"When you stir your rice pudding, Septimus, the spoonful of jam spreads itself round making red trails like the picture of a meteor in my astronomical atlas. But ***if you stir backwards, the jam will not come together again****. Indeed, the pudding does not notice and continues to turn pink just as before. Do you think this is odd?"*

Tom Stoppard: Arcadia

Time-reversibility in atmospheric dispersion

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HARMO17

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Backward trajectory calculation and chaos

- Backward calculation → e.g. source identification
  - backward trajectory computation
  - inverse modeling

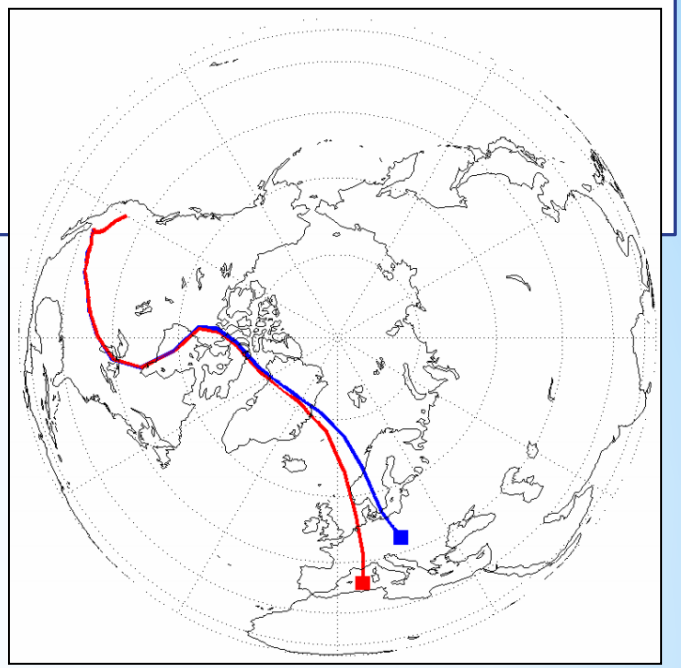
$$\frac{dx}{dt} = u_{air}(x, y, z, t)$$
$$\frac{dy}{dt} = v_{air}(x, y, z, t)$$
$$\frac{dz}{dt} = w_{air}(x, y, z, t) + w_{term}$$

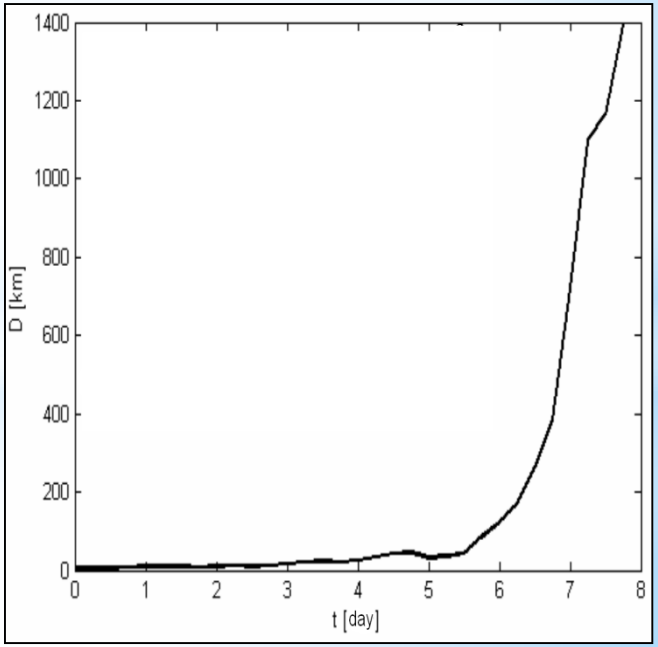
- Backward trajectory computation
  - how precisely a cloud of particles returns to its position when after monitoring over a few days, time is reversed and the backward simulation runs for the same time-period
  - only advection →
    - particle trajectories are based on the 3D velocity field
    - ***deterministic*** trajectories: 3 time-dependent differential equations
    - 3D passive advection has 3 degrees of freedom → ***chaotic behavior*** (sensitivity to the initial conditions, appearence of complex formations: folded, elongated filaments) [Ottino, 1989]

Backward trajectory calculation and chaos

- ***Chaotic behavior*** →
  - motion of a particle is unpredictable for long times (close particles diverge ***exponentially***)
  - sensitivity to slight deviations

- ***Aim:*** Investigating the accuracy of backward trajectory simulations (the degree of fulfillment of time-reversibility) in the view of ***chaotic advection***.
  - matching of the ***backward (BWC)*** and ***forward (FWC)*** clouds (considered here as the "measured" position of the pollutants)
  - statistical parameters
  - → accuracy of backward tracking ***decreases exponentially*** in time





Data and Methods

- **Model: *RePLaT*** (Real Particle Lagrangian Trajectory) model (Haszpra and Tél, 2013)
  - tracking particles with a realistic radius (r) and density.
  - ideal tracers and gaseous contaminants: $r = 0$ → in this work
  - only advection, turbulent diffusion is neglected in the simulations

