DATA ASSIMILATION AT LOCAL SCALE TO IMPROVE CFD SIMULATIONS OF DISPERSION AROUND INDUSTRIAL SITES AND URBAN NEIGHBOURHOODS

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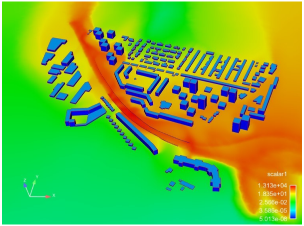
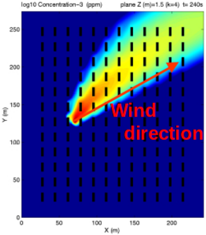
Harmo18 - Mathematical problems in air quality modelling

MICRO-METEOROLOGY APPLICATIONS

- Dispersion in built up environment

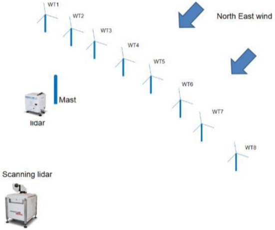


City of Toulouse



MUST experiment

- Estimation of local wind fields



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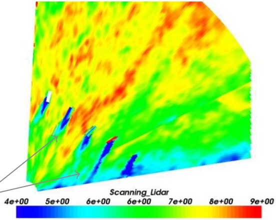
Turbine wakes

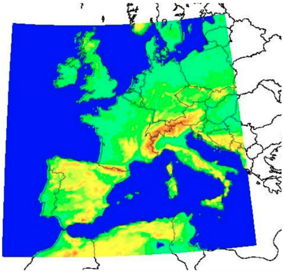


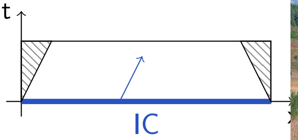
CONTEXT

- Atmospheric dispersion modelling requires meteorological inputs (wind, turbulence, etc.)
- Local wind fields (urban neighbourhoods, surroundings of industrial sites, etc.) have very complex structures ⇒ difficult to simulate with CFD
- CFD simulations could be improved using available observations
- Objective: Develop local-scale data assimilation methods

LOCAL CFD SIMULATIONS

Mesoscale simulations
(e.g. WRF, ALADIN)
$\Delta x \approx 10\text{km}$, $\Delta z \approx 10\text{m}$
$L \approx 3000\text{km}$, $\frac{L}{U} \approx 7$ days



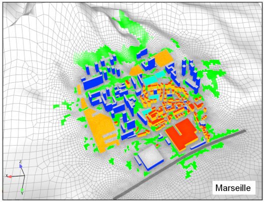


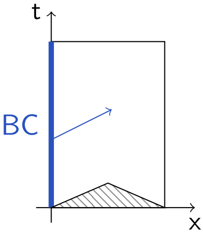
Boundary conditions

Data assimilation

Observations

Local simulations
(e.g. *Code_Saturne*)
$\Delta x \approx 10\text{m}$, $\Delta z \approx 1\text{m}$
$L \approx 5\text{km}$, $\frac{L}{U} \approx 17\text{min}$





INTRODUCTION TO DATA ASSIMILATION

- $\mathbf{z}^a$: analysis = best estimate of control variables $\mathbf{z}$, given all available information
  - model $\mathcal{M}$,
  - observations $\mathbf{y}^o$,
  - prior knowledge $\mathbf{z}^b$,
  - etc.

