CLASSIC PAPERS ON DISPERSION IN THE SURFACE LAYER

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Motivated by recent interest in near road impacts of vehicle emissions

Nieuwstadt, F.T.M., van Ulden, A.P., 1978. A numerical study of the vertical dispersion of passive contaminants from a continuous source in the atmospheric surface layer. Atmos. Environ. 14, 267–269. doi:10.1016/0004-6981(80)90288-7

van Ulden, A.P., 1978. Simple estimates for vertical diffusion from sources near the ground. Atmos. Environ. 12, 2125–2129. doi:10.1016/0004-6981(78)90167-1

Nieuwstadt, F.T.M., van Ulden, A.P., 1978. A numerical study of the vertical dispersion of passive contaminants from a continuous source in the atmospheric surface layer.

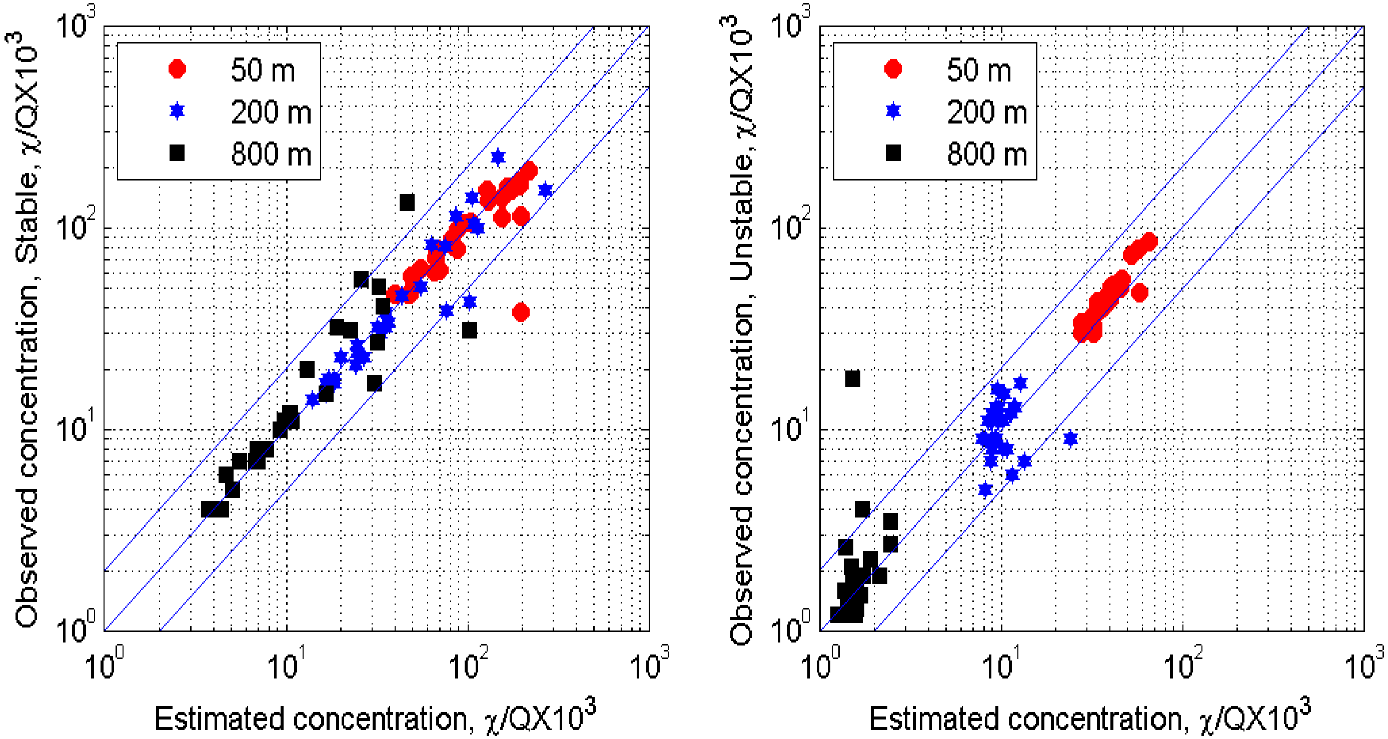
First paper to show that the solution of the eddy diffusivity equation provides concentration estimates that compare well with measurements made in Prairie Grass experiment (Barad, 1958)

$$U\frac{\partial C}{\partial x} = \frac{\partial}{\partial z}\left(K_H \frac{\partial C}{\partial z}\right)$$

$$K_H = \frac{k u_* z}{\phi_H\left(z/L\right)}$$

$\phi_H\left(z/L\right)$ based on heat transfer, Businger(1973)

Nieuwstadt, F.T.M., van Ulden, A.P., 1978. A numerical study of the vertical dispersion of passive contaminants from a continuous source in the atmospheric surface layer.