- **Input data**
  - 3D reanalysis wind fields of the ERA-Interim database (ECMWF)
  - 32 pressure levels (1000-100 hPa), 1.5°×1.5°, 6 h of time resolution

- forward simulation for 9 days +
  backward simulations initiated at t_b = 9, 8, …, 1 days

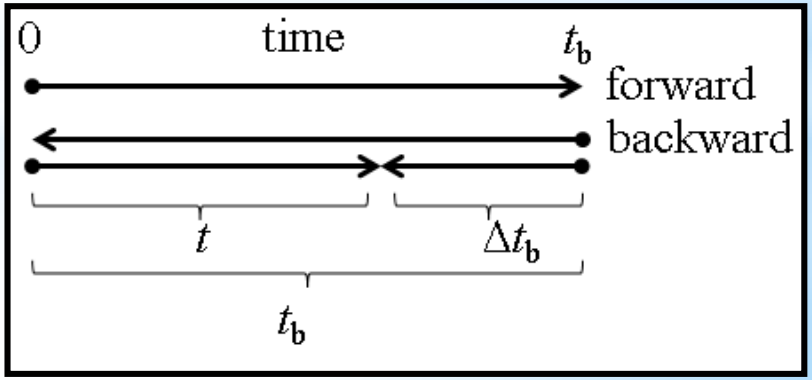
Data and Methods

- **Statistical parameters**
  BWCs and FWCs at the appropriate time instants are compared.
  - n/n_0: proportion of the particles that returned to the initial volume
  - ΔCM_h: horizontal deviation of the center of mass
  - $\Delta\sigma_h$: horizontal difference of the standard deviation
  - **FMS**: figure of merit in space (overlap)
  - **PCC**: Pearson's correlation coefficient

$$FMS = 100 \times \frac{A_{FWC} \cap A_{BWC}}{A_{FWC} \cup A_{BWC}}$$

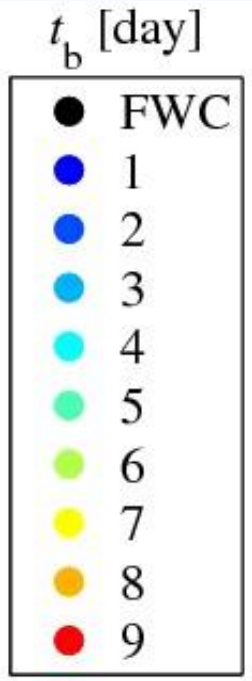
$$PCC = \frac{\sum_{i=1}^{K}(c_{FWC}(i)-\overline{c_{FWC}})(c_{BWC}(i)-\overline{c_{BWC}})}{\left(\sum_{i=1}^{K}(c_{FWC}(i)-\overline{c_{FWC}})^2 \sum_{i=1}^{K}(c_{BWC}(i)-\overline{c_{BWC}})^2\right)^{1/2}}$$

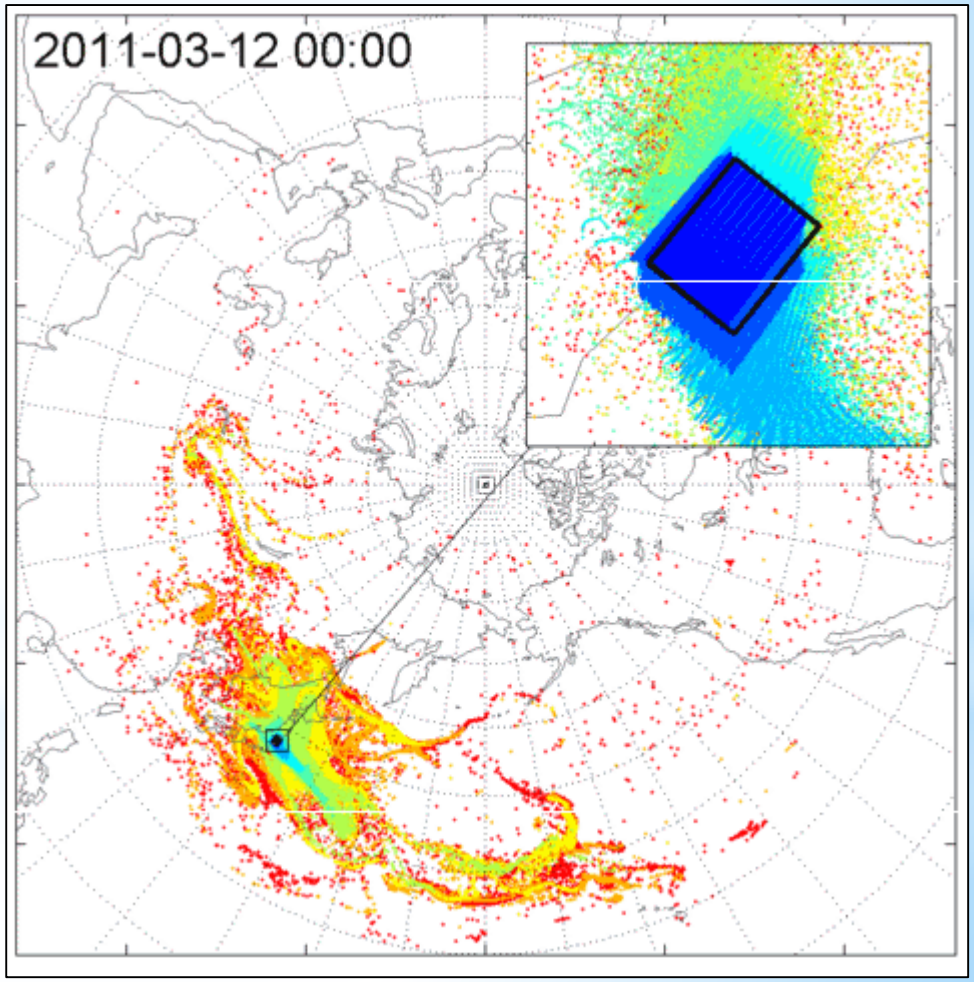
  - The deviations between the corresponding BWC and FWC are considered as a function of the ***time interval of the backward simulation*** $\Delta t_b = t_b - t$
    [t_b: initialization time of the BWC (t_b = 1, 2,…,9 days)
    t: time instant of the comparison (t = 0, 1…,8 days)]



