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MONIN-OBUKHOV SIMILARITY IN THE URBAN INERTIAL SUBLAYER UNDER UNSTABLE CONDITIONS

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Abstract: The present paper examines the applicability of Monin-Obukhov similarity theory for heat and momentum in urban areas under a wide range of unstable conditions, using turbulence data measured on a tower in Himeji City at a height of $z=6.4h$ ($=54$ m) above the ground, where h denotes the average building height. The present result shows that Monin-Obukhov similarity theory is valid on the whole for calculating heat flux. However, integrated momentum stability correction function Ψ_m obtained in Himeji is equal to 0 for $-2.6 \leq z'/L < -1$, which deviated from Businger's similarity function.

Key word: field observation, heat, inertial sublayer, momentum, Monin-Obukhov similarity theory, tower, urban area

1. INTRODUCTION

Monin-Obukhov similarity theory has not been verified in the urban inertial sublayer under wide range of atmospheric stability conditions, because of the lack of observations. Observations of turbulence in Tokyo (Moriwaki and Kanda, 2006) and in Hamburg (Gryning et al., 2007) conducted under $-L > 100$ m did not provide the data under very unstable conditions. The average values of ϕ_h and ϕ_m calculated from those data were close to the empirical form presented by Businger et al. (1971). The present paper discusses the applicability of Monin-Obukhov similarity theory for heat and momentum in urban areas under the wide range of unstable conditions for $-L > 20$ m ($-2.6 \leq z'/L < 0$) where $z' = z - d$, d is the zero-plane displacement.

2. METHODS

2.1. Measurement of wind and temperature

We used turbulence data measured on a tower in Himeji City at a height of $z=6.4h$ ($=54$ m) above the ground, where h denotes the average building height (8.5 m). The turbulence was measured using a 3-D ultrasonic anemometer-thermometer (SONIC, SAT-550). The sampling interval was 10 Hz and a 60-min-averaging time was used for each run. The temperature was observed at two heights of $z=18$ m and 70 m using thermistor thermometers (Espec, RT-30). The details were described by Kono et al. (2008). The data used for the analysis were measured for April–August in 2006 and June–July in 2007. For the analysis 80 hours of data were selected from all the observed data. The selected data were under favorable wind directions of south to southwest ($170^\circ - 230^\circ$) in which measurements were not affected by the tower and hills around. The average building height $h=8.5$ m and the zero-plane displacement $d=2.8$ m are the averaged values in the sector within the directions of south to southwest and within 5 km from the tower and the corresponding surface roughness $z_0 \doteq 1.6$ m. The z_0 value was determined using the logarithmic wind profile and the values of wind velocity u and friction velocity u_* which were obtained at the tower for neutral conditions. In the present paper, the measurement height of turbulence $z=54$ m $= 6.4h$, is in the inertial sublayer, though it was treated as being in the upper roughness sublayer in the previous paper by Kono et al. (2008). It is resulted from the detail survey of the building height in the sector mentioned above and according to the estimate of roughness sublayer height which is $2h-5h$ by Raupach et al. (1991).

2.2. Scaling velocity for L

It is known that in the roughness sublayer, the momentum flux decreases with height. It has maximum value approximately at $1.5h$ for uniform building arrangements. (Kastner-Klein and Rotach, 2004; Oikawa and Meng, 1995; Moriwaki and Kanda, 2006; Uehara et al., 1997) According to Moriwaki and Kanda (2006), the maximum friction velocity u_{*r} was used as the scaling velocity (surface scaling) to obtain the Monin-Obukhov length, L . We estimated that $u_{*r} = 1.5u_*(z=6.4h)$ from the wind tunnel experiments (Uehara et al, 1997) and the field experiments (Oikawa and Meng, 1995). On the other hand, the heat flux observed at $z=6.4h$ (local value) was used to estimate L , according to Moriwaki and Kanda (2006).