Comparison of numerical solution of diffusion equation with measurements from Prairie Grass

Nieuwstadt, F.T.M., van Ulden, A.P., 1978. A numerical study of the vertical dispersion of passive contaminants from a continuous source in the atmospheric surface layer.

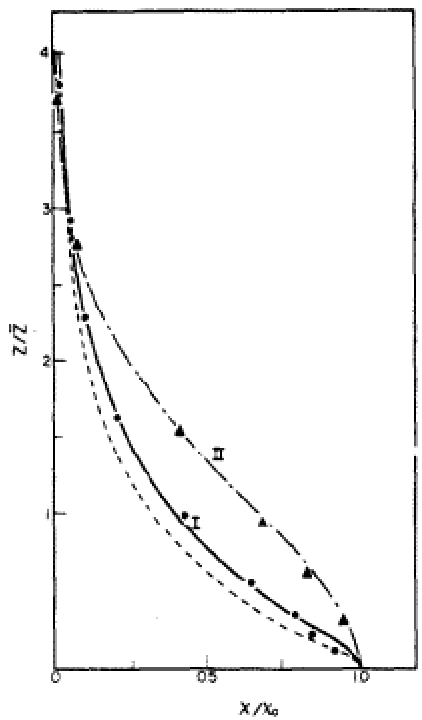
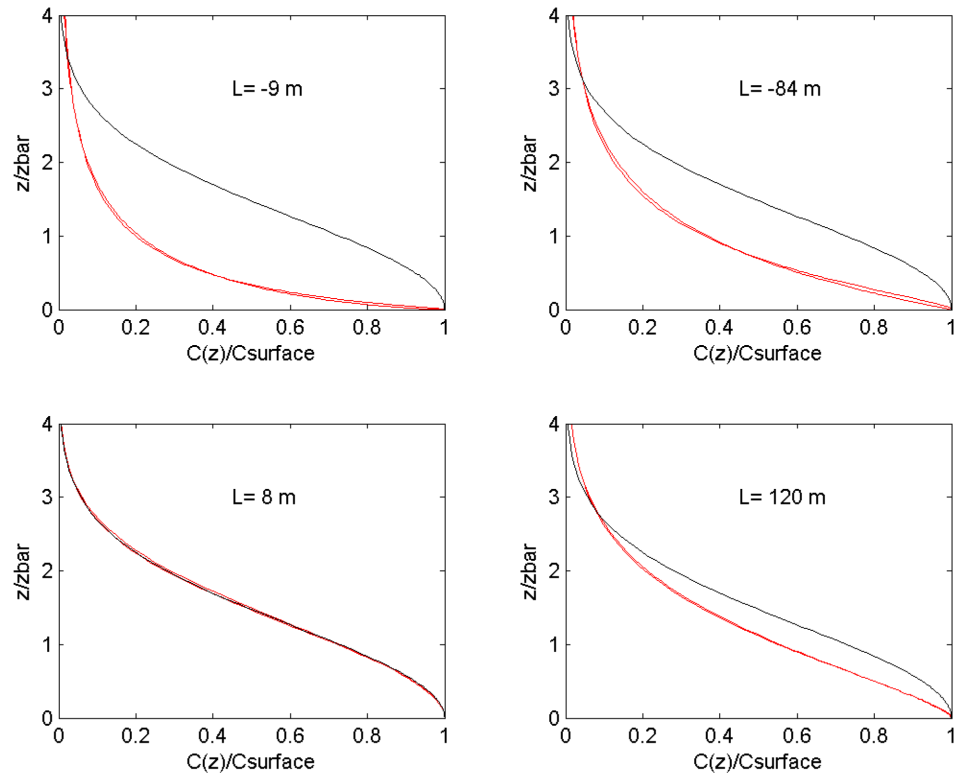


Fig. 3. Comparison between the shape of the measur and calculated concentration profile; I. Run 26: $L^{-1} = -0.031\ m^{-1}$; ● measurement, — computation with $K_z = 1.35 K_M$, --- computation with $K_z = K_H$; II. Run 59: $L^{-1} = 0.095\ m^{-1}$, ▲ measurement, -·- computation with $K_z = 1.35 K_M$ and $K_z = K_H$.