Case study
Advection of the FWC and the BWCs

- FWC (9 days) + BWCs (t_b = 1, ...,9 days)
- FWC: on 12 March 2011 at 00 UTC
  - 1°×1°×100 hPa
  - λ_0 = 141°E, φ_0 = 37.5°N, p_0 = 500 hPa

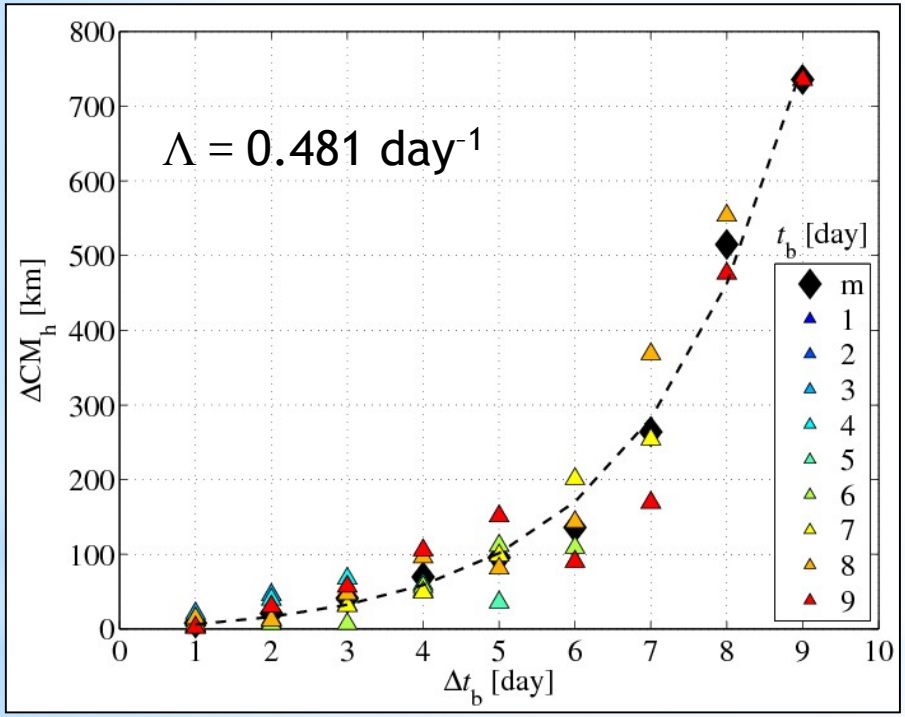




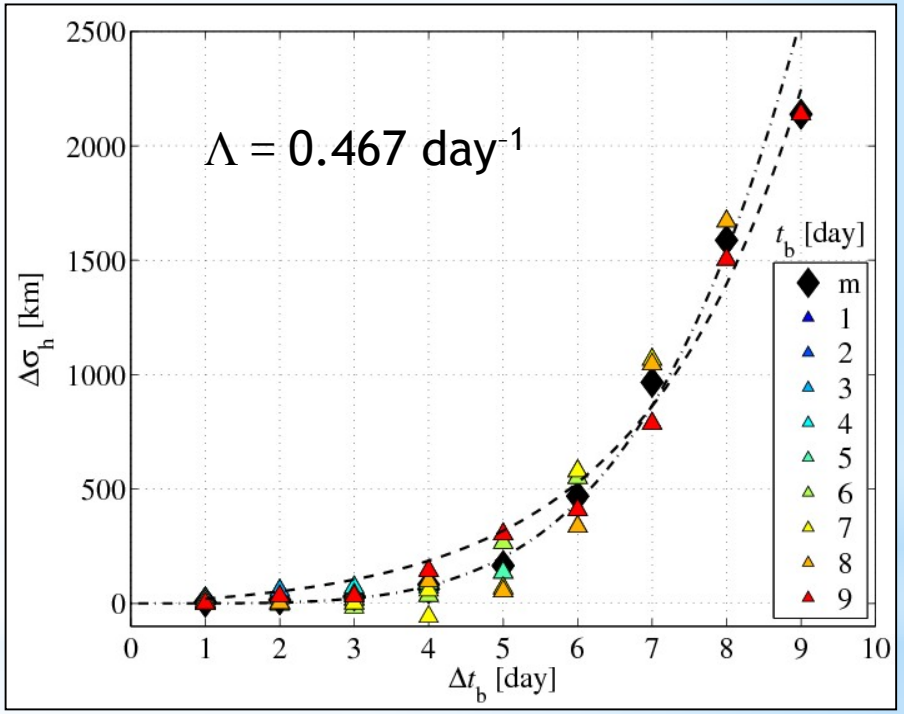
- Δt_b = 1–3 days: reasonable agreement
- Δt_b > 4-5 days: poor agreement, a clear breakdown of time-reversibility is found

