THE INERTIAL SUBRANGE KOLMOGOROV CONSTANT CO IN STOCHASTIC DISPERSION MODELS: ASSESSMENT BASED ON WIND TUNNEL EXPERIMENTAL DATA

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INTRODUCTION

Some of the most common types of dispersion models are the Lagrangian Particle models, which are based on the assumption that the evolution of a tracer particle's state (velocity-position) is a Markovian process. The particle's displacement is expressed as $dx_i = u_i(t)dt$ where ui is the Lagrangian velocity, xi is the particle's position coordinate in the ith direction, t is time and dt is the time increment. The velocity increment is calculated statistically from the Langevin equation (Thomson, 1987):

$$du_i = a_i(\mathrm{x},\mathrm{u},t)\,dt + b_{ij}(\mathrm{x},\mathrm{u},t)\,dW_j = a_i(\mathrm{x},\mathrm{u},t)\,dt + d_{ij}\sqrt{C_o e}\;dW_j \quad (1)$$

where ai is the drift term in the direction i, dij is Kronecker's delta and bijdWj is a random forcing caused by the fluctuating pressure gradients and molecular diffusion with dWj being the one-dimensional increments of a Wiener process – a Gaussian random forcing with zero mean and variance dt. In order for the Langevin equation to be consistent with the Kolmogorov's theory of local isotropy at high Reynolds numbers in the inertial subrange (Monin and Yaglom, 1975, pp. 358-359), the term bij is expressed in equation (1) as function of the mean dissipation rate of turbulent kinetic energy e, and the constant Co (Thomson, 1987; Luhar and Britter, 1989). The constant Co is called "inertial subrange Kolmogorov constant" or "universal constant for the Lagrangian structure function". According to Kolmogorov's hypothesis (Du, 1997), "this value is supposed to be universal, i.e. it should take the same value for any turbulent flow, provided that the Reynolds number is sufficient high (so as to ensure an inertial subrange is present)". However, contrary to the term "universal constant" there is a considerable uncertainty about the value of Co, which presents a matter of discussion. Over the last thirty years different investigators indicate different values of Co ranging mainly between 2 and 6 based on theoretical assumptions, on Lagrangian and Eulerian measurements, on observed dispersion of fluid particles and from numerical experiments (e.g., Du, 1997; Lien and D'Asaro, 2002; Rizza et al., 2006). Heinz (2002) in order to explain the reasons for the observed variations considers the velocity fluctuations as the sum of contributions due to anisotropy, acceleration fluctuations and stochastic forcing that are controlled by the Kolmogorov constant. He argues that the effects of anisotropy and acceleration fluctuations are responsible for the significant variations of Co and claims that Co is near 2 for flows, where anisotropy and acceleration fluctuations contribute to the energy budget (for the real non isotropic, non-homogeneous atmospheric turbulent flows) and near 6 if such contributions disappear.

In this work, to evaluate the most appropriate values of C_o for use in atmospheric dispersion, the 3-dimensional Lagrangian particle dispersion model DIPCOT (*Davakis et al.* 2005, *Davakis et al.* 2007; *Andronopoulos et al.*, 2005) was applied to simulate a wind tunnel experiment, using the Langevin equation (1) with values of C_o in the range between 1 and 6.

EXPERIMENTAL APPARATUS AND MODEL DESCRIPTION

A cylinder was placed parallel to a flat plate and normal to the flow (Figure 1). The cylinder was positioned above the boundary layer, so that the lower part of the wake was interacting with the boundary layer, producing a complex quasi two-dimensional flow at the upper part of the boundary layer and above it. Near the plate a fully non-homogeneous turbulent flow is developed. Heat was supplied to the boundary layer flow by means of an electrically heated wire. Since the Prandtl and Schmidt numbers for gases are both close to unity and the heat dispersion exhibits the same features as the mass dispersion, this study is considered equivalent to atmospheric dispersion over complex terrain. Hot-wire anemometry was used for the measurements with a triple-wire probe. During the experiment, the triple-wire probe recorded simultaneously the streamwise and the normal to the wall velocity components and the temperature. The probe was traversed normal to the plate, taking measurements at 41 observation heights, at five measuring downwind distances *x/D* (*D* is the cylinder diameter) in the streamwise direction. A schematic representation of the flow field is given in Figure 2.