2.3. Calculation of heat flux by K-theory

Heat flux $H = C_p \rho \overline{\theta' w'}$ was calculated using the K-theory of equation (1) where $d\theta/dz$ is the potential temperature gradient; C_p is specific heat; and ρ is the density of air. Eddy diffusivity for heat K_h was calculated from equation (2) where ϕ_h is the Monin-Obukhov similarity function of temperature profile and von Karman constant $k=0.35$. ϕ_h is calculated from empirical equation (3) given by Businger et al. (1971).

$$H = C_p \rho \overline{\theta' w'} = -C_p \rho K_h \frac{d\theta}{dz} \quad (1)$$

$$K_h = \frac{kz u_*}{\phi_h} \quad (2)$$

$$\varphi_h = 0.74\{1 - 9(z'/L)\}^{-0.5} \tag{3}$$

2.4. Estimation of M-O similarity function

Using equation (4) φ_h is calculated from potential temperature gradient between two height and T_* .

$$\varphi_h = \frac{kz}{T_*} \frac{d\theta}{dz} \tag{4}$$

where $T_* = -\overline{\theta'w'}/u_*$ is the temperature scaling parameter for Monin-Obukhov scaling. According to Moriwaki and Kanda (2006), Local T_* value was used in the present paper, because they mentioned that T_* did not show remarkable change with height. Integrated momentum stability correction function Ψ_m is calculated from equation (5).

$$\frac{u}{u_*} = \frac{1}{k} \left(\ln \frac{z'}{z_0} - \Psi_m \right) \tag{5}$$

$$\Psi_m \equiv \int_{z_0}^z (1 - \varphi_m) \zeta^{-1} d\zeta \tag{6}$$

where $\zeta = z'/L$. Using $\varphi_m = (1 - 15z/L)^{-1/4}$ of Businger’s similarity function, the following expression for Ψ_m was given by Paulson (1970)

$$\Psi_m = \ln \left[\left(\frac{1+x^2}{2} \right) \left(\frac{1+x}{2} \right)^2 \right] - 2 \arctan x + \frac{\pi}{2} \tag{7}$$

where $x = (1 - 15z/L)^{1/4}$.

3. RESULTS AND DISCUSSION

3.1. Intensity of turbulence in the vertical direction

The intensity of turbulence in the vertical direction σ_w/u_* observed on the tower in Himeji is shown in Fig. 1 where u_* represents local values. The formula presented by Panofsky et al. (1977), $\sigma_w/u_* = 1.25(1 - 3z/L)^{1/3}$, and an empirical function in the roughness sublayer ($2.5 < z/h < 3.2$), $\sigma_w/u_* = 1.15(1 - 2.09z/L)^{0.33}$ presented by Roth (2000) are shown in the figure. σ_w/u_* are plotted for two types of z'/L where L is scaled using surface u_{*r} and local u_* values respectively. When we used local u_* values to calculate L , σ_w/u_* was less than Panofsky’s formula (Kono et al., 2008). However, present data in which surface u_{*r} was used to calculate L are found to be close to both of the Panofsky’s formula and Roth’s empirical function. The results verify that Panofsky’s formula is applicable to the urban inertial sublayer. The result means that the surface scaling using u_{*r} to calculate L is superior to the local scaling in the urban inertial sublayer.

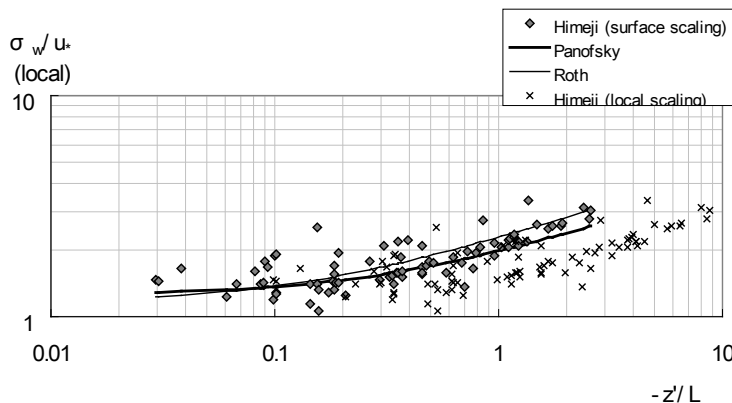


Figure 1. Intensity of turbulence in the vertical direction σ_w/u_* observed at 54m in height in Himeji.