Red is numerical solution. Black line is Gaussian

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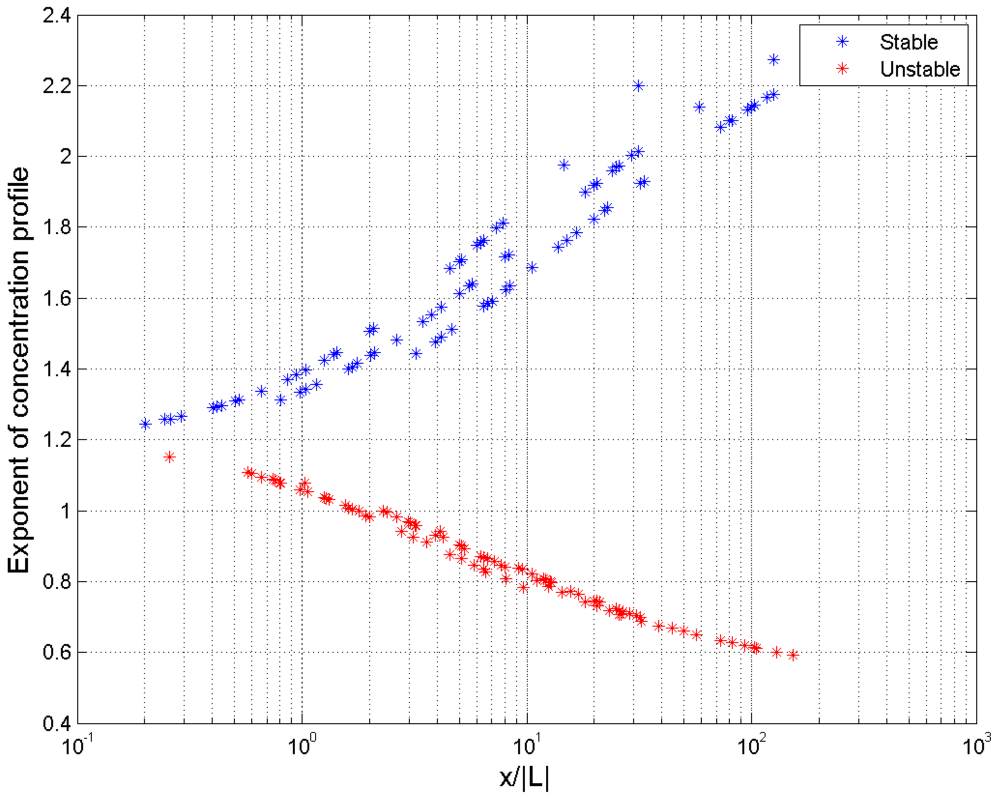

$$C = C_0 \exp\left(-\left(\frac{Bz}{\bar{z}}\right)^p\right)$$

Gryning, S. -E, van Ulden, P., Larsen, R.E., 1983. proposed an analytical expression for exponent

van Ulden, A.P., 1978. Simple estimates for vertical diffusion from sources near the ground. Atmos. Environ. 12, 2125–2129. doi:10.1016/0004-6981(78)90167-1

An elegant analytical approximation for the numerical solution

Assuming $U(z) = U_1 z^m \qquad K(z) = K_1 z^n$

$$\frac{C}{Q} = \frac{S}{\bar{U}\bar{z}} \exp\left(-\left(\frac{Bz}{\bar{z}} \right)^p \right)$$

where S and B are functions of $p = m - n + 2$

$$\bar{U} = \frac{\int_0^\infty U(z) C(z)\, dz}{\int_0^\infty C(z)\, dz} \qquad \bar{z} = \frac{\int_0^\infty z C(z)\, dz}{\int_0^\infty C(z)\, dz}$$