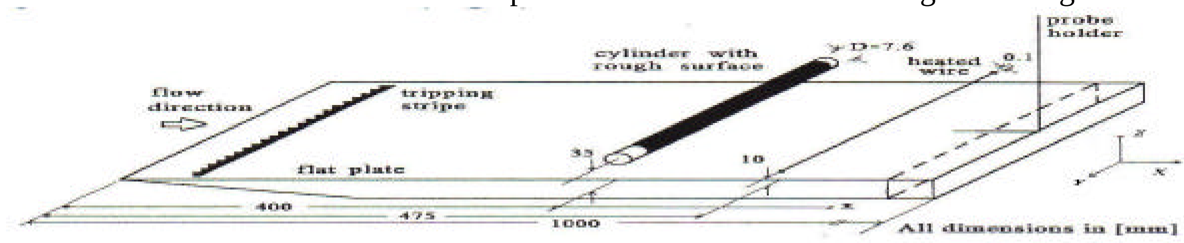

Fig 1; The experimental setup.

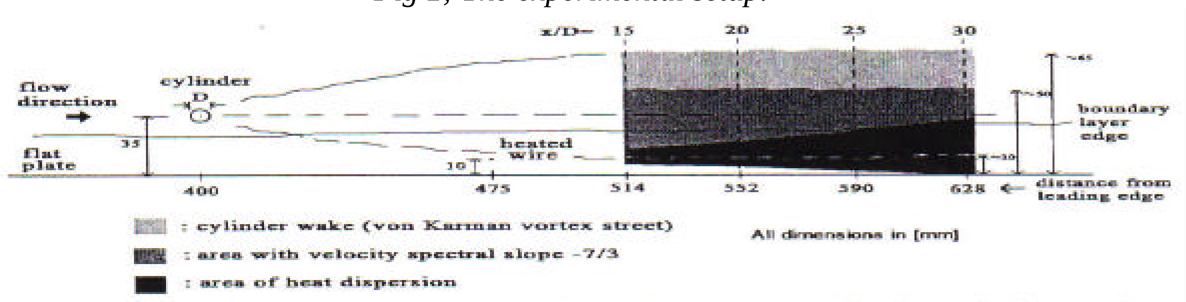

Fig. 2; Schematic representation of the experimental flow created in the wind tunnel.

EVALUATION PROCEDURE FOR “BEST PERFORMING” CO VALUES

To evaluate the most appropriate values for C_o, the model-calculated and measured mean temperature rise ΔT at each observation point were inter-compared. Well-known statistical tools (e.g., *Mosca et al.*, 1998) were used to quantify the level of agreement, such as: the Fractional Bias (*FB*) and the Geometric Mean bias (*MG*) with their 95% confidence limits, the Normalized Mean Square Error (*NMSE*), the Geometric Variance (*VG*), and the FACTor of 2 (*FACT2*). The cases where model results agreed best with the observations indicated the “best performing” C_o values.

The main study to evaluate the effects of the C_o values on the model results was carried out using the formulation of the drift term *a* in Langevin equation (1) proposed by *Franzese et al.* (1999). In addition, and to examine whether and to what extend the formulation of the drift term, *a*, can affect the choice of the “best performing” C_o values, the study was repeated (with less detail) using the drift terms proposed by *Luhar and Britter* (1989) and by *Weil* (1990).

RESULTS

In Table 1 the *FACT2* values obtained for the different values of C_o and drift term models are presented. The best model performance is indicated by the higher *FACT2* values which occur when C_o is between 2 and 4.

Table 1. FACT2 values (%) for different C_o values for the three formulations of the drift term in Langevin equation.

C_o → Drift term model	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0	6.0
Franzese et al.	56	77	84	85	84	84	82	74	66
Luhar and Britter	38		66		67		67	63	
Weil	57		71		70		67	67	

Figures 3a and 4a lead to the same conclusions: the model presents the smaller deviations from the measurements (i.e., the lower *NMSE* and *VG* values) for C_o between 2 and 4. Moreover the *FB* and *NMSE* values are closer to zero and unity and have the smaller confidence interval. The use of different formulations for the drift term does not change the above outcomes. In the other two cases that were examined (*Luhar and Britter*, *Weil*), the model performs better when C_o is between 2 and 4. However, looking at the statistical indices in table 1 and figure 3 and 4 (cases b and c), it can be seen that the value of C_o equal to 5 can also be included in the accepted range.