3.2. Power spectra

Power spectra of vertical wind velocity were obtained from FFT (Fig. 2). Averaged values in the same stability classes are shown in the figure. The number of runs for the average n is 29 for $-0.3 < z'/L < 0$, 22 for $-1 < z'/L < -0.3$ and 16 for $-2.6 \leq z'/L < -1$.

For comparing urban and rural power spectra, the Kansas experiments (Kaimal et al., 1972) are shown in the figure. Note that the spectral density in Himeji experiments is normalized by σ_w^2 but that in Kansas experiments by $u_*^2 \phi_\epsilon^{2/3}$ where ϕ_ϵ is normalized dissipation function. The non dimensional frequency of peak energy and the shape of the spectra in Himeji experiments are consistent with those of Kansas experiments under near neutral conditions ($-0.3 < z'/L < 0$). For unstable conditions of $-2.6 \leq z'/L < -0.3$, the spectrum has a broad peak over the non dimensional frequency ranges between 0.1 and 0.7 in Himeji experiments. This resulted from superposing two peaks of high frequency fluctuations of mechanical turbulence and low frequency fluctuations of convective turbulence. Kansas spectra for $-2.0 < z/L < -0.3$, the shaded area, were not arranged according to z/L and the non dimensional frequency of peak energy ranged between 0.1 and 0.5 (after Kaimal et al., 1972). That indicates the spectra in urban and in rural areas become similar when they are scaled with z/L .

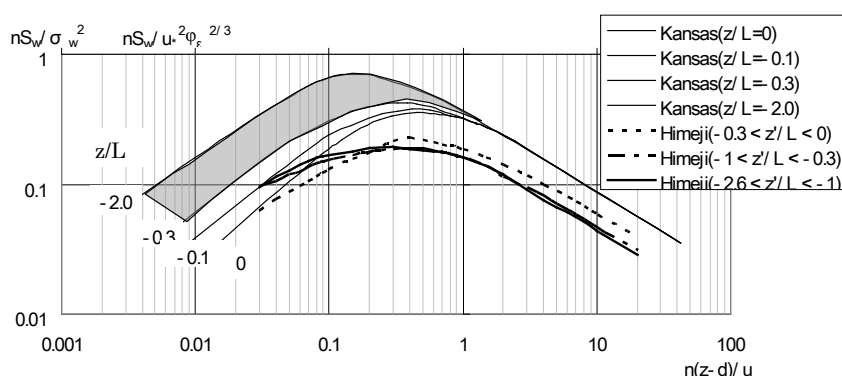


Figure 2. Normalized w spectra observed in Himeji versus Kansas.

3.3. Heat flux estimation and ϕ_h

The comparison on heat flux between calculations using Businger's similarity function ϕ_h and measurements is shown in Fig. 3. As the precision of thermistor thermometer is 0.1 K, the data for small heat flux may be contaminated by errors due to the instrument. If the data of $d\theta < 0.1$ K are excluded, most of the calculations are within 1/2 to 2 times the measurements. However, the calculation is found to overestimate observations for heat flux in the range larger than 250 Wm^{-2} .

The calculated ϕ_h using the data in an urban area of Himeji were compared with Businger's similarity function ϕ_h and calculated ϕ_h using the data of Kansas ($z_0=2.4\text{cm}$) (Izumi, 1971) in Fig. 4. The data of $d\theta < 0.1$ K were excluded from Fig. 4. The values of ϕ_h in the urban area scatter more than the rural area. Because the large scatter in Himeji is consistent with observations in Tokyo by Moriwaki and Kanda (2006) and in Türich by Rotach (1993), it is probably characteristics in urban areas. The ensemble mean of ϕ_h using the data of Himeji is 1.6–2.3 times Businger's similarity function.