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Solution becomes useful with

$$\frac{d\bar{z}}{dx} = \frac{K\left(q\bar{z}\right)}{U\left(q\bar{z}\right)q\bar{z}} \qquad q = \left(B^{p}\, p\right)^{\frac{1}{1-p}}$$

van Ulden derives equations that provide estimates of

$\bar{z}$ as an implicit function of x / L and z_0

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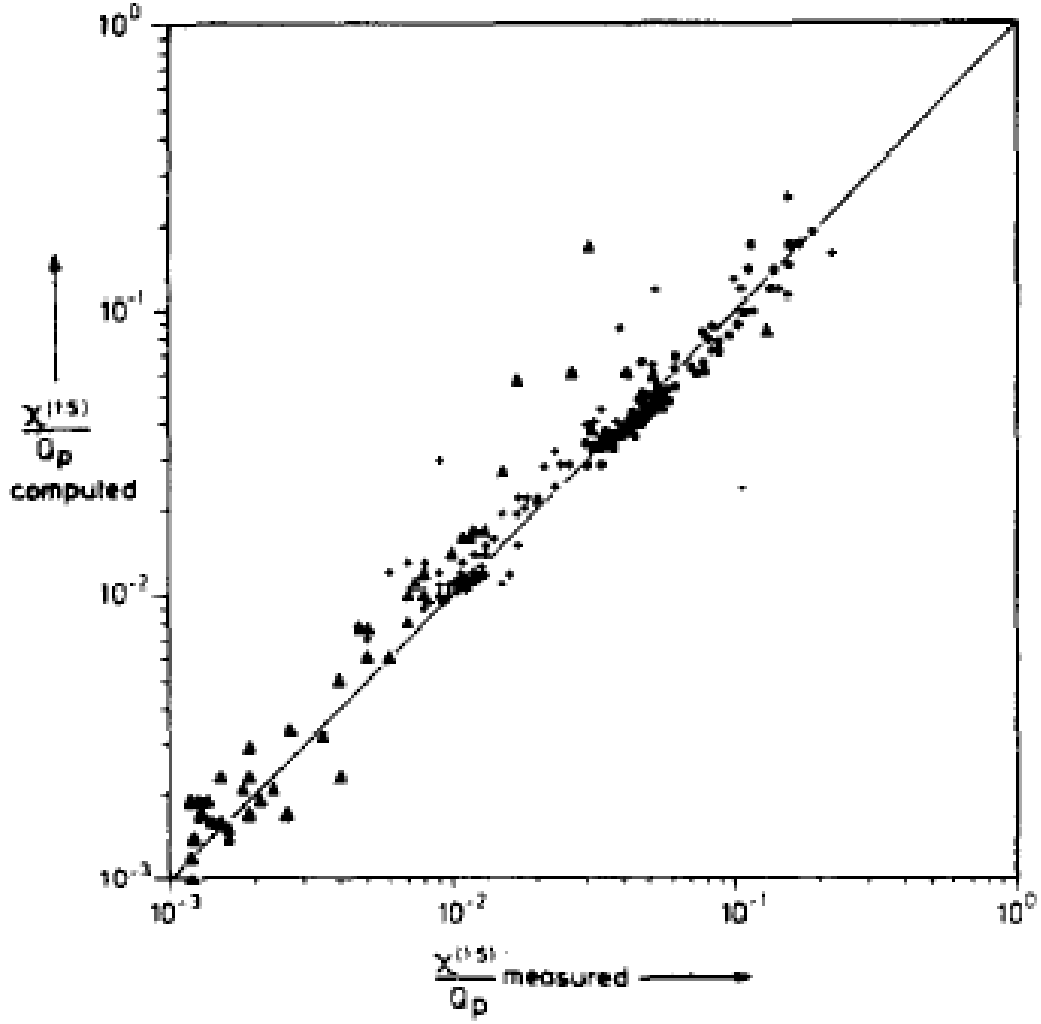