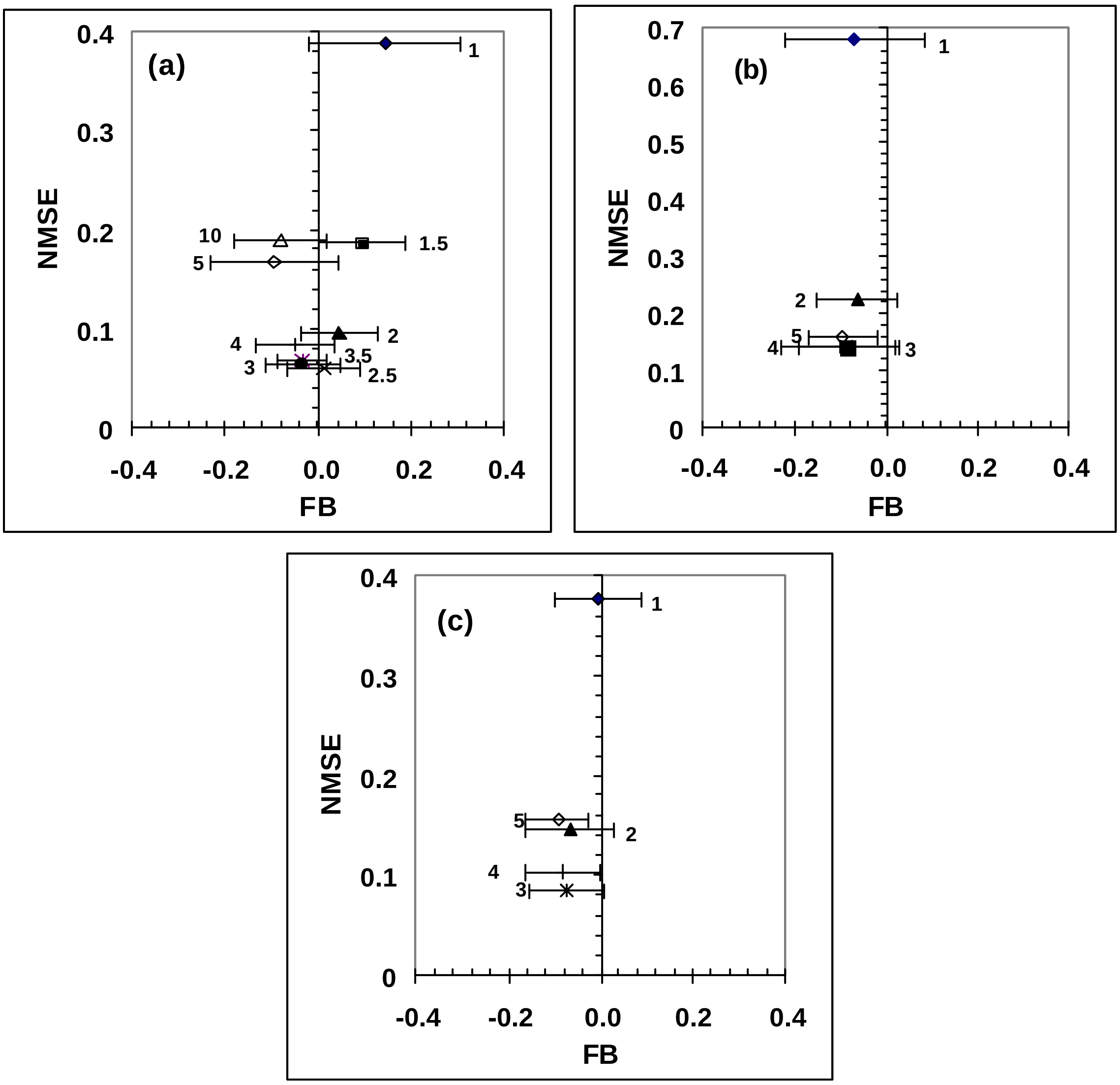

Fig. 3; NMSE vs FB (with their 95% confidence limits) values of the model performance for different C_o, for the three used formulations of the drift term in Langevin equation: (a) Franzese et al., (b) Luhar and Britter (1989) and (c) Weil (1990).