Case study
Statistical parameters

Deviation of the ***center of mass*** of the FWC and BWC



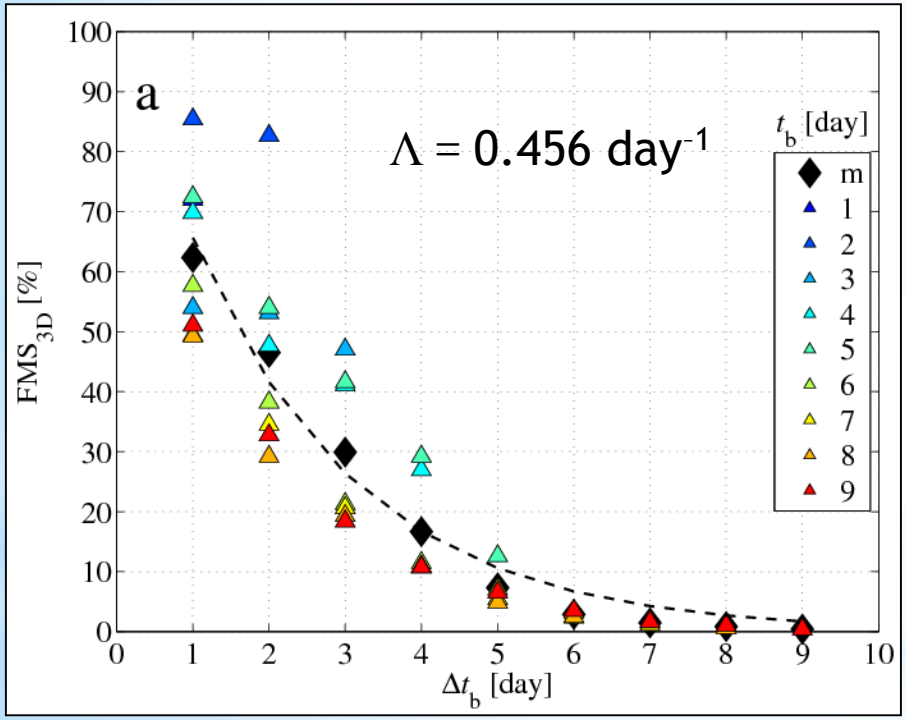
Deviation of the ***standard deviation*** of the FWC and BWC



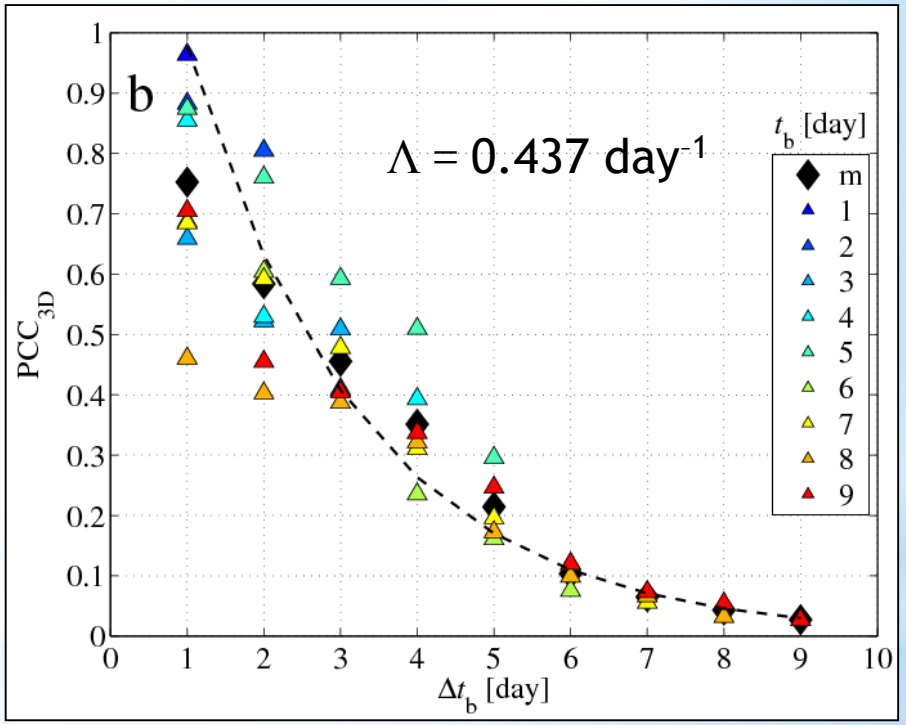
$\Delta\sigma_h > 0$ → BWCs extend to a larger area than the corresponding FWC (sign of the chaotic behavior of the advection)

