Uncertainty Estimation in the Retrieval of an Atmospheric Point Release

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Context

- Accidental/intentional release scenarios.
- Estimation of origin and strength of releases.
- Estimation of uncertainty in the source parameters.
  - ✓ Sampling approaches.
  - ✓ Hessian.
  - ✓ A posteriori standard error.
- Knowledge of a priori statistics related to the measurements and release.

Objective: To develop an uncertainty estimation methodology in the framework of renormalization inversion

Source-receptor relationship

$$\boldsymbol{\mu} = \mathbf{A}\mathbf{s} + \boldsymbol{\varepsilon}$$

- $\boldsymbol{\mu} = [\mu_1, \mu_2, \ldots, \mu_m]$ is the measured concentrations,
- $\mathbf{A} \in \mathbb{R}^{m \times N}$: sensitivity matrix, $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_N]$ where $\mathbf{a}_i \in \mathbb{R}^m$
- $\mathbf{s} \in \mathbb{R}^N$: discretized source vector.
- $\boldsymbol{\varepsilon}$: error.

Basic concepts of Renormalization

Unique feature is the diagonal weight matrix.

$$\boldsymbol{\mu} = \mathbf{A}_w \mathbf{W} \mathbf{s}$$

- New sensitivity matrix $\mathbf{A}_w = \mathbf{A}\mathbf{W}^{-1}$.
- Optimal weights characterized by $\mathbf{a}_w^T(\mathbf{x})\mathbf{H}_w^{-1}\mathbf{a}_w(\mathbf{x}) = 1$ where $\mathbf{H}_w = \mathbf{A}_w \mathbf{W} \mathbf{A}_w^T$.

$$\hat{\mathbf{s}} = \mathbf{A}_w^T \mathbf{H}_w^{-1} \boldsymbol{\mu}$$

Features of renormalization inversion

- $\max(\hat{\mathbf{s}})$ coincides with the location of the point source $(\mathbf{x}_e)$.
- Source strength is determined as $\hat{\mathbf{s}}(\mathbf{x}_e)/w(\mathbf{x}_e)$.
- $\mathbf{H}_w = \mathbf{A}_w \mathbf{W} \mathbf{A}_w^T$: describe dispersion in sensitivity vectors.

$$E\left[\boldsymbol{\mu}\boldsymbol{\mu}^T\right] = \sigma \mathbf{H}_w$$

$$\text{Var}[\hat{\mathbf{s}}] < \sigma$$

Estimation of confidence bounds

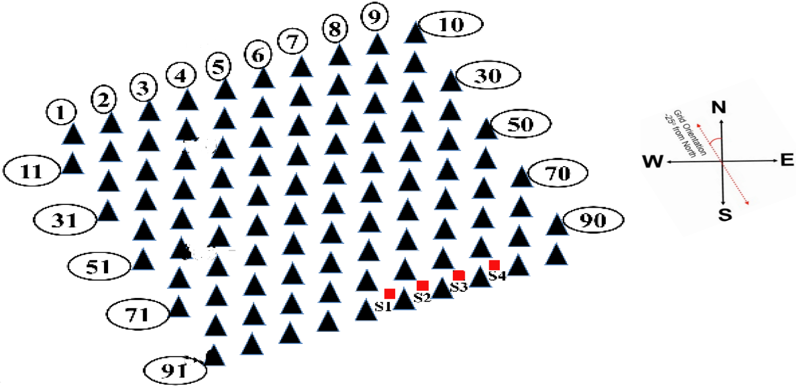
A posteriori standard error is derived as,

$$\hat{\sigma} = \frac{\boldsymbol{\varepsilon}^T \mathbf{H}_w^{-1} \boldsymbol{\varepsilon}}{m-1}$$

- $\boldsymbol{\varepsilon}$: Measurement - prediction.
- $P\left(-t_{m-1,\alpha/2} \leq \frac{s(\mathbf{x}_e)-s(\hat{\mathbf{x}}_e)}{\hat{\sigma}} \leq t_{m-1,\alpha/2}\right) = 1-\alpha$
- $P\left(-t_{m-1,\alpha/2} \leq \frac{q-\hat{q}}{\hat{\sigma}/w(\mathbf{x}_e)} \leq t_{m-1,\alpha/2}\right) = 1-\alpha$

Fusion Field Trials

A short scale point source dispersion experiment with tracer Propylene released 10 min (Storwald, 2007).



Fusion Field Trials

- 100 samplers arranged in 475 m$\times$ 450 m area.
- Distance between two samplers is approx. 50m.
- Source and samplers height were 2 m.
- 11 trials of single continuous releases are considered.
- Wind is predominant from south-east to north-west direction.
- Average wind speed $U > 2$ ms^{-1}.
- First 4 min of concentration data ignored to establish steady state.
- Average concentration measurements are considered.

Numerical computation

- Computational domain 1200 m$\times$ 1200 m is chosen and discretized into 399$\times$399 cells.
- Computation of sensitivity from adjoint of dispersion model.
- Estimation of point source parameters with Renormalization technique.
- Estimation of confidence bounds for release location and strength.
- Comparison with residual bootstrap method.

Resolution features of retrieved source

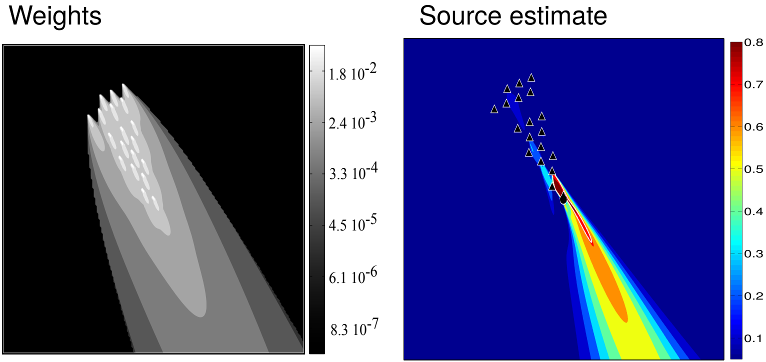


Figure : Trial 7.