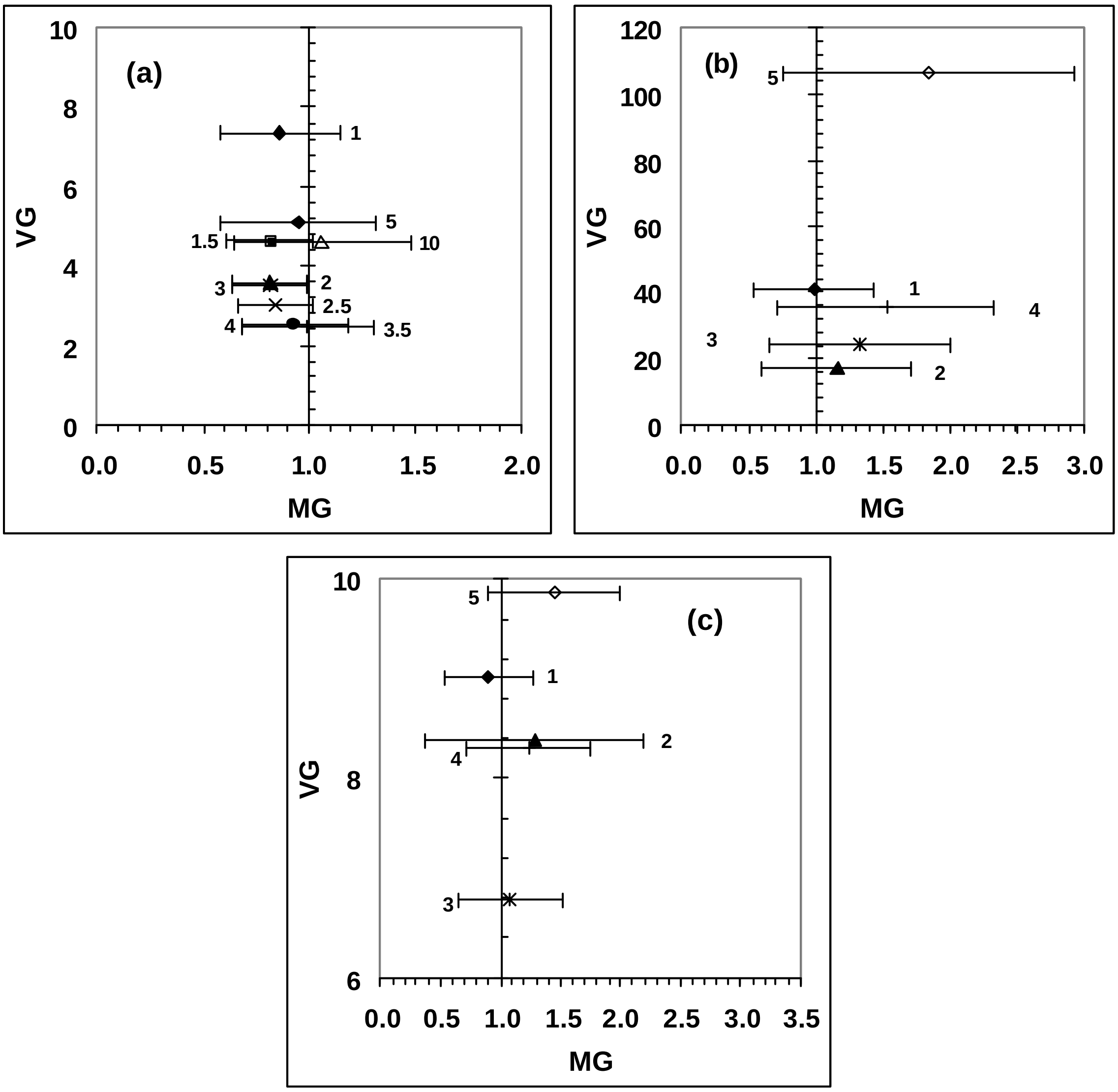

Fig. 4; VG vs MG (with their 95% confidence limits) values of the model performance for different C_o, for the three formulations of the drift term in Langevin equation: (a) Franzese et al., (b) Luhar and Britter (1989) and (c) Weil (1990).

However there isn't a single C_o value that can be pointed out, giving the best fitting between experimental and theoretical mean concentrations (temperatures). For the formulation of *Franzese et al.* (1999), *FACT2, FB* and *NMSE* point at C_o=2.5, while *MG* and VG suggest that the model behaves better when C_o=3.5. This could be explained because *FB* and *NMSE* give more weight to the higher values while *MG* and *VG* to lower and FACT2 expresses the overall performance. Moreover, for the same statistical indices the best C_o value depends also on the formulation of the drift term: *MG* and *VG* for the *Luhar and Britter* (1989) and *Weil* (1990) models indicate different C_o values (2 and 3, respectively). Nevertheless, suggested values for C_o according to the present study lie in the range between 2 and 3, since in the vast majority of the examined cases the statistical indices indicate the best agreement of model predictions with measurements.

Summarizing the results of this work, we can argue that the choice of C_o crucially affects the performance of Lagrangian dispersion models that use the Langevin equation. The analysis indicates that for atmospheric dispersion C_o should be between 2 and 4, with more weight between 2 and 3, since for these values the model performance is optimized. This conclusion is compatible with the suggestions of other researchers for atmospheric flows (e.g., *Thomson*, 1987; *Anfossi et al.*, 2000; *Pope and Chen*, 1990; *Du*, 1997; *Heinz*, 2002; *Rizza et al.*, 2006).

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