3.4. Integrated momentum stability correction function Ψ_m

Values of u_*/u observed in Himeji and calculated u_*/u using Businger's Ψ_m (equation (7)) are shown in Fig. 5. Although the data of Himeji shows large scatter, the ensemble mean does not deviate much from the calculation using Businger's Ψ_m for $-0.3 < z'/L < 0$. On the other hand, the calculated value using Businger's Ψ_m increases with decreasing z'/L for $-2.6 \leq z'/L < -0.5$, while the observed values are almost the same as those under neutral condition.

Values of Ψ_m calculated from the data of Himeji is shown in Fig. 6. The calculated values show large scatter and deviate from Businger's Ψ_m . The ensemble mean of Ψ_m approaches 1 at $z'/L = -0.3$ as Businger's Ψ_m increases with decreasing the stability within $-0.3 < z'/L < 0$. However, the variation of the ensemble mean of Ψ_m with stability for $-2.6 \leq z'/L < -0.3$ is completely different from that for $-0.3 < z'/L < 0$. For $z'/L < -1$, the ensemble mean of Ψ_m approaches 0 which is the same value under neutral condition. The correlation coefficient for uw , R_{uw} and θw , $R_{\theta w}$ obtained both in Himeji experiments and in Kansas experiments are shown in Fig. 7. The data for R_{uw} and $R_{\theta w}$ in Kansas experiments were observed at 5 m, 11 m and 23 m in height. The ensemble mean of R_{uw} and that of $R_{\theta w}$ in Kansas experiments are almost consistent with those in Himeji experiments. It is noteworthy that the absolute value $|R_{uw}|$ decreases with the decrease of stability from $z'/L = -0.3$ and it is less than 0.2 for $z'/L < -1$. The decrease of $|R_{uw}|$ reduces u_* . The variation of the ensemble mean of Ψ_m with z'/L is similar to that of R_{uw} . The mixture of mechanical turbulence and convective turbulence probably reduces the momentum transport as well as the correlation between u and w for $z'/L < -1$.

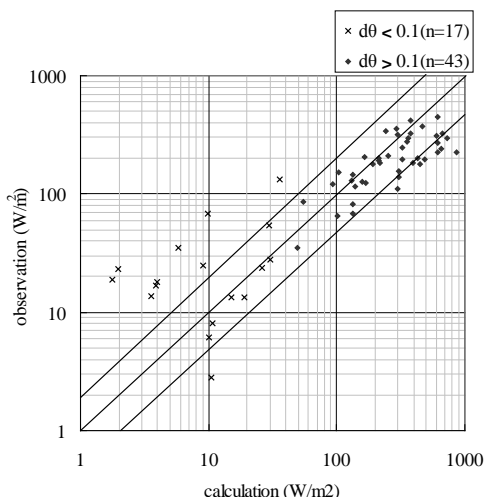


Figure 3. Comparison between calculated heat flux and observed heat flux in Himeji.

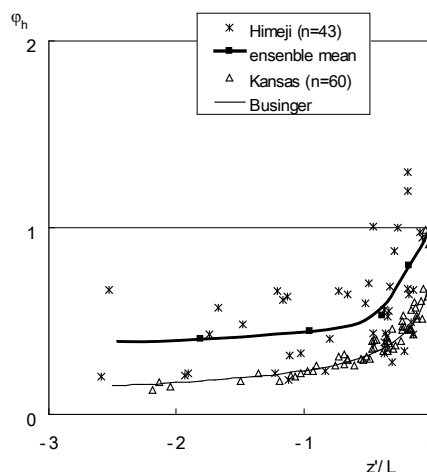


Figure 4. Comparison of calculated ϕ_h using the data of Himeji with those of Kansas and Businger's ϕ_h function.