Case study
Statistical parameters

Figure of merit in space (overlap)

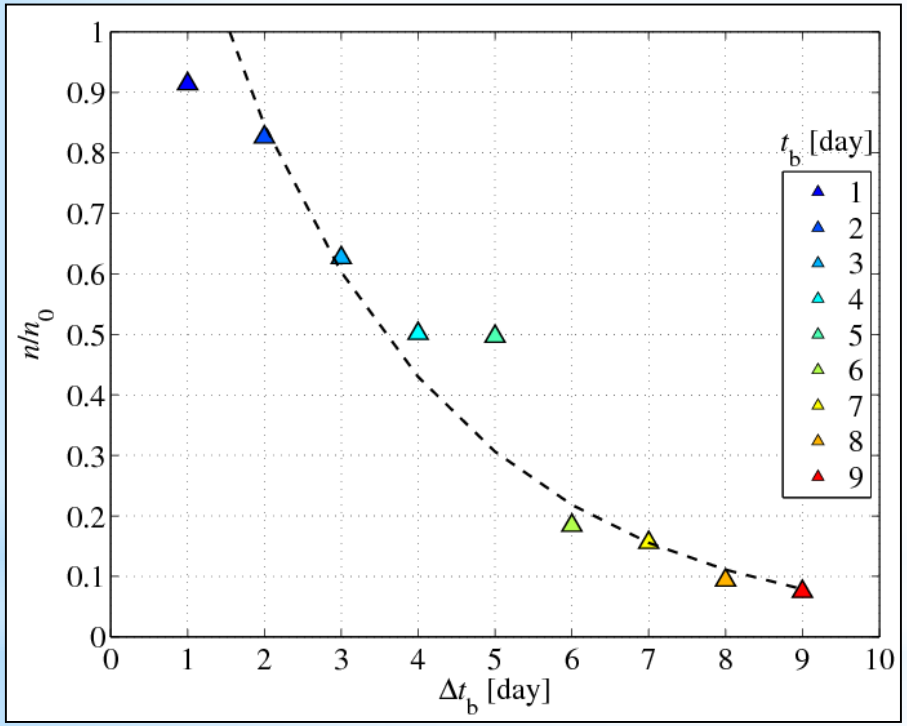


Pearson's correlation coefficient



Case study
Statistical parameters

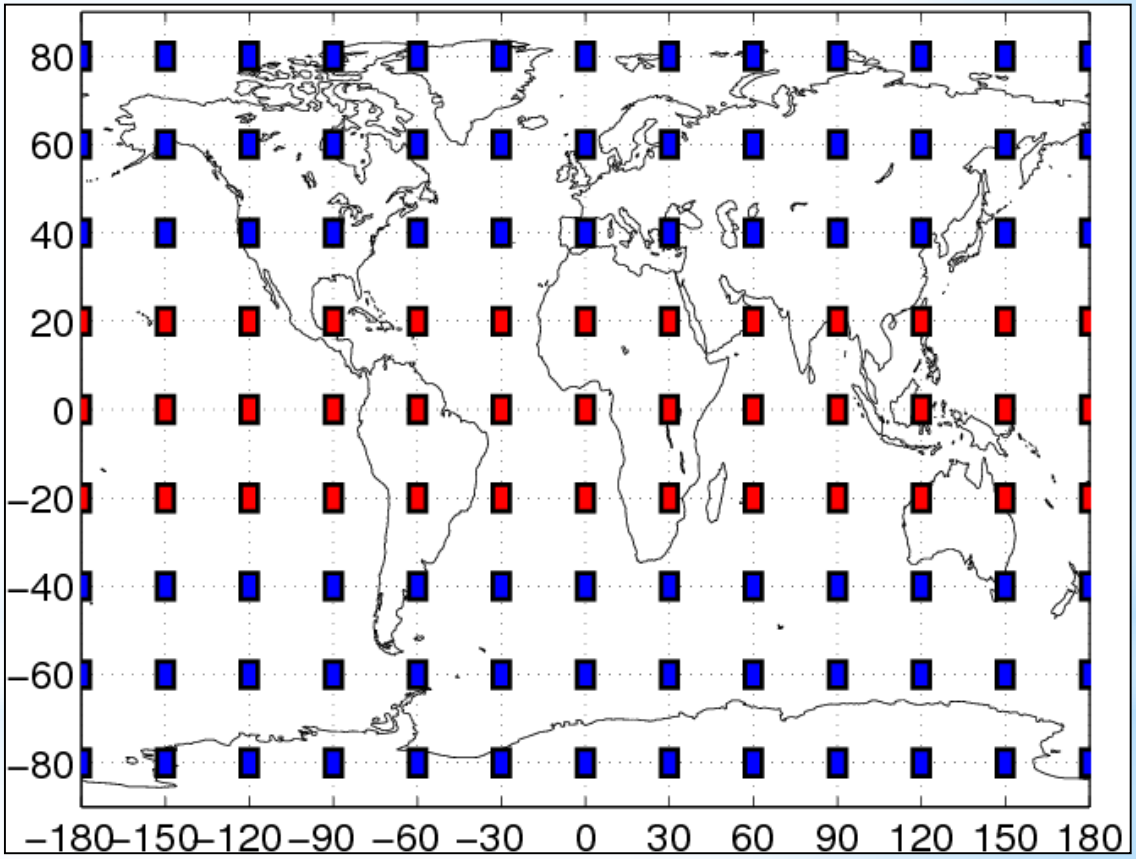
Proportion of the particles that return to the initial volume