- Nudging: add relaxation term to dynamical equations
  - **Back and forth nudging** (BFN)

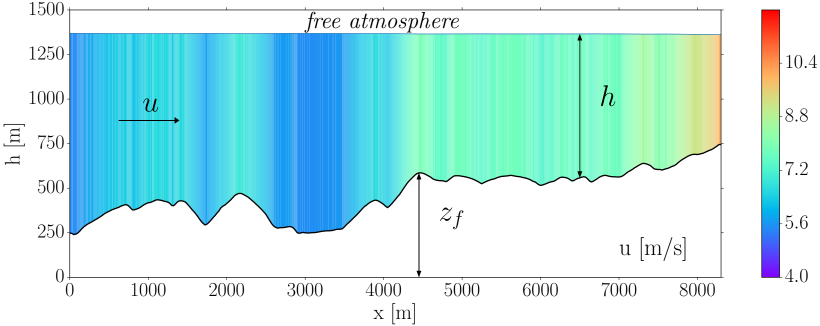
- Filtering methods (e.g. Kalman filter) and Variational methods (e.g. 3D-Var)
  - Ensemble variational methods: **iterative ensemble Kalman smoother/filter** (IEnKS, IEnKF)

SHALLOW WATER MODEL

- 'Level' models $\Longleftrightarrow$ 'Layer' models
  Vertical finite-difference approximation — Multi-layer SWE

- Vertically averaged equations: $\frac{\partial \mathbf{X}}{\partial t} + \mathbf{M}\frac{\partial \mathbf{X}}{\partial x} = \mathbf{S}$

$$\mathbf{X} = \begin{pmatrix} h \\ u \end{pmatrix}, \quad \mathbf{M} = \begin{pmatrix} u & h \\ g' & u \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} 0 \\ -g'\frac{\partial z_f}{\partial x} \end{pmatrix}, \quad \text{and} \quad g' : \text{ reduced gravity}$$



Simulation with 1D shallow water model over topography.

BACK AND FORTH NUDGING ALGORITHM

Iterative algorithm of forward and backward integrations with nudging [1]:

forward (f) or backward (b)

Observation operator

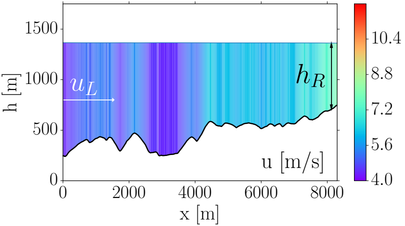
$$\textbf{(F)}\ \frac{\partial \mathbf{X}_k^f}{\partial t} + \mathbf{M}^f \frac{\partial \mathbf{X}_k^f}{\partial x} = \mathbf{S} + \mathbf{K}\left[\mathbf{y}^o - \mathcal{H}(\mathbf{X}_k^f)\right] \quad \text{for } 0 \leq t \leq T,\ \delta t > 0$$

$$\textbf{(B)}\ \frac{\partial \mathbf{X}_k^b}{\partial t} + \mathbf{M}^b \frac{\partial \mathbf{X}_k^b}{\partial x} = \mathbf{S} - \widetilde{\mathbf{K}}\left[\mathbf{y}^o - \mathcal{H}(\mathbf{X}_k^b)\right] \quad \text{for } T \geq t \geq 0,\ \delta t < 0$$

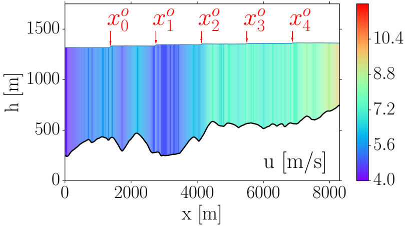
k: BFN iteration

[1] Auroux and Blum (2005, 2008); Auroux et al. (2013)

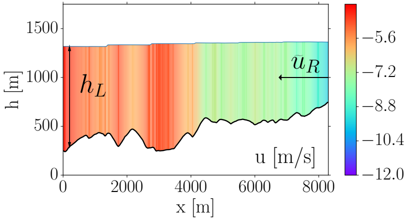
BOUNDARY CONDITIONS FOR BFN ALGORITHM



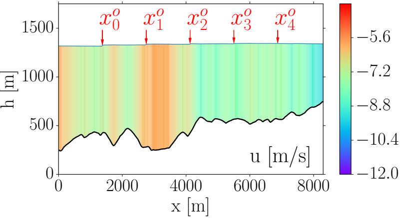
Forward



$$\widetilde{u} = -u$$
$$\widetilde{u}_R^k = -u^k(x = L)$$
$$\widetilde{h}_L^k = h^k(x = 0)$$