Confidence region for source location and Comparison with bootstrap

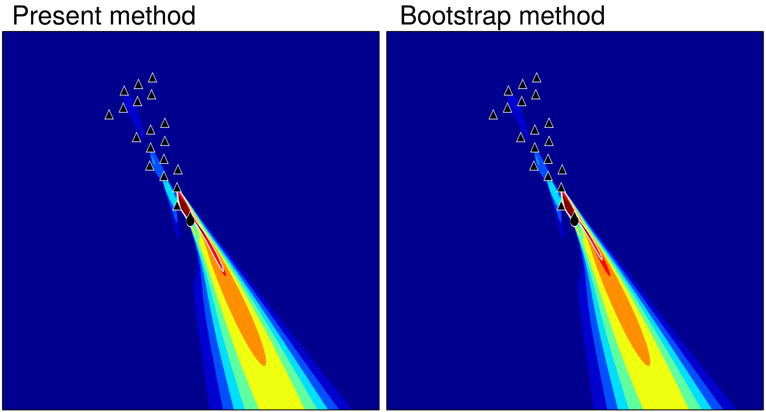


Figure : Trial 7.

Confidence region for source location and Comparison with bootstrap

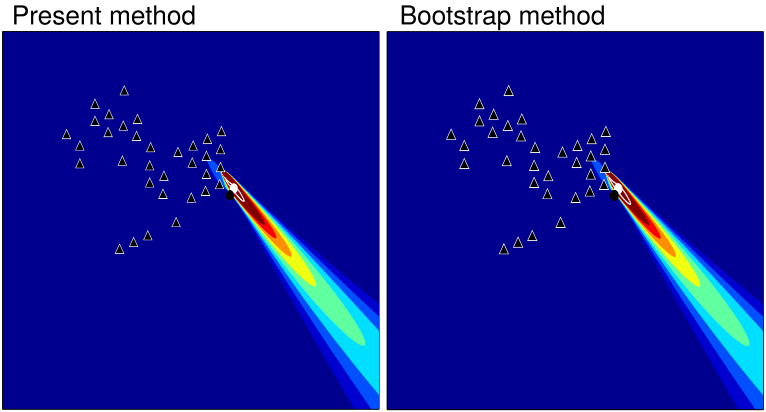


Figure : Trial 15.

Confidence interval for location error and Comparison with bootstrap

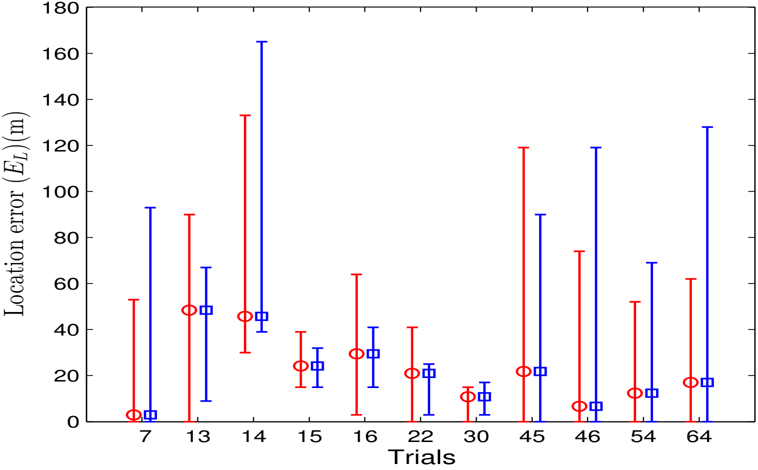


Figure : Red and blue colors denote the present method and bootstrap respectively.

Confidence interval for factor of retrieved strength and Comparison with bootstrap

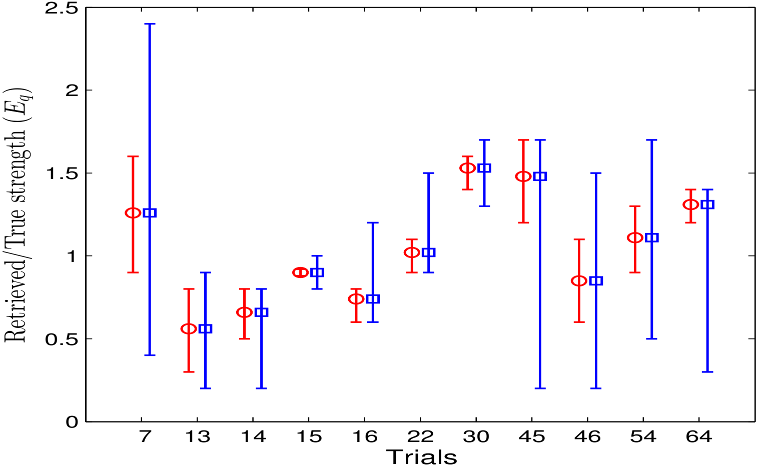


Figure : Red and blue colors denote the present method and bootstrap respectively.

Conclusion

- Estimates are proposed for uncertainty charcaterization.
- Evaluation is shown in Fusion Field Trials.
- Uncertainty in reconstruction is subjected to the number of measurements and design of monitoring network.
- A large uncertainty may occur besides the accurate retrieval.
- Present method is simpler and computationally efficient than Hessian and sampling approaches.
- The results are comparable to bootstrap sampling method.

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Thanks
for
kind attention