- the exponents can be considered as specific ***Lyapunov exponents*** of the advection
- $\Lambda \approx$ atmos. Lyapunov exp. of adv. [e.g., Pierrehumbert and Yang, 1993; von Hardenberg et al., 2000]
- independent of methods and data: the deviation between the BWC and FWC becomes significant: ***integration time > Lyapunov time of advection*** $(1/\Lambda)$
- general property of ***chaotic processes*** and is in harmony with the observation that the Lyapunov time is the characteristic time interval beyond which predictions become unreliable

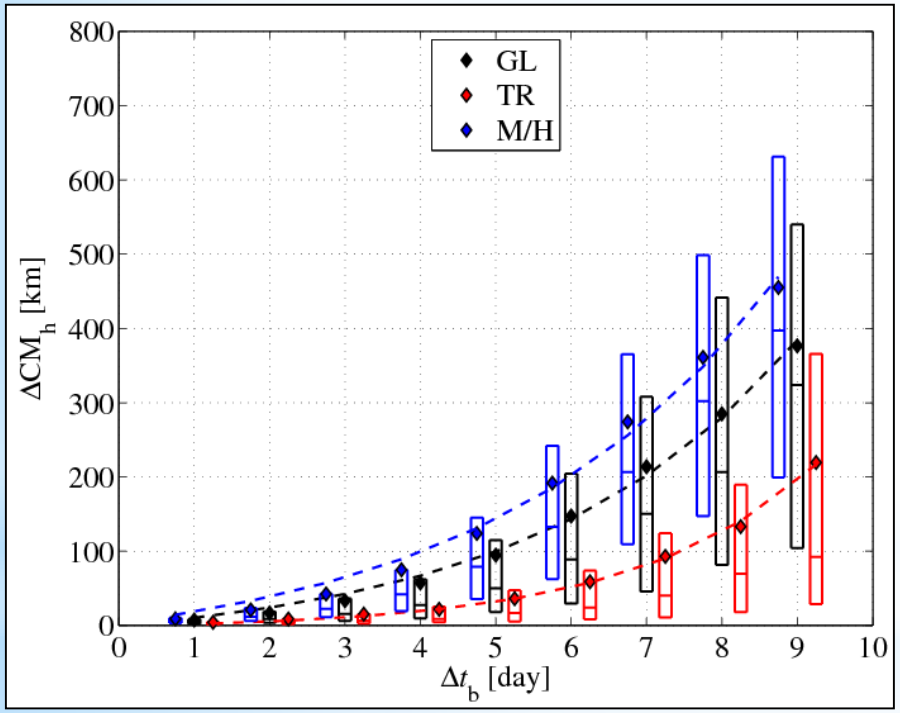
Global results

- FWC for 9 days + BWCs
- FWC:
  12 March 2011, 00 UTC
  1°×1°×100 hPa,
  λ_0, φ_0, p_0 = 500 hPa
- 12×9 pollutant clouds uniformly distributed
  - **TR:** tropical region (φ_0 = 0°, 20° N/S)
  - **M/H:** mid- and high latitudes (φ_0 = 40° N/S, 60° N/S, 80° N/S)
  - **GL:** global results