Some Consequences

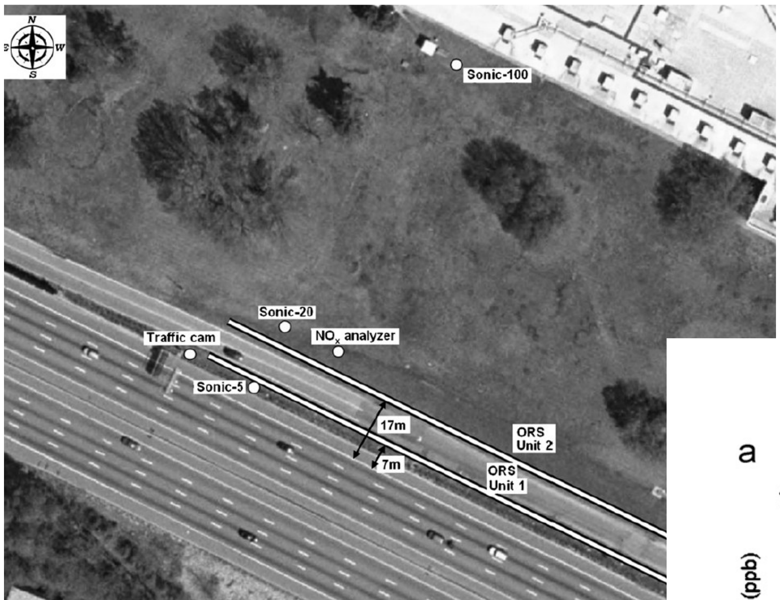
$$U\sigma_z \frac{d\sigma_z}{dx} \;\square\; \left(\quad _z \right)$$

For a neutral boundary layer $K = ku_* z$, $\quad u(\sigma_z) \sim u_* \ln\left(\frac{\sigma_z}{z_0}\right)$

$$\sigma_z \left[\frac{u}{u_*} - 1 \right] + z_0 \sim x \qquad\qquad \sigma_z u \sim u_* x$$

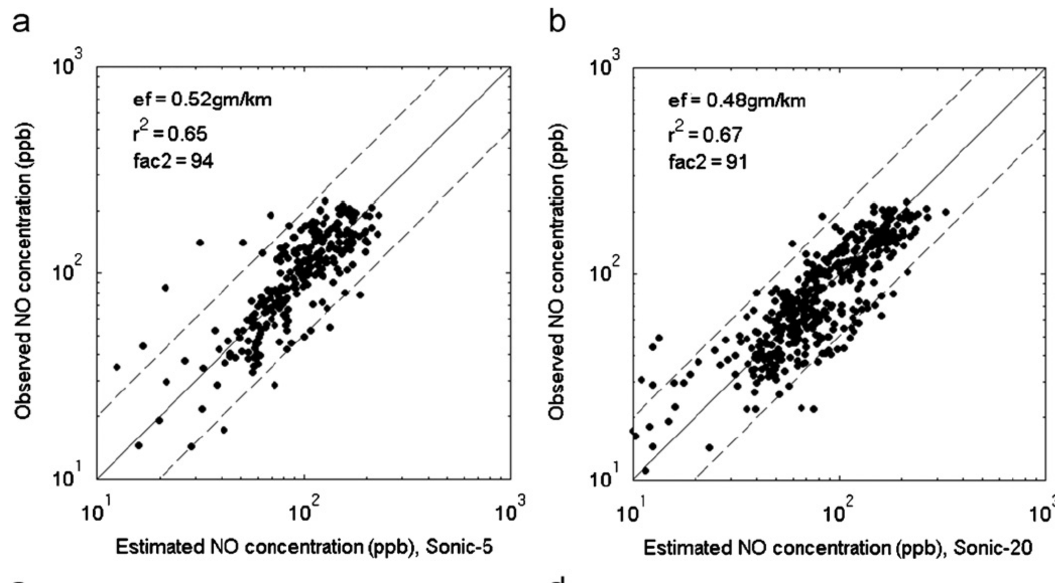
$$\overline{C}^y \sim \frac{Q}{u_* x}$$

Application



$$C(x) \sim \frac{T_r e_f}{u_* W} \ln\left(1 + \frac{a u_* W}{x + \frac{h_0 U}{a u_*}}\right)$$

A. Venkatram et al. / Atmospheric Environment 41 (2007) 9481–9497



Vertical Plume Spread (RLINE)

$$\sigma_z \square \frac{u\, x}{U(z)}$$ *Neutral*

$$\sigma_z \square \left(\frac{u}{U}\right)^2 \frac{x^2}{|L|}$$ *Unstable*

$$\sigma_z \square$$ *Stable*

$$\sigma_z = 0.57\frac{u_*}{U}x\left(1+3\frac{u_*}{U}\left(\frac{x}{L}\right)^{2/3}\right)^{-1}$$

$$\sigma_z = 0.57\frac{u_*}{U}x\left(1+1.5\left(\frac{u_*}{U}\frac{x}{|L|}\right)\right)$$

