Estimation of the Cesium-137 Source Term from the Fukushima Daiichi Power Plant Using Air Concentration and Deposition Data

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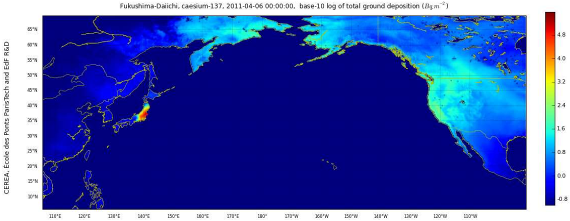
The Fukushima Daiichi accident

- Chronology: March 12: R 1 venting + explosion; March 13-14: R 3 venting + explosion; March 15: R 2 venting + explosion; March 20-22: R 2 R 3 spraying - smokes.

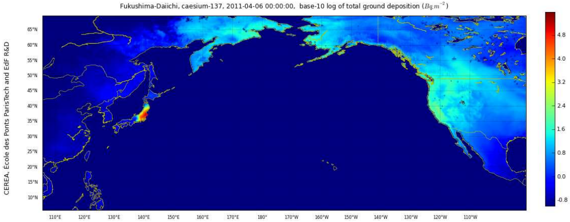
$\longrightarrow$ Source term of major interest for risk/health agencies, NPP operators

Observations of the Fukushima atmosperic dispersion



- Available data:
  1. Very few observations of activity concentrations in the air: A few hundreds of observations over Japan publicly released.
  2. Several thousands of observations from the CTBO IMS network, only partly publicly available, far away from the release site (except for Tokyo station).
  3. Activity deposition: a few hundreds, but more difficult to exploit (mainly ^{137}Cs).
  4. Hundreds of thousands of gamma dose measurements available on the web: very difficult to exploit. ($\longrightarrow$ Saunier et al., tomorrow 11:30, Room 1002)

Observations of the Fukushima atmosperic dispersion



- Inverse modeling of ^{137}Cs with three different data sets:
  1. Activity concentration in the air over Japan (104 observations).
  2. Daily measurements of deposited ^{137}Cs in the prefectures (198).
  3. Total deposited ^{137}Cs (2180).

Dispersion model and physics

$$\frac{\partial c}{\partial t} + \underbrace{\mathrm{div}\,(\mathbf{u}c)}_{\text{wind advection}} = \underbrace{\mathrm{div}\,(\mathbf{K}\nabla c)}_{\text{turbulent diffusion}} - \underbrace{\boldsymbol{\Lambda} c}_{\substack{\text{wet scavenging}\\ \text{radioactive decay}}} + \underbrace{\sigma}_{\text{source}}$$

- Polair3D eulerian model of the platform Polyphemus [Quélo 2007]
- Resolution: $0.05^\circ \times 0.05^\circ$; $N_x = 270$, $N_y = 260$, $N_z = 15$.
- Mesoscale WRF simulation.
- Simplest reasonable numerical scheme (for demanding DA algorithms)
  - Wet scavenging: related to rainfalls $\Lambda^s = 3.5 \times 10^{-5}.p^{0.8}$ [Maryon, 1991]
  - Dry deposition: constant deposition velocity, $v_\mathrm{d} = 0.15$ cm.s^{-1} for $^{137}\mathrm{Cs}$.
  - Vertical turbulent diffusion (K_z): Louis scheme [Louis 1979].
  - Radioactive decay.

source-receptor relationship and cost function minimization

Cost function

- Posing the inverse problem (estimate σ knowing μ) with the Jacobian $\mathbf{H}$:

$$\mu = \mathbf{H}\sigma + \varepsilon \,. \tag{1}$$

  $\mathbf{H}$ computed with the forward or adjoint model + observation operator.
- Traditional methodology inspired from geophysical data assimilation:

$$\mathscr{J} = \frac{1}{2}(\mu - \mathbf{H}\sigma)^{\mathrm{T}}\mathbf{R}^{-1}(\mu - \mathbf{H}\sigma) + \frac{1}{2}(\sigma - \sigma_b)^{\mathrm{T}}\mathbf{B}^{-1}(\sigma - \sigma_b)\,, \tag{2}$$

About the first guess σ_b

- A regularisation is absolutely necessary if the dataset is not overwhelming!
- In the case of accidental release inverse modeling, such a prior does not exist, or is difficult to establish. Moreover its uncertainty is even more difficult to assess.
- Choices for the first guess: $\sigma_b = \mathbf{0}$ (rather than a σ_b estimated from nuclear physics model).

