WELL MIXED CONDITION VERIFICATION IN WINDY AND LOW WIND SPEED CONDITIONS

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INTRODUCTION

Dispersion in low wind speed (LWS) conditions is mostly governed by meandering (low frequency horizontal wind oscillations). Even when the stability reduces the vertical dispersion, meandering disperses the plume over rather wide angular sectors. Thus, the resulting ground level concentration is generally much lower than that predicted by standard Gaussian plume models.

Very recently (Anfossi et al., 2005; Oettl et al.., 2005) by studying the low wind speed turbulence and dispersion characteristics, found that: the autocorrelation functions (AF) of the horizontal wind components shows an oscillating behavior with the presence of large negative lobes due to the meandering. The following relationship,

$$R(\tau)=e^{-\frac{\tau}{(m^2+1)T_3}}\cos\frac{m\tau}{(m^2+1)T_3} \qquad (1a)$$

or

$$R(\tau)=e^{-p\tau}\cos(q\tau) \qquad (1b)$$

where : $p=\frac{1}{(m^2+1)T_3}$ and $q=\frac{m}{(m^2+1)T_3}$ (1c)

proposed, respectively, by to Frenkiel (1953) and Murgatroyd (1969) in a different context, containing two parameters: one (p or T_3) associated to the classical integral time scale and the second (q or m) to the meandering characteristic, very well fitted the observed AF, $R(\tau)$.

Besides providing a physical explanation of the meandering occurrence, they also proposed the following system of two coupled Langevin equations (SCLE) for the horizontal wind fluctuation components describing the LWS ($\overline{u}<1.5\ ms^{-1}$)dispersion in homogeneous conditions:

$$du=-(pu+qv)dt+\sigma\sqrt{2pdt}\,\xi_u \qquad (2a)$$

$$dv=-(-qu+pv)dt+\sigma\sqrt{2pdt}\,\xi_v \qquad (2b)$$

where u and v are the horizontal components of the wind velocity, ξ_u and ξ_v are random Gaussian variates having zero mean and unit variance, $\sigma=\sigma_u=\sigma_v$ is the standard deviation of the horizontal wind components assumed equal in LWS (Anfossi et al., 2005; Oettl et al.., 2005).

In this paper, we propose a new SCLE for the more general case of inhomogeneous turbulence and for the total velocity. We also propose a new SCLE for the windy situations ($\overline{u} > 1.5\ ms^{-1}$). Both SCLEs are based on the so-called "Thomson simplest solution" (Thomson, 1987; Rodean, 1996). This last will be presented in the next Section, whereas our new solutions will be introduced in the following Section. In the last Section we will show that all these solutions (windy and low wind cases) satisfy the "well-mixed condition.

NEW SCLE FOR WINDY CONDITIONS ($\overline{u} > 1.5\ ms^{-1}$)

We start from the well-known "simplest solution" of the 3D Langevin equations for inhomogeneous Gaussian turbulence proposed by Thomson (1987)

$$a_i = -\left(\frac{C_0\,\varepsilon}{2}\right)\Gamma_{ik}\left(u_k - \overline{u_k}\right) + \frac{\Phi_i}{g_a} \qquad \text{(3a)}$$

where

$$\frac{\Phi_i}{g_a} = \frac{\partial \overline{u_i}}{\partial t} + \overline{u_l}\frac{\partial \overline{u_i}}{\partial x_l} + \frac{\partial \overline{u_i}}{\partial x_j}\left(u_j - \overline{u_j}\right) + \frac{1}{2}\frac{\partial \tau_{il}}{\partial x_l} + \frac{\Gamma_{lj}}{2}\frac{\partial \tau_{il}}{\partial t}\left(u_j - \overline{u_j}\right) + \frac{\Gamma_{lj}}{2}\left(\overline{u_m}\frac{\partial \tau_{il}}{\partial x_m}\right)\left(u_j - \overline{u_j}\right) + \frac{\Gamma_{lj}}{2}\frac{\partial \tau_{il}}{\partial x_k}\left(u_j - \overline{u_j}\right)\left(u_k - \overline{u_k}\right) \qquad \text{(3b)}$$

in which x is the position of each particle, u_i and $\overline{u_i}$ their Lagrangian velocity (total and average, respectively), ε is the dissipation rate of turbulent kinetic energy, C_0 is a numerical constant and Γ_{lj} is the inverse of the Reynolds stress tensor τ_{ij}. Assuming stationarity ($\frac{\partial}{\partial t}=0$), 3-D inhomogeneity ($\frac{\partial}{\partial x_1} \neq \frac{\partial}{\partial x_2} \neq \frac{\partial}{\partial x_3} \neq 0$), 3-D mean flow ($\overline{u_1} \neq 0, \overline{u_2} \neq 0, \overline{u_3} \neq 0$), no cross-correlations ($\sigma_{1,2}^2 = \sigma_{1,3}^2 = \sigma_{2,1}^2 = \sigma_{2,3}^2 = \sigma_{3,1}^2 = \sigma_{3,2}^2 = 0$; $\Gamma_{lj} = \frac{1}{\sigma_{lj}^2}$ and $\tau_{il} = \sigma_{il}^2$) and using $T_{Li} = \frac{2\sigma_i^2}{C_0\,\varepsilon}$ (Hinze, 1975; Tennekes, 1982), we obtain the following equations:

$$du = \left\{-\frac{\left(u-\overline{u}\right)}{T_{Lu}} + \frac{\partial \overline{u}}{\partial x}u + \frac{\partial \overline{u}}{\partial y}v + \frac{\partial \overline{u}}{\partial z}w + \sigma_u\frac{\partial \sigma_u}{\partial x} + \frac{\left(u-\overline{u}\right)}{\sigma_u}\left[\frac{\partial \sigma_u}{\partial x}u + \frac{\partial \sigma_u}{\partial y}v + \frac{\partial \sigma_u}{\partial z}w\right]\right\}dt + \left(\frac{2\,dt}{T_{Lu}}\right)^{1/2}\sigma_u\,\xi_u \qquad \text{(4a)}$$

$$dv = \left\{-\frac{\left(v-\overline{v}\right)}{T_{Lv}} + \frac{\partial \overline{v}}{\partial x}u + \frac{\partial \overline{v}}{\partial y}v + \frac{\partial \overline{v}}{\partial z}w + \sigma_v\frac{\partial \sigma_v}{\partial y} + \frac{\left(v-\overline{v}\right)}{\sigma_v}\left[\frac{\partial \sigma_v}{\partial x}u + \frac{\partial \sigma_v}{\partial y}v + \frac{\partial \sigma_v}{\partial z}w\right]\right\}dt + \left(\frac{2\,dt}{T_{Lv}}\right)^{1/2}\sigma_v\,\xi_v \qquad \text{(4b)}$$

$$dw = \left\{-\frac{\left(w-\overline{w}\right)}{T_{Lw}} + \frac{\partial \overline{w}}{\partial x}u + \frac{\partial \overline{w}}{\partial y}v + \frac{\partial \overline{w}}{\partial z}w + \sigma_w\frac{\partial \sigma_w}{\partial z} + \frac{\left(w-\overline{w}\right)}{\sigma_w}\left[\frac{\partial \sigma_w}{\partial x}u + \frac{\partial \sigma_w}{\partial y}v + \frac{\partial \sigma_w}{\partial z}w\right]\right\}dt + \left(\frac{2\,dt}{T_{Lw}}\right)^{1/2}\sigma_w\,\xi_v \qquad \text{(4c)}$$

In 2-D equations (4) become:

$$du = \left\{-\frac{\left(u-\overline{u}\right)}{T_{Lu}} + \frac{\partial \overline{u}}{\partial x}u + \frac{\partial \overline{u}}{\partial y}v + \sigma_u\frac{\partial \sigma_u}{\partial x} + \frac{\left(u-\overline{u}\right)}{\sigma_u}\left[\frac{\partial \sigma_u}{\partial x}u + \frac{\partial \sigma_u}{\partial y}v\right]\right\}dt + \left(\frac{2\,dt}{T_{Lu}}\right)^{1/2}\sigma_u\,\xi_u \qquad \text{(5a)}$$

$$dv = \left\{-\frac{\left(v-\overline{v}\right)}{T_{Lv}} + \frac{\partial \overline{v}}{\partial x}u + \frac{\partial \overline{v}}{\partial y}v + \sigma_v\frac{\partial \sigma_v}{\partial y} + \frac{\left(v-\overline{v}\right)}{\sigma_v}\left[\frac{\partial \sigma_v}{\partial x}u + \frac{\partial \sigma_v}{\partial y}v\right]\right\}dt + \left(\frac{2\,dt}{T_{Lv}}\right)^{1/2}\sigma_v\,\xi_v \qquad \text{(5b).}$$

In this work we will limit our considerations to the horizontal 2D model because the systems proposed for LWS (equations 2 and 6) apply to the two horizontal components only.

NEW SCLE FOR WINDY CONDITIONS ($\bar{u} < 1.5\ ms^{-1}$)

The equations for LWS corresponding to equations (5) are:

$$du = \left\{ -p(u-\bar{u}) - q(v-\bar{v}) + \frac{\partial \bar{u}}{\partial x}u + \frac{\partial \bar{u}}{\partial y}v + \sigma_u \frac{\partial \sigma_u}{\partial x} + \frac{(u-\bar{u})}{\sigma_u}\left[\frac{\partial \sigma_u}{\partial x}u + \frac{\partial \sigma_u}{\partial y}v\right]\right\}dt + \sqrt{2\,p\,dt}\,\sigma_u \xi_u \qquad (6a)$$

$$dv = \left\{ q(u-\bar{u}) - p(v-\bar{v}) + \frac{\partial \bar{v}}{\partial x}u + \frac{\partial \bar{v}}{\partial y}v + \sigma_v \frac{\partial \sigma_v}{\partial y} + \frac{(v-\bar{v})}{\sigma_v}\left[\frac{\partial \sigma_v}{\partial x}u + \frac{\partial \sigma_v}{\partial y}v\right]\right\}dt + \sqrt{2\,p\,dt}\,\sigma_v \xi_v \qquad (6b)$$

It is easy to verify that equations (6), by setting *m = 0* (from which $p = 1/T_L$) collapse on equations (5) and, by setting $\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$ and $\bar{u} = 0, \bar{v} = 0$, collapse on equations (2).