Horizontal Plume Spread

Eckman, 1994

$$\frac{d\sigma_y}{dx} = \frac{\sigma_v}{U(\sigma_z)}$$

Neutral Conditions

$$\frac{d\sigma_y}{dx} \approx \frac{\sigma}{u_*}\frac{u}{U(\sigma_z)} = \frac{\sigma_v}{u_*}\frac{d\sigma_z}{dx}$$

$$\sigma_y \approx \frac{\sigma}{u_*}$$

Horizontal Plume Spread

Eckman, 1994

$$\frac{d\sigma_y}{dx} \approx \frac{\sigma_v}{U}$$

$$\sigma_y \approx \frac{\sigma_v}{u_*} z \qquad \textit{Neutral}$$

$$\sigma_y \approx \frac{\sigma_v}{u_*} \left(z |L| \right)^{1/2} \qquad \textit{Unstable}$$

$$\sigma_y \approx \frac{\sigma_v}{u_*} \frac{\sigma_z^2}{L} \qquad \textit{Stable}$$

Formulation in RLINE

Stable Conditions

$$\sigma_z = 0.64 \frac{u_*}{U} x \left(1 + 3 \frac{u_*}{U} \left(\frac{x}{L} \right)^{2/3} \right)^{-1}$$

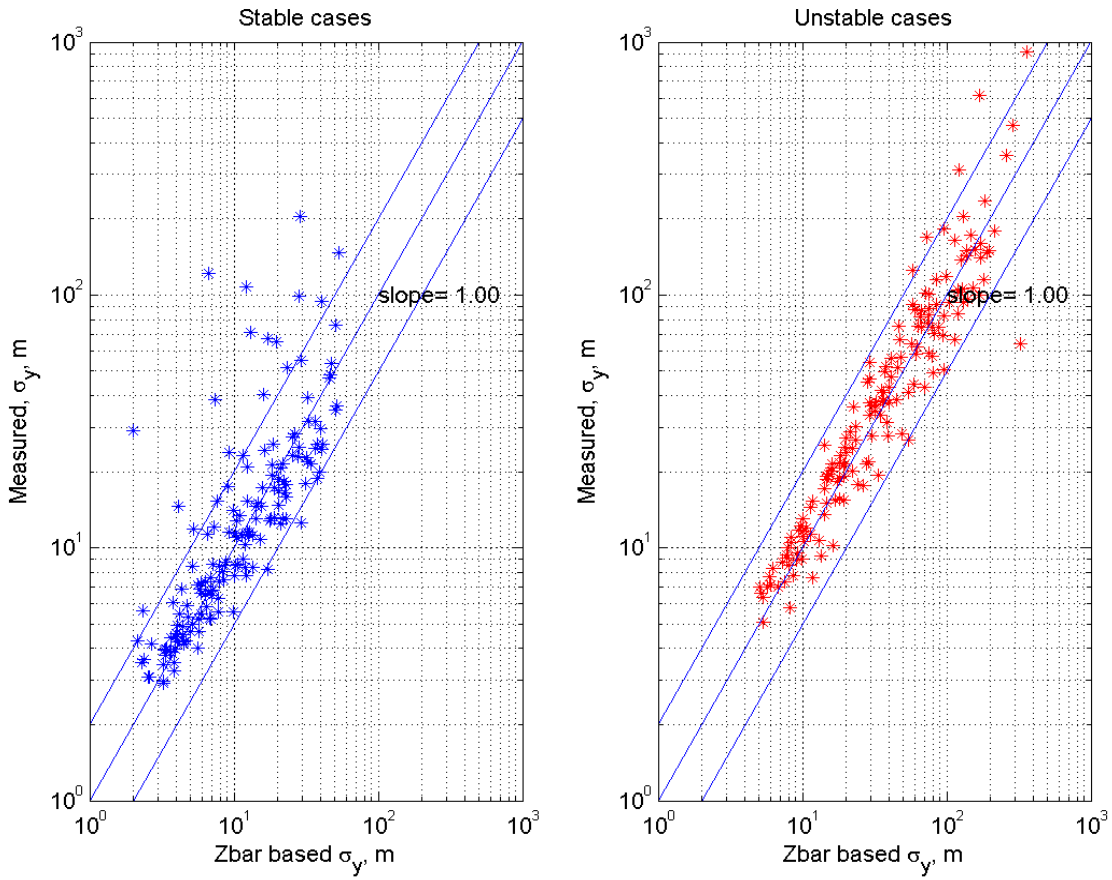
$$\sigma_y = 1.6 \frac{\sigma_v}{u_*} \sigma_z \left(1 + 1.5 \frac{\sigma_z}{L} \right)$$

