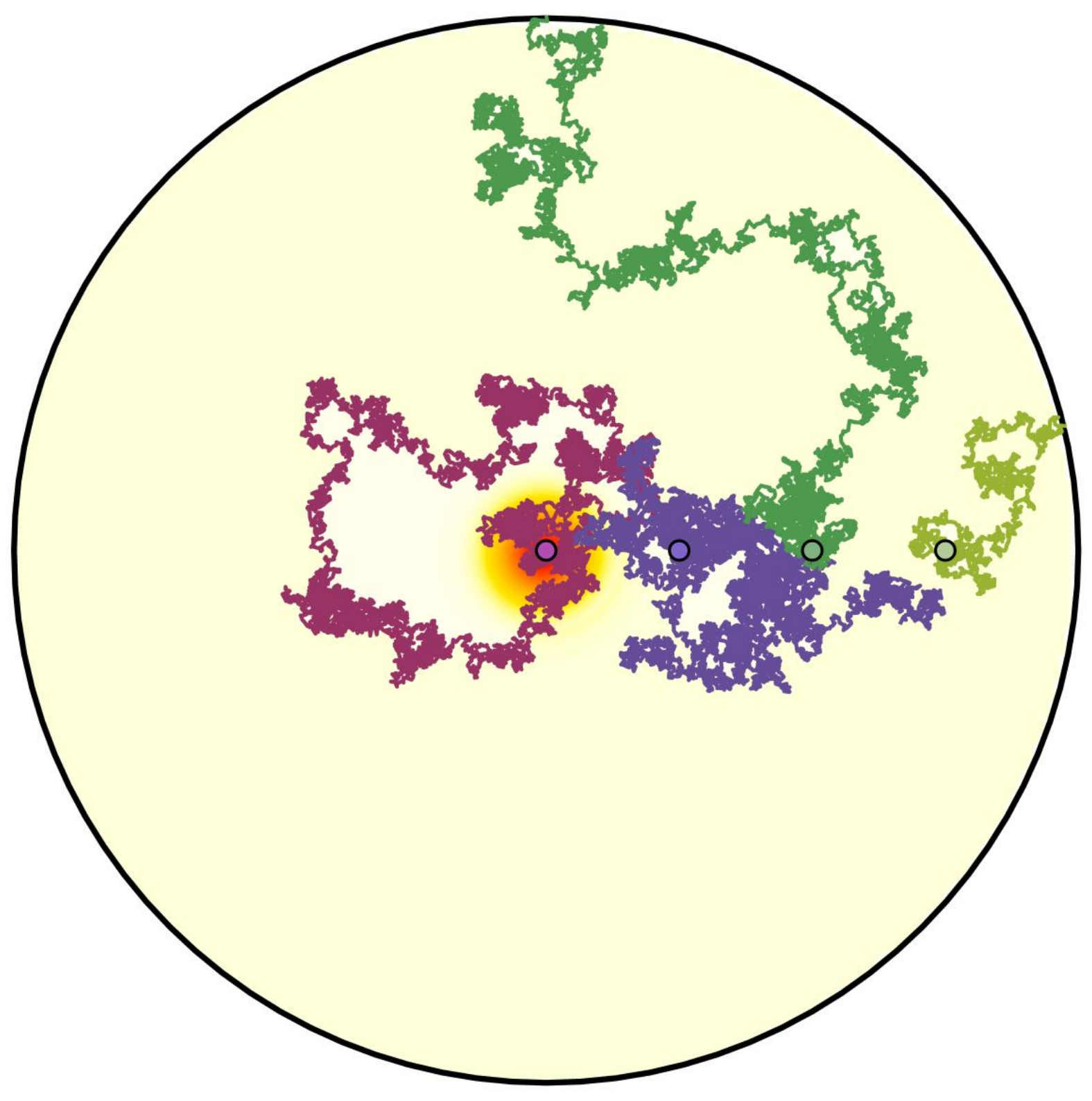

Figure 1. The domain is circular and restricted by a border that terminates all paths that extends beyond it. A symmetric two-dimensional Gaussian concentration field is located at the center of the domain, causing exposure of the, initially, uniformly distributed population. Four example paths are shown.

$$\Theta = \log \int_0^T c(t, X(t))^n \, dt$$

$$Q_r = P(\alpha + \beta\Theta \geq \Psi)$$

Θ	logarithmic toxic load
c	concentration
t, T	time
$X(t)$	random paths
Q_r	population injury
Ψ	tolerance, i.e. standard normal distribution
α, β, n	substance toxicological parameters