Backward



$$u = -\widetilde{u}$$
$$u_L^{k+1} = -\widetilde{u}^k(x = 0)$$
$$h_R^{k+1} = \widetilde{h}^k(x = L)$$

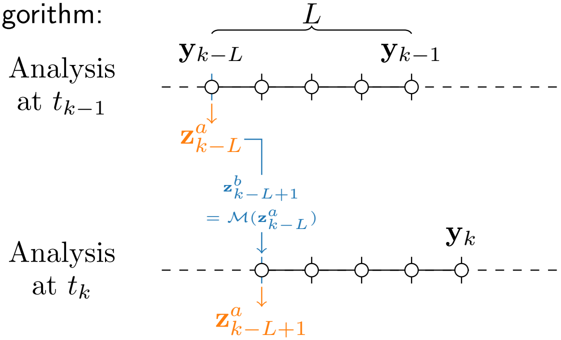
ITERATIVE ENSEMBLE KALMAN SMOOTHER [1]

- Cost function:
  $\mathcal{J} = \|\text{distance to prior}\|_{\mathbf{P}^{-1}} + \|\text{distance to observations}\|_{\mathbf{R}^{-1}}$

- Ensemble method $\rightarrow$ estimation of error covariance matrices

- Iterative minimisation of $\mathcal{J}$ with Gauss-Newton algorithm

- 2 cycles of IEnKS algorithm:

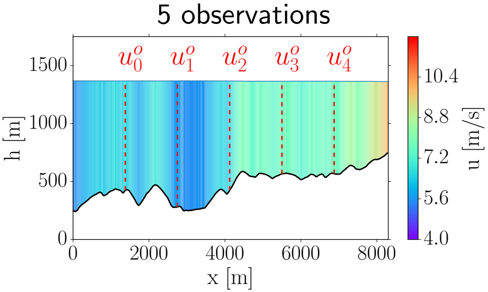


[1] Sakov et al. (2012); Bocquet and Sakov (2014)

EXPERIMENTS

True BCs
$u_L^t = 5.5\mathrm{m/s}$
$h_R^t = 617\mathrm{m}$

Reference simulation →



perfect obs. → Experiment 1

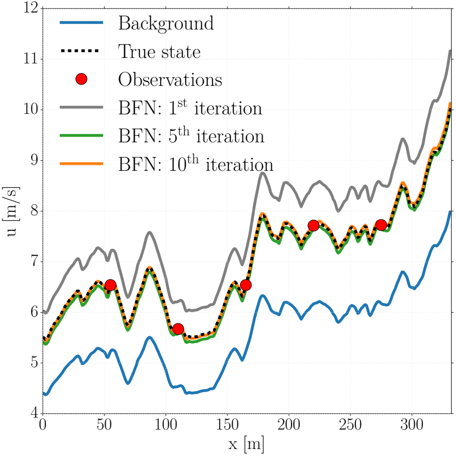
noisy obs. → Experiment 2

A priori BCs
$u_L^b = 4.4\mathrm{m/s}$
$h_R^b = 617\mathrm{m}$

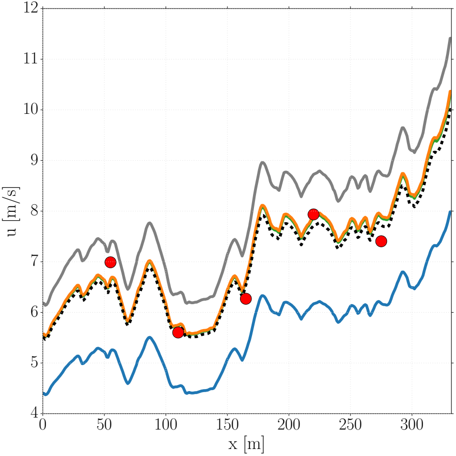
→ Experiment 1
→ Experiment 2

BFN RESULTS

- $\mathbf{K} = \widetilde{\mathbf{K}} = k\mathbf{H}^{\mathrm{T}}$ where $k\Delta t = 0.1$
- Convergence in $\sim$ 5 iterations