Unstable Conditions

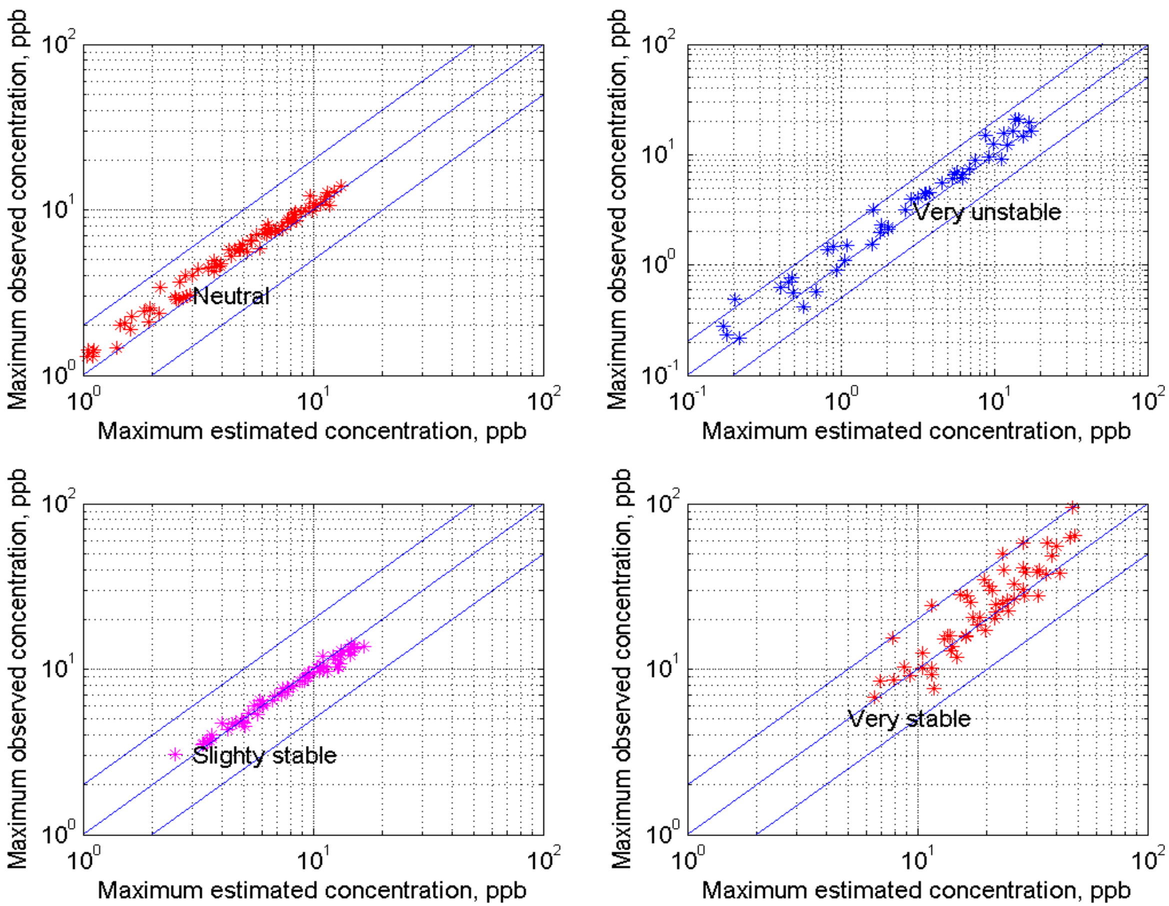
$$\sigma_z = 0.64 \frac{u_*}{U} x \left(1 + 1.5 \left(\frac{u_*}{U} \frac{x}{|L|} \right) \right)$$

$$\sigma_y = 1.6 \frac{\sigma_v}{u_*} \sigma_z \left(1 + 0.5 \frac{\sigma_z}{|L|} \right)^{-1/2}$$

