STATISTICS OF ABSOLUTE AND RELATIVE DISPERSION IN THE ATMOSPHERIC CONVECTIVE BOUNDARY LAYER

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INTRODUCTION

To predict the dispersion of compounds in the Atmospheric Boundary Layer, it is very important to determine statistics of plume position such as mean, variance and skewness. In fact, the variance of the plume position is a direct measure of the plume spread (dispersion parameter), whereas third-order moments, such as the skewness of the plume position, provide information on the structure and the shape of the plume, and quantify the asymmetry of the plume distribution. In the Convective Boundary Layer (CBL), the main responsible of this asymmetric distribution is the inhomogeneous and non Gaussian large-scale turbulent motion. Moreover, a further contribution to the skewness of the plume shape is given by the reflection of the plume at the CBL boundaries, which tend to accumulate the scalar near the surface and in the entrainment zone at the top of the CBL.

In our study, a Large-Eddy Simulation (LES) is used to calculate statistics of a plume dispersing in the CBL, in two different coordinate systems. By analyzing dispersion in *absolute* framework, i.e. in a coordinate system relative to a fixed point (e.g. the source location), the statistical properties are influenced by the full spectrum of turbulent eddies. In the *relative* framework, the coordinate system moves with the instantaneous plume centerline position. In this framework, therefore, the (vertically) inhomogeneous large-scale meandering motion is removed, and the concentration statistics are only dependent on the small turbulent eddies, which are homogeneous and isotropic.

Large-scale (meandering) and small-scale (relative diffusion) motions are usually assumed to be statistically independent (*Gifford*, 1959; *Hanna*, 1986). From the analysis of LES results we show that the reflection of the plume by the CBL boundaries generates the presence of non-linear cross-correlation terms in the balance equation for the third-order moments of the plume position. As a result, the third-order moment of the absolute position is not balanced by the sum of the third-order moments of the meandering and relative plume position.

THEORETICAL BACKGROUND

Let c=c(x,y,z,t) be the plume instantaneous concentration. At each time *t* and any downwind distance *x*, let (y_m, z_m) be the position of the plume centerline. It is now possible to define the system of coordinates relative to the instantaneous plume centerline position as:

$$y_r=y-y_m\,; \qquad z_r=z-z_m \qquad (1)$$

Consequently, for each downwind position *x*, the relative concentration is defined as

$$c_r\,(x,y_r,z_r,t)=c(x,y_m+y_r,z_m+z_r,t). \qquad (2)$$

Let *z* be the instantaneous (vertical) position of a particle in the plume. From the instanttaneous plume centerline position $z_m(t)$ and the mean plume height $\bar{z}$, the fluctuation of the absolute (z'), relative (z'_r) and centerline (z'_m) positions are defined as follows:

$$\begin{cases} z'=z-\bar{z} \\ z'_r=z-z_m=z_r \\ z'_m=z_m-\bar{z}. \end{cases} \qquad (3)$$

From these quantities, dispersion parameters and skewness of plume position in the different coordinate systems are calculated as:

$$\begin{cases} \sigma_z^2=\overline{z'^2} & (4a) \\ S_z=\overline{z'^3}/\sigma_z^3\,. & (4b) \end{cases}$$

NUMERICAL SETUP

Model description

The LES code used here is the parallelized version of the one described by *Dosio et al.* (2003), in which a set of filtered prognostic equations for the dynamic variables (wind velocity, potential temperature, turbulent kinetic energy) is solved on a staggered numerical grid. The space and time integrations are computed with a Kappa and leap-frog numerical schemes respectively. A conservation equation for a passive tracer is added to the governing set of equations. The horizontal numerical domain covers an area of 5.120 *Km* x 5.120 *Km*. In order to solve the largest spectrum range of motion, a very fine numerical grid is prescribed, with a resolution of 10 *m* in all the directions. The aspect ratio, that is, the ratio between the horizontal domain dimension to the CBL height z_i, is around 6.6. Lateral periodic boundary conditions are imposed for all the variables. A time step of 0.25 s. is used.