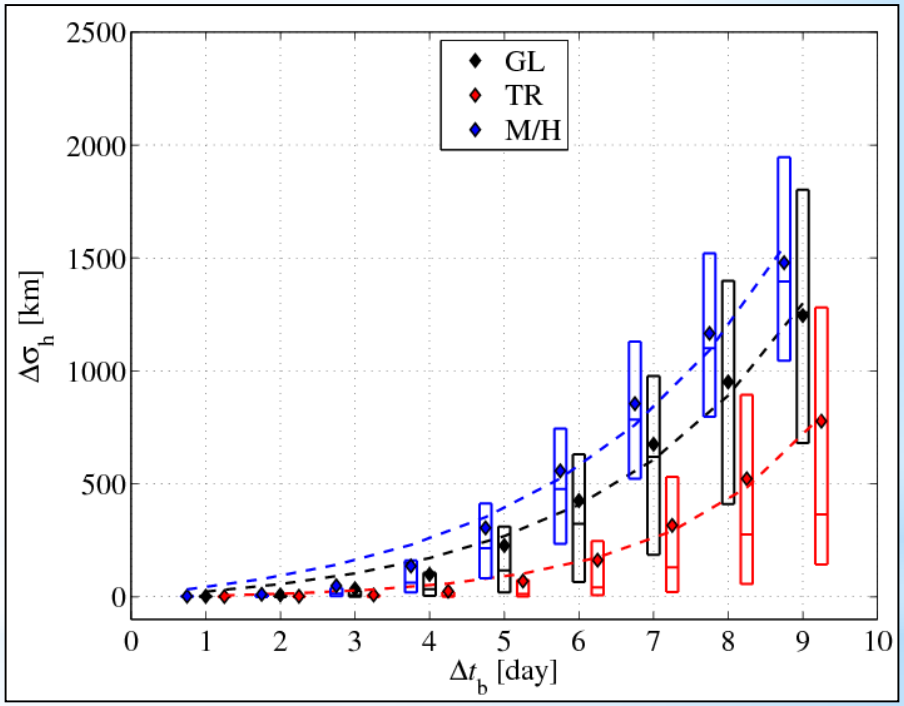


Global results
Statistical parameters

Deviation of the ***center of mass*** of the FWC and BWC



Deviation of the ***standard deviation*** of the FWC and BWC

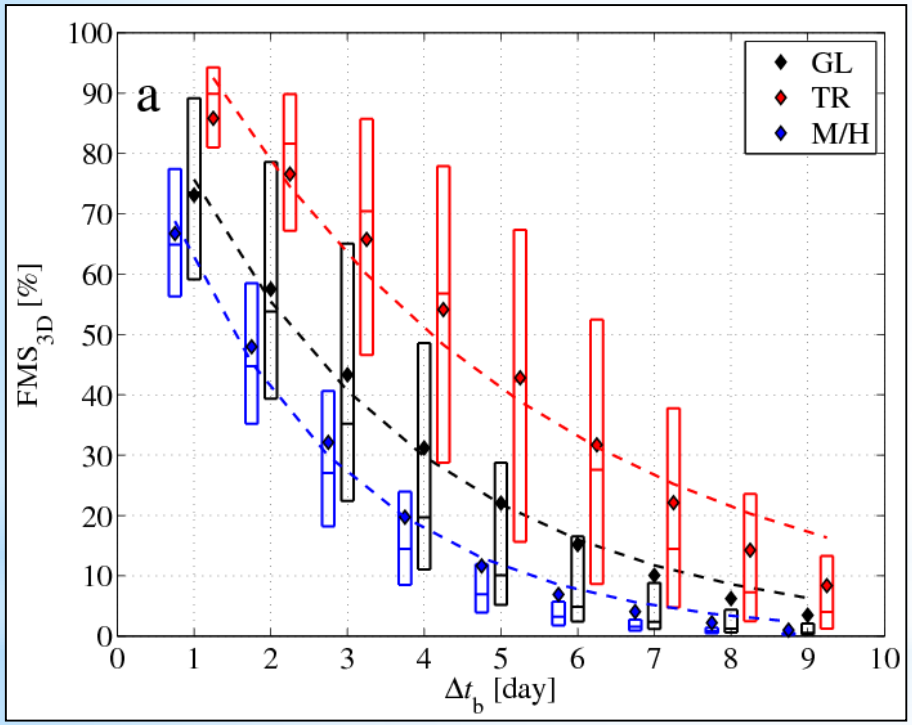


- ***TR:*** fewer cyclones → weaker shearing → weaker chaoticity
- pollutant clouds in ***TR*** can become mixed into ***M/H***
  pollutant clouds initiated in ***M/H*** might get into ***TR***

} properties can be mixed together

Global results
Statistical parameters

Figure of merit in space (overlap)