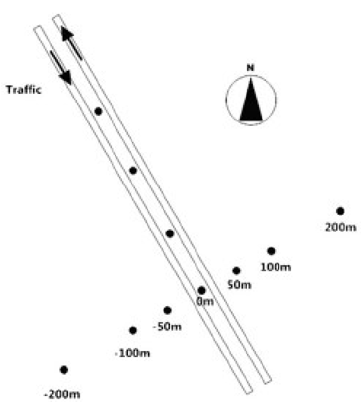
Horizontal Spread Prairie Grass Data

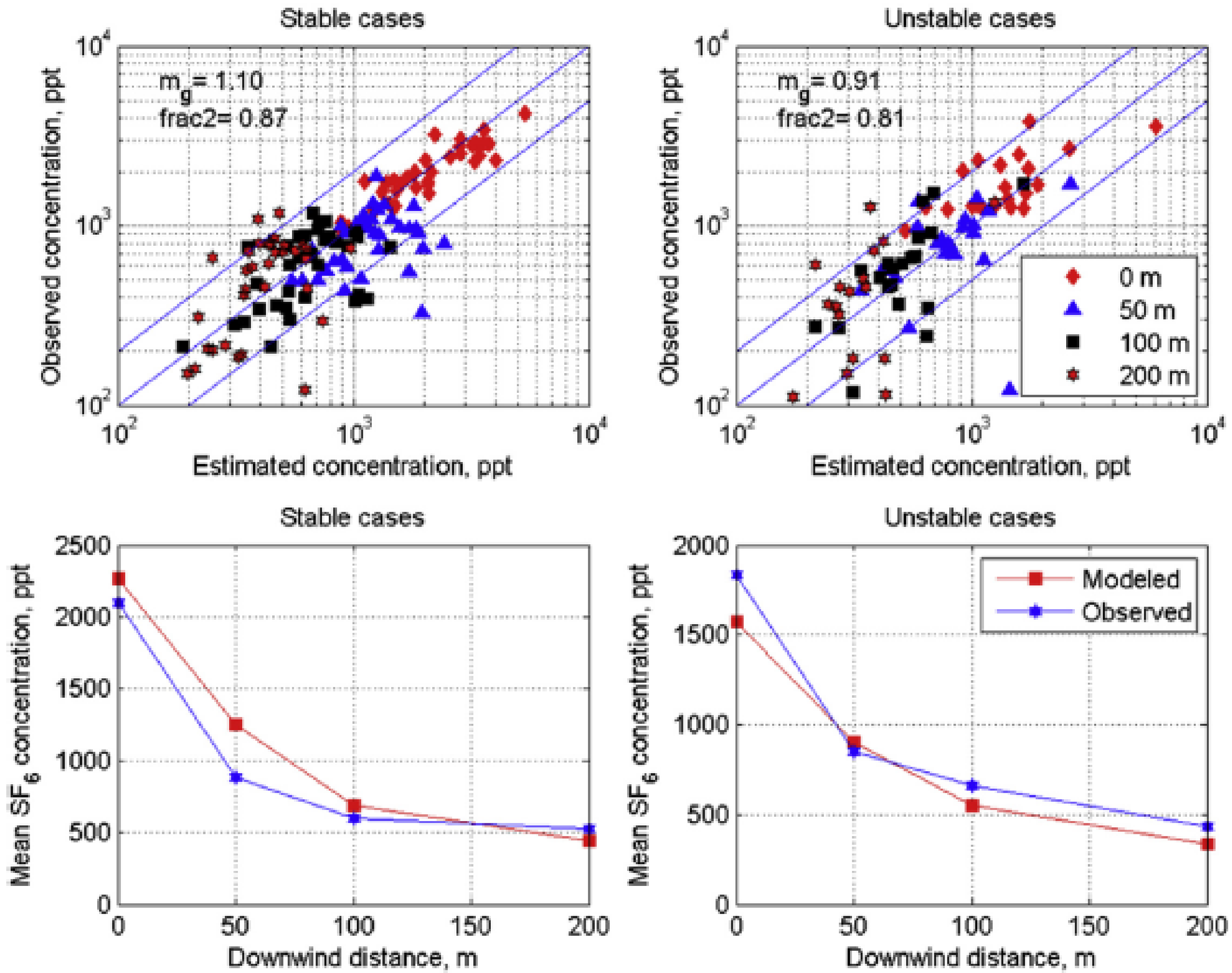


Concentrations
Idaho Falls Data



Application to Real World (Highway 99, Sacramento, 1981-1982)





Conclusions

- Paper by Nieuwstadt and van Ulden showed that the eddy diffusivity formulation provides “real world“ estimates
- van Ulden provided an analytical solution that forms the basis of plume spreads used in currently used models such as AERMOD.