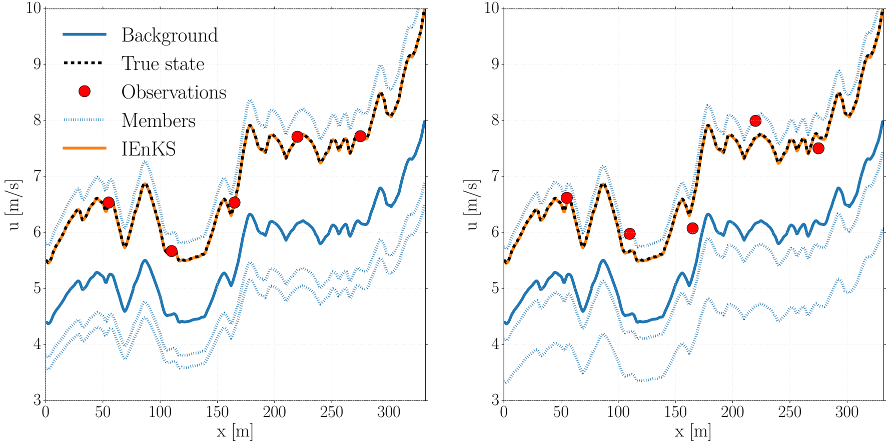
Exp. 1: Perfect observations



Exp. 2: Noisy observations

IEnKS RESULTS

- Background ensemble: 3 members
- $\mathbf{P} = \mathbf{I}$ and $\mathbf{R} = 0.1\mathbf{I}$
- Fast convergence (2-3 iterations)



Exp. 1: Perfect observations

Exp. 2: Noisy observations

CONCLUSIONS & PERSPECTIVES

- Both BFN algorithm and IEnKS help correcting BCs
- IEnKS more efficient here (less model integrations)
- Next steps:
  - More complex cases:
    - SW model: 2D
    - *Code_Saturne*: Vertical profiles of u
  - Localization or reduction of control vector size (e.g. principal component analysis)
  - Realistic cases with *Code_Saturne* (buildings, obstacles, etc.)

THANK YOU FOR YOUR ATTENTION

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IEnKS ALGORITHM

Background ensemble: $\mathbf{E}_0 = \underbrace{\mathbf{z}_0^{(0)}\mathbf{1}^{\mathrm{T}}}_{\text{mean}} + \underbrace{\mathbf{A}_0}_{\text{anomalies}}$. Initialisation: $\mathbf{w} = 0$

(BCs)

$$\boxed{\mathbf{w}} \longrightarrow \mathbf{z}'_0 = \mathbf{z}_0^{(0)}\mathbf{1}^{\mathrm{T}} + \mathbf{A}_0\mathbf{w} \longrightarrow \mathbf{E}'_0 \xrightarrow{\text{Model}} \mathbf{E}'_L \xrightarrow{\text{Obs. operator}} \mathbf{y}_L \xrightarrow{\text{Obs. } \mathbf{y}^o} \mathrm{d}\mathbf{y}$$

$$\mathrm{d}\mathbf{y} \longrightarrow \nabla\mathcal{J},\ \mathbb{H} \longrightarrow \Delta\mathbf{w} \xrightarrow{+} \mathbf{w}$$

$\Delta\mathbf{w}$ ↓ *until* $\|\Delta\mathbf{w}\| < e$

$$\mathbf{w}^a, \quad \mathbf{z}_0^a = \mathbf{z}_0^{(0)} + \mathbf{A}_0\mathbf{w}^a, \quad \mathbf{E}_0^a = \mathbf{z}_0^a\mathbf{1}^{\mathrm{T}} + \mathbf{A}_0\mathbb{H}^{-1/2}$$