Flow characteristics

At the top of the CBL, an inversion strength of 5 *K* is imposed, which maintains the height of the CBL fairly constant with time. A geostrophic wind of 2 *m/s* aligned in the *x* direction and a heat flux of 0.1 *K m/s* are prescribed as constant forcing. The simulation is run for an initialization period of 2 hours. After this period, the gradients of the mean variables are independent on time and the turbulent kinetic energy has become constant. The average values of the convective velocity scale $w*$ is 1.38 *m/s* and the shear/buoyancy ratio $u*/w*$ is equal to 0.14. The value of the stability parameter $-z_i/L$ is around 136.

Plume concentration calculation

After the initialization period, an instantaneous line source (ILS) of scalar (non-buoyant tracer) is emitted {along the *x* axis} at $z/z_i = 0.28$. The line source measures one grid spacing both in the vertical and in the horizontal direction. As the numerical grid moves with the mean wind along the *x* direction, the ILS results at time *t* can be interpreted as those for a continuously released point source (CPS) at corresponding downwind distances $x = Ut$ (where *U* is the mean wind speed in the CPS case), as explained by *Dosio et al.*(2003). In order to obtain statistically sound results, nine different realizations are performed, in which the horizontal position of the instantaneous release is changed. The results are subsequently ensemble-averaged over the different realizations.

RESULTS AND DISCUSSION

Dispersion in absolute coordinate system

Figure shows the vertically-integrated concentration as a function of the normalized distance $t*=(w* z_i)t$. In the horizontal plane, the mean concentration has a Gaussian shape. The crosswind-integrated concentration (Fig. 1b), shows a ground-level maximum at $t*=0.65$, in close agreement with the water-tank experiment. The elevated maximum, due to the fast rise of the plume caught by the updrafts, occurs at $t*=1.7$, and the correspondent surface minimum is present around $t*=1.75$. The total horizontal and vertical dispersion parameters σ_y and σ_z are shown in Fig. 1c, where they are compared with laboratory data and other LES results, showing a satisfactory agreement. The skewness of the vertical position S_Z is shown in Fig. 1d. In a flow characterized by the inhomogeneity and skewness of the turbulence, S_Z quantifies the asymmetry of the plume with respect to its mean position (first-order moment). The skewness is positive between $0< t*<1.4$. In fact, since the CBL is characterized by a positively skewed vertical velocity, the plume is more likely to be transported towards the surface. As a result, the plume is more likely to be below its mean position $\bar{z}$, as shown by the maximum plume concentration, which is closer to the ground than the plume mean height (Fig. 1b). For $1.3< t*<2.5$ the skewness has a negative value. As Fig. 1b shows, when the plume reaches the ground, it is reflected by the ground. Near the ground the (downward) vertical motion is transformed into horizontal, and the plume remains close to the surface until it is transported upwards by the thermals. For $1.4<t_*<2.2$ the position of the maximum plume concentration lies above its mean position and an elevated maximum is present. The skewness, therefore, must have a negative value. At $t_*=2.6$ the skewness becomes slightly positive, correspondingly to the descent of the plume maximum concentration. The evolution of S_z is in very good agreement with the results of *Luhar et al.* (2000) calculated from their Lagrangian particle model.

Meandering component

An example of trajectories of the plume centerline position z_m is shown in Fig. 2a. Close to the source ($t_*<0.8$), the shape of the ensemble of trajectories is similar to the mean cross-integrated concentration (Fig. 1b), because meandering is the main contribution to the plume motion. The spread of the instantaneous plume centerline position reaches its maximum around $t_*=0.5$, and then it slowly diminishes. Far from the source, when the plume is nearly uniformly well mixed vertically, the instantaneous plume position becomes similar to the mean plume height $\bar{z}$, which has reached the asymptotic value of 0.5.

The second-order moment of the horizontal and vertical centreline position (σ_{ym} and σ_{zm}) calculated by the LES agrees satisfactorily with previous laboratory experiments and numerical simulation (Fig. 2b). The vertical meandering component σ_{zm} reaches a maximum value around $t_*=0.5$, and then it decays quickly to a very small value when the plume vertical motion is constrained by the CBL boundaries.