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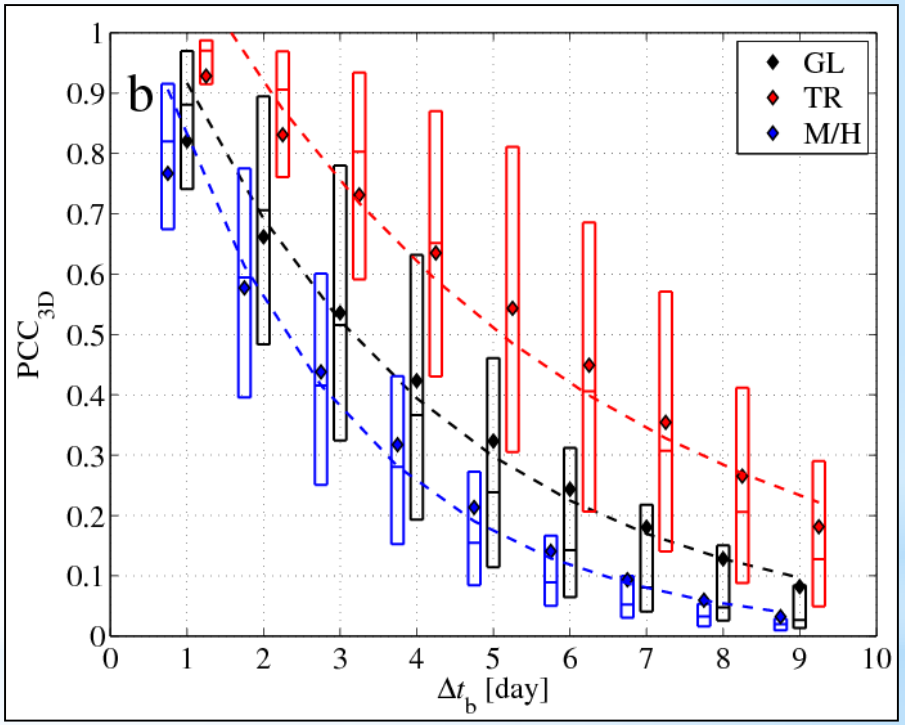
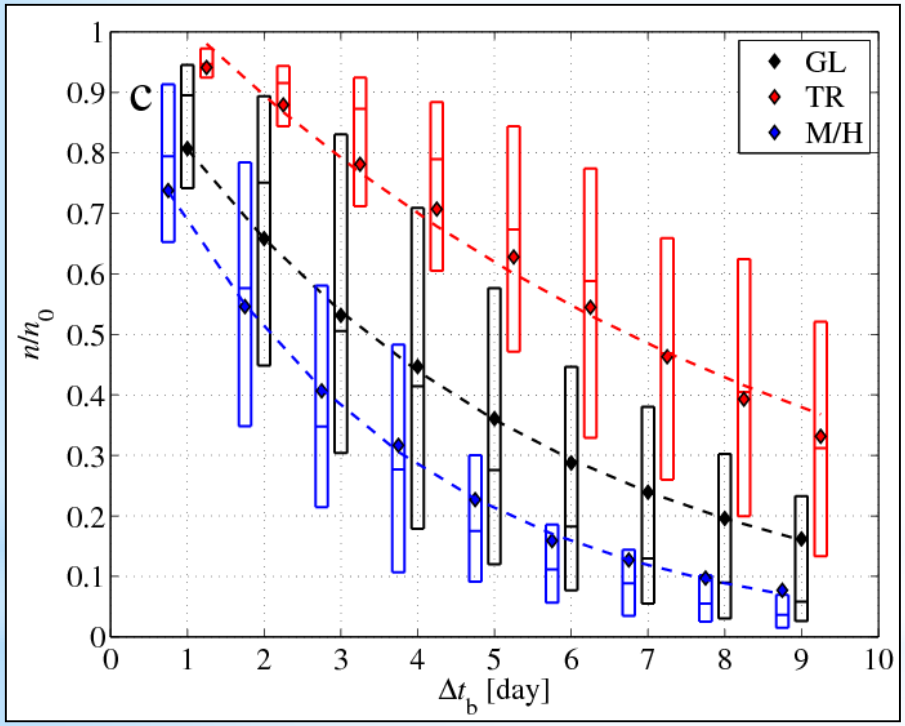


Global results
Statistical parameters

Proportion of the particles that return to the initial volume



- the time-dependence of the different statistical parameters can be described by similar exponents

Λ [day^{-1}]	GL	TR	M/H
ΔCM_h	0.290	0.267	0.422
$\Delta\sigma_h$	0.358	0.330	0.495
FMS	0.311	0.217	0.417
PCC	0.281	0.196	0.390

Summary

- investigating the accuracy of ***backward trajectory simulations***
- Δt_b = 1–3 days: reasonable agreement
  Δt_b > 4–5 days: poor agreement, a clear breakdown of time-reversibility

- The accuracy of backward simulations are found to ***decrease exponentially***.
  - **M/H**: faster decrease ← more intense cyclonic activity
  - **TR**: slower decrease
  - time interval over which the accuracy of time-reversibility is found to be reasonably good ~ predictability time of atmospheric advection ($1/\lambda$).

- applying more sophisticated num. schemes + met. data with finer resolution? → it is not expected to basically alter the dynamics of the time-dependence due to the unavoidable ***exponential degradation*** of the accuracy of dispersion simulations owing to the ***chaotic nature*** of atmospheric advection.

- [*Haszpra (2016) Atmosphere, 7(1), 11.*]

Thank you for your attention!