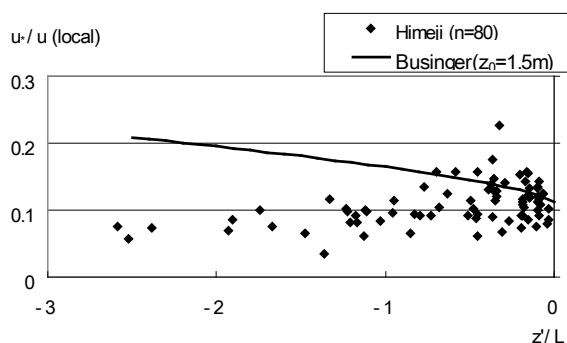


Figure 5. u^*/u observed at $z=54m=6.4h$ in Himeji versus calculated u^*/u using Businger's Ψ_m function

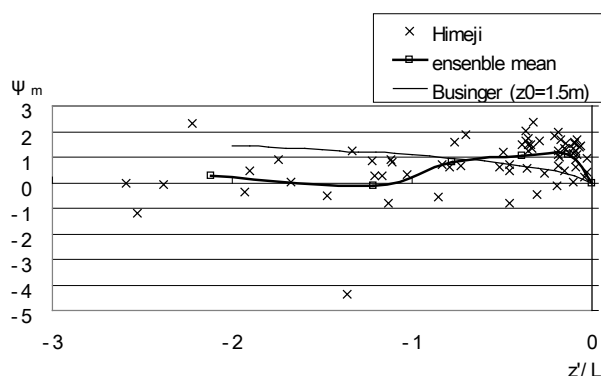


Figure 6. Ψ_m calculated from the data of Himeji and their ensemble mean versus Businger's Ψ_m function.

4. CONCLUSIONS

- (1) The friction velocity at approximately 1.5h in height, u_{*r} is better than a local value as the scaling velocity for Monin-Obukhov length, L .
- (2) Both σ_w/u_* and power spectra of vertical wind velocity obey Monin-Obukhov similarity theory in urban areas as well as rural areas.
- (3) Calculated heat flux using Businger's similarity function overestimated observations for heat flux in the range larger than 250 Wm^{-2} , however most of the calculations were within 1/2 to 2 times observations. Therefore Monin-Obukhov similarity theory is generally valid for calculation of heat flux in the urban inertial sublayer. However more data are necessary to estimate exact ϕ_h in urban areas.
- (4) The mixture of mechanical turbulence and convective turbulence probably reduces momentum transport in urban areas and rural areas as well. It reduces the correlation between u and w for $z'/L < -1$.
- (5) For $z'/L < -1$, the ensemble mean of Ψ_m approaches 0 which is the same value under neutral condition. The results correspond to the conclusion (4). It seems that the difference in Ψ_m between Himeji and Kansas for $-2.6 \leq z'/L < -1$ is caused by the strong mechanical turbulence in urban areas at low measurement height of $z/h=6.4$.

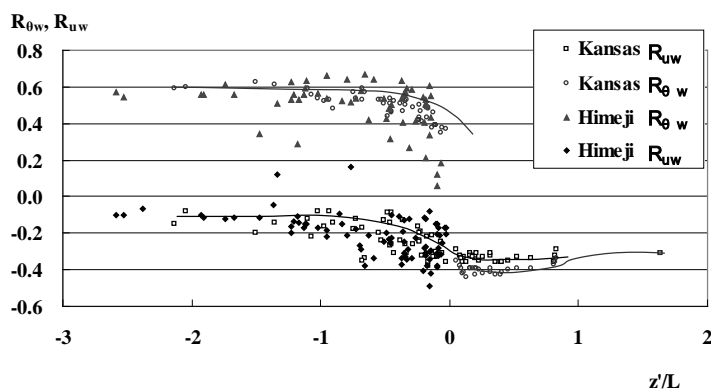


Figure 7. Correlation coefficient R_{uw} and R_{0w} observed in Himeji and Kansas, the solid line is the ensemble mean of Kansas.

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