The evolution of the skewness of the instantaneous centreline (vertical) position S_{zm} is shown in Fig. 2c. The skewness of the meandering position provides information on the distribution of the plume as transported by the large-scale turbulent motions. As pointed out by *Luhar et al* (2000), there are no currently available data to validate the evolution of S_{zm}. The LES results provide an estimation of the downwind variation of the meandering skewness and they can be useful to derive a suitable parameterization. Close to the source, as meandering is the main contribution to plume dispersion, the concentration distribution is

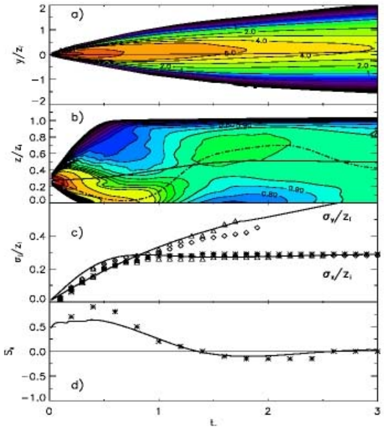

Fig. 1; a) Normalized vertically-integrated mean concentration. b) Normalized cross-integrated mean concentration. The continuous line is the normalized mean plume height and the dashed line is the position of the maximum concentration. c) Normalized horizontal and vertical dispersion parameters. The following experimental and numerical data are also shown: Willis and Deardorff (1978) (triangles); Hibberd (2000) (*); Nieuwstadt (1992) (diamonds). d) Skewness of the vertical plume position. The model results of Luhar et al. (2000) are also shown (*).

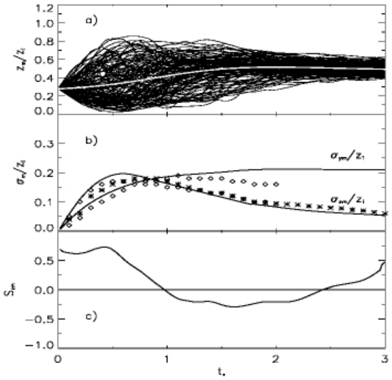

Fig. 2; a) Example of trajectories of the plume instantaneous centerline position zm. The plume mean height position is also shown.

b) Normalized horizontal and vertical variances of the plume centerline position (meandering). The laboratory data by Hibberd (2000) (*) and the LES data by Nieuwstadt (1992) (diamonds) are also shown.

c) Skewness of the vertical meandering position.

mainly affected by the large-scale motions. As a result, at t∗<0.5 the meandering skewness is very similar to the total skewness S_z. The meandering skewness follows closely the motion of the plume carried by the large-scale eddies, because, by definition, meandering represents the contribution of the large scale motion to the plume total dispersion. In particular, S_{zm} is positive when the plume is transported downward by the downdrafts (t∗<1 and t∗>2.4), and it becomes negative when the plume is transported upward by the thermals, as shown by the position of the maximum concentration in Fig. 1b. The meandering skewness behaves differently than the total skew-ness at distances t∗>0.5. For example, at t∗=1.1, S_{zm} is negative, whereas S_z is still positive.

Dispersion in relative coordinate system

Figure 3a shows the normalized vertically-integrated relative concentration. The vertically integrated concentration has a Gaussian shape similar to the one in absolute coordinates (Fig. 1a). The crosswind-integrated concentration is shown in Fig. 3b. Since in the relative coordinate system the meandering, which characterized the large-scale inhomogeneous and skewed motion, has been removed, one expects that the relative concentration has a more homogeneous and Gaussian distribution. This is evident especially close to the source (t∗<1), where the plume is still very narrow and it has not reached the boundaries yet. At t∗=1 the plume in relative coordinates has reached the CBL boundaries and it still shows a quasi-Gaussian distribution but the maximum concentration is somehow below the plume centerline. This deviation form the Gaussian distribution is an effect of the reflection of the plume by the ground, which occurs at t∗=0.5 (Fig. 1b), and causes the relative concentration to be positively skewed (Fig. 3d). At larger distances (t∗>1.2), the maximum lies above the relative mean height. Both the absolute and relative concentration present an elevated maximum around t∗=2 caused by the reflection with the CBL top. It is noteworthy that the relative concentration shows a distribution very similar to the absolute concentration for t∗>1.5. Since at large distances the meandering component becomes small, the relative diffusion is the main contribution to the plume dispersion as shown by Fig. 3c, where the vertical and horizontal relative dispersion parameters are shown. As a result, the concentration distribution in absolute coordinates is influenced mainly by the in-plume, small-scale motions, and the absolute concentration is very similar to the relative concentration for t∗>1.5.