WELL MIXED CONDITION VERIFICATION

Finally we verified that the two equation systems (5 and 6) satisfy the well mixed condition. To do this, a simulation series were performed. A domain of 500 x 200 m^2 was considered. Periodic boundary conditions were used. The values of $\bar{u}(x,y)$, $\bar{v}(x,y)$ and $\sigma(x,y)$ were prescribed according to the following expressions:

$$\bar{u} = A + B\sin\left(\frac{\pi}{L}(C\,x + y)\right) \qquad (7a)$$

$$\bar{v} = -C\,\bar{u} \qquad (7b)$$

and

$$\sigma = D + E\sin\left(\frac{\pi}{L}(C\,x + y)\right) \qquad (7c)$$

where the numerical values of the coefficients are shown in Table 1.

Table 13. Coefficients of equations (7)

A (ms^{-1})	B (ms^{-1})	C	D (ms^{-1})	E (ms^{-1})	L (m)
0.2	0.7	0.4	0.4	0.2	100

It is easy to verify that equations (7a and 7b) give a non-divergent wind field. The Lagrangian time scale $(T_{Lu} = T_{Lv})$ was set equal to 150 s for the windy case, while *p* and *q* were computed according to the following empirical relationships (estimated from a fit to the Graz data set):

$$m = \frac{8.5}{(\bar{u}+1)^2}\ , \qquad T_3 = \frac{m(200\,m + 350)}{2\pi(m^2+1)} \quad \text{and then} \quad p = \frac{1}{(m^2+1)T_3},\quad q = m\,p \qquad (8)$$

150,000 particles were released uniformly distributed in the domain, then they were moved for 4,000 iterations ($dt = 0.5\,s$) by means of the two equation systems (5 and 6) and, at the end, the final particle distributions were verified to be well mixed.

Figures 1, 2 and 3 represent, as an example, the initial (Figure 1) and final particle distributions, versus *x* and *y*, of the well mixed condition verification of equations 6 (Figure 2) and of equation 5 (Figure 3). The computation domain was divided in 24 layers (either along *x* or along *y*), the number of particles in each layer was computed and then divided by the expected well mixed value. A perfectly well mixed model should give 1 at

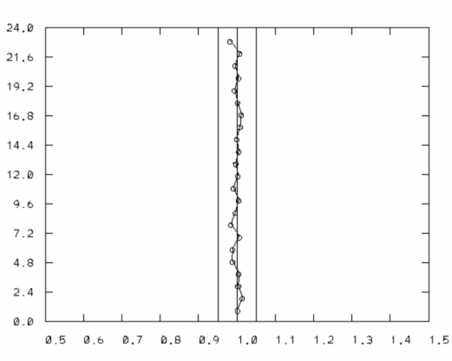

Fig. 1 – Initial distribution

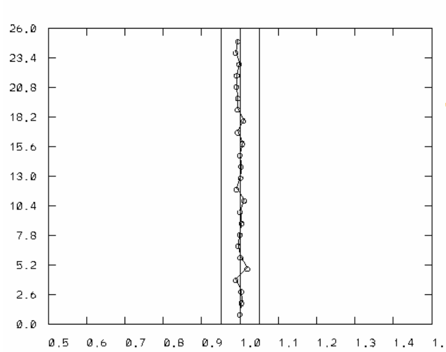

Fig.2 – final distribution (LWS –eq. 6)

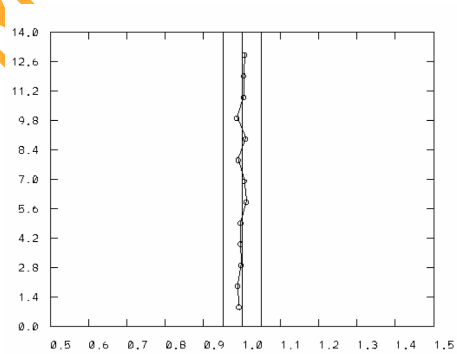

Fig.3 – final distribution (windy – eq. 5)

each level. The three vertical lines correspond to a perfectly well mixed model (1.0) and to an error of ± 5%. It can be clearly seen that the agreement is very good. All the other well mixed condition verification gave similarly good results.

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