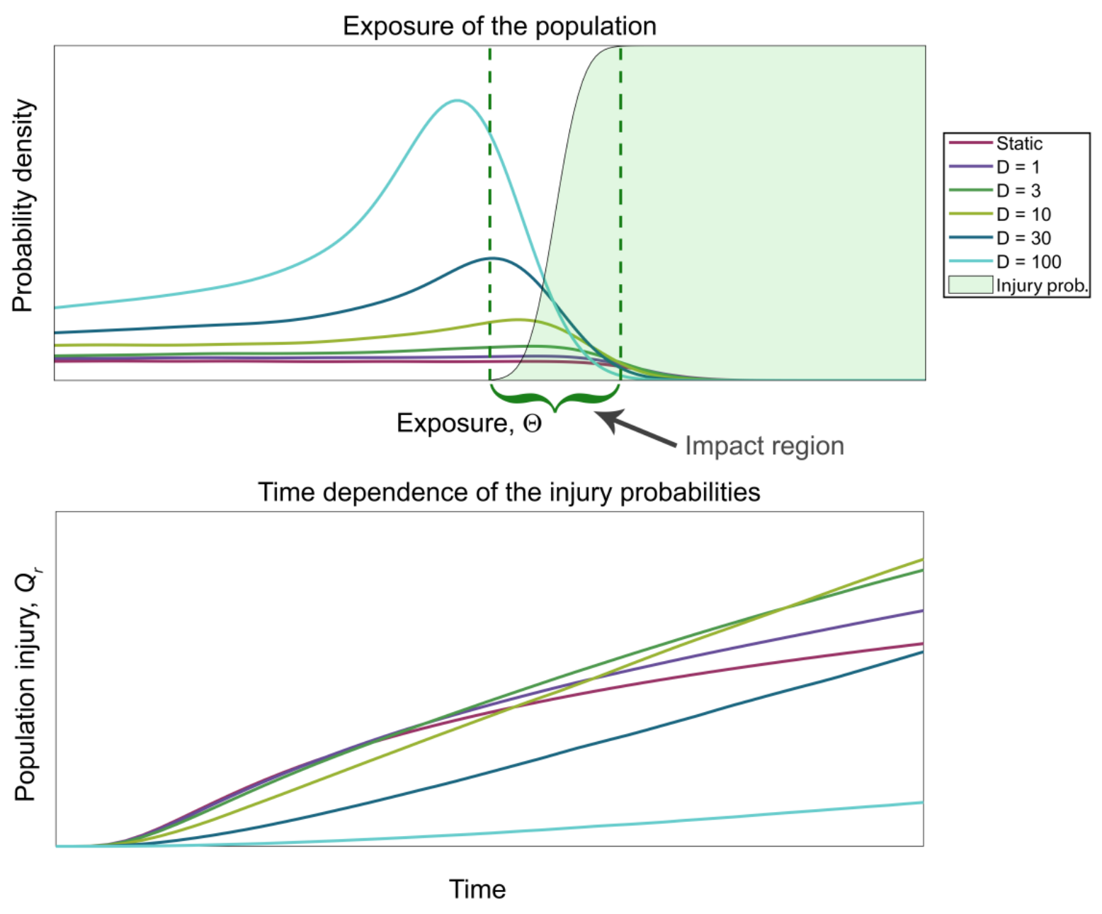

Figure 2. Top panel, a snapshot of the distributions of Θ for different movement speeds at a certain point in time. The impact region depicts the exposure where the population becomes injured. Bottom panel, the time dependency of the population injury.

Approach

Our ambition is to improve consequence analyses by unraveling the main properties of the interaction between a toxic plume and population dynamics. The initial approach is to identify and investigate the most fundamental and relevant features derived from population dynamics within the framework of probit analysis.
We describe the dynamics as Itô diffusion acting on a 2D domain. Even though this is not a realistic movement pattern of a population, it provides distinct mathematical properties that allow for a wider generalization.
An integrated toxic load model is applied on a population that are allowed to move within a domain containing a concentration field. Static population constitutes the baseline for the study and is found as a special case where the movement speed is reduced to zero. The population is assumed to be unaware of the exposure and do therefore not react in any way to the concentration field. The main metric of interest is the population injury, Q_r, which is defined as the fraction of the entire population that becomes injured.

Numerical method

The core of the problem is the issue of how to calculate the time-dependent distribution of Θ depending on the starting position, concentration field and movement speed. This can be accomplished either with comprehensive and time consuming Monte Carlo-simulations (MC) or by a more sophisticated method using partial differential equations (PDE). The mathematical solution of the latter method and its application is the focus of this work.

In short, assuming that the population movement can be expressed in terms of Itô diffusion, the famous Feynman-Kac formula enables the problem to be cast in partial differential equation form. The solution of the PDE is the moment generating function (MGF) of the toxic load, which is related to the distribution of Θ by a deconvolution problem.

Hence, the numerical challenge of the proposed method consist of solving:

1. 2-dimensional PDE problems to obtain the MGF and
2. the deconvolution problem by posing it as a sequential least squares programming problems, to obtain the distribution of Θ.

Initial results

1. The PDE-method suggested here provides satisfying result while showing significant higher computational efficiency than MC. The statistical issues with MC are avoided, but the need for a well-designed deconvolution process is recognized.
2. The distribution of Θ, top panel in Figure 2, must be considered to avoid a biased injury estimate.
3. An investigation using the prerequisites shown in Figure 1 provided interesting qualitative results. It was found that the movement speed that maximize Q_r changes with time, illustrated in the lower panel of Figure 2. Slow or static population will give rise to the highest exposure levels while higher speed results in more people visiting the toxic plume, but for a shorter period of time.

Conclusion

It is worth noticing that, for any given starting position, the calculated injury probability may be considerable different if only the expectation value of Θ is used instead of the complete distribution thereof. For low exposures, the injury risk is underestimated when using the expectation value, while the opposite is true for high exposures.

The impact of the movement speed on Q_r varies substantially over time. This phenomenon might be non-intuitive at first, and it underlines the complexity of this field. Its direct implication is that there is no constant factor available to translate the population injury between different movement speeds. Instead, this finding motivates further and more thorough studies on the subject.