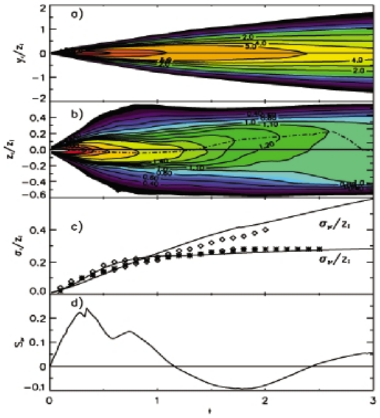

Fig. 3; a) Normalized vertically-integrated relative mean concentration. b) Normalized cross-integrated relative mean concentration. The dashed line is the position of the maximum concentration. c) Normalized horizontal and vertical dispersion parameters. The following experimental and numerical data are also shown: Hibberd (2000) (*); Nieuwstadt (1992) (diamonds). d) Skewness of the vertical plume relative position.

This result is corroborated by the analysis of the skewness of the vertical relative plume position S_{zr}, shown in Fig. 3d. Generally the relative skewness has a small value, because the relative concentration distribution is driven mainly by the small-scale motion, which is homogeneous and Gaussian. The reflection of the plumeby the ground is the cause of the small positive value of S_{zr} at distances 0< t∗<1.1. At larger distances (t∗>1.5), the relative skewness becomes negative due to the reflection by the CBL top, and S_{zr} becomes very similar to total skewness S_z. The effect of the CBL boundaries on the asymmetry (skewness) of the plume can be studied by analyzing the conservation equation for the third order moments. From the fluctuation of the plume positions (3), it can be shown that

$$\overline{z'^3} = \overline{z_m'^3} + \overline{z_r'^3} + \overline{3z'_m z_r'^2}, \quad (5)$$

where the cross-correlation term can be rewritten as follows:

$$C1 = \overline{3z'_m z_r'^2} = \overline{3z_m z_r'^2} - 3\bar{z}\sigma_{zr}^2 \qquad (6)$$

If meandering is independent of relative dispersion, then the cross-correlation terms C1is zero. Our LES results, on the other hand, show that the value of C1 with the distance from the source. Close to the source (t∗<0.2), before the plume reaches the ground, the total dispersion is dominated by the meandering, because the plume is very narrow. As a result, z'$_r$=0. When the plume reaches the ground, the scalar is reflected (0.2<t∗<2). The concentration distribution changes becoming skewed. The main contribution to the third-order moment is given by the cross-correlation term C1. The cross term reaches a maximum around t∗=0.6, when the plume is closest to the ground. At large distances from the source (t∗>2) the plume is confined between the bottom and the top of the CBL, and the instantaneous plume centerline position z$_m$ approaches its mean value. As a result, z'$_m$ becomes zero and the main contribution to the vertical spread is given by the relative diffusion.

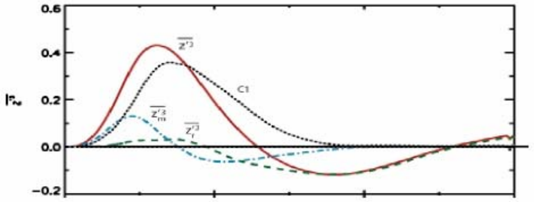

Fig. 4; Evolution of the third-order moments of the total (z'^{3}, continuous line), meandering (z'$_m$^{3}, dashed-dotted line) and relative (z'$_r$^{3}, dashed line) vertical plume position. In the figure, the the non-linear cross-correlation term C1 is also shown as dotted line. Note that, for clarity, all the terms have been divided by the factor 10^6.

The value of the cross-term C1 varies with the distance from the source, because the reflection of the plume by the boundaries strongly affects the value of the local (instantaneous) relative width z'$_r$^{2} with respect to its average value σ$_{zr}$^{2}. When the plume reaches the ground and the scalar is reflected and the concentration distribution changes becoming skewed. As a result, the local (instantaneous) relative variance departs from its average value; consequently, C1 becomes different than 0.

The variation of the cross term C1 with distance from the source can be interpreted as an effect of the meandering (i.e. position of the mean plume height) on the relative dispersion (value of the local relative variance), which, therefore, are not statistically independent in case of reflection of the plume by the boundaries. This result needs to be taken into account to calculate higher-order concentration statistics by means of operational models that are currently based on the assumption of statistically independence of meandering on relative dispersion.

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