Reconstruction of the Fukushima Daiichi source term

Cost function

Retrieval of the cesium-137 source term $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_{504})$ $(\Delta t = 1\text{h})$ using

$$\mathscr{J} = \frac{1}{2}(\mu - \mathbf{H}\sigma)^{\mathrm{T}}\mathbf{R}^{-1}(\mu - \mathbf{H}\sigma) + \frac{1}{2}\sigma^{\mathrm{T}}\mathbf{B}^{-1}\sigma\,, \tag{3}$$

Gaussian statistics - BLUE theory

First, Gaussian assumptions on the background (no positivity constraint).
In this case, the estimated source term is given by the Best Linear Unbiased Estimator (BLUE):

$$\sigma_{\mathrm{BLUE}} = \mathbf{B}\mathbf{H}^{\mathrm{T}}(\mathbf{H}\mathbf{B}\mathbf{H}^{\mathrm{T}} + \mathbf{R})^{-1}\mu \tag{4}$$

and its analysis error covariance matrix:

$$\mathbf{P}_{\mathrm{BLUE}} = \mathbf{B} - \mathbf{B}\mathbf{H}^{\mathrm{T}}(\mathbf{H}\mathbf{B}\mathbf{H}^{\mathrm{T}} + \mathbf{R})^{-1}\mathbf{H}\mathbf{B} = (\mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{H} + \mathbf{B}^{-1})^{-1} \tag{5}$$

Estimation of prior errors

Design of **R** and **B** matrices

The reconstructed source is very sensitive to the error covariance matrices. **R** is decomposed in $\mathbf{R}_i = r_i^2 \mathbf{I}_{d_i}$ (one variance for each data set), $\mathbf{B} = m^2 \mathbf{I}_N$.

Estimation of the prior errors statistics hyper-parameters

- The reconstructed source is very sensitive to the hyper-parameters that characterize the prior errors statisctics $\left(r_1, \ldots, r_{N_d}, m\right)$.
  - Example: for a fixed **R**, $m = 10^{10.5}$ give $M = 3.4\mathrm{PBq}$, $m = 10^{11}$ give $M = 8.4\mathrm{PBq}$, $m = 10^{11.5}$ give $M = 14\mathrm{PBq}$).
- Two methods have been used to estimate the hyper-parameters:
  - The value screening of the likelihood, to find the parameters that maximize the likelihood.
  - Desroziers' scheme to numerically localize the maximum likelihood parameters.

Maximum likelihood principle

Likelihood of the observation set

The likelihood of the observation set

$$p(\mu) = \int p(\mu|\sigma)p(\sigma)d\sigma \tag{6}$$

is actually a function of the hyper-parameters set $\theta = \left(r_1, \ldots, r_{N_d}, m\right)^{\mathrm{T}}$.
In the Gaussian context,

$$p(\mu|\theta) = \frac{e^{-\frac{1}{2}\mu^{\mathrm{T}}\left(\mathbf{HBH}^{\mathrm{T}}+\mathbf{R}\right)^{-1}\mu}}{\sqrt{(2\pi)^d|\mathbf{HBH}^{\mathrm{T}}+\mathbf{R}|}} \tag{7}$$

Maximization of the likelihood

Two strategies :

- A numerical scheme [Desroziers, 2001], which converges to a fixed point.
- The exhaustive value screening of the likelihood.

Maximum likelihood principle (2/3): Desroziers's scheme

The maximization of the likelihood lead to $N_d + 1$ scalar equations

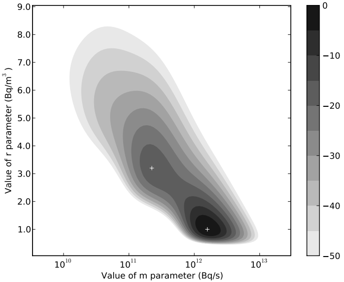
$$m^2 = \frac{\boldsymbol{\sigma}_a^{\mathrm{T}} \boldsymbol{\sigma}_a}{N - \mathrm{Tr}(\mathbf{P}_{\mathrm{BLUE}} \mathbf{B}^{-1})} \tag{8}$$

$$r_i^2 = \frac{(\mathbf{H}_i \boldsymbol{\sigma}_a - \boldsymbol{\mu}_i)^{\mathrm{T}} (\mathbf{H}_i \boldsymbol{\sigma}_a - \boldsymbol{\mu}_i)}{d_i - \mathrm{Tr}(\mathbf{H}_i \mathbf{P}_{\mathrm{BLUE}} \mathbf{H}_i^{\mathrm{T}} \mathbf{R}_i^{-1})} \tag{9}$$

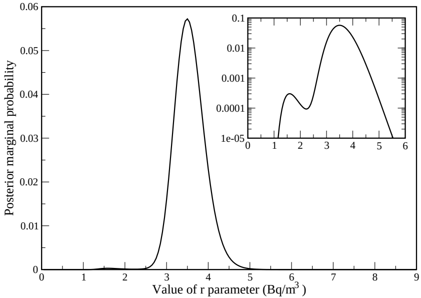
that can be used in an iterative scheme that will converge to a (local) maximum of the likelihood (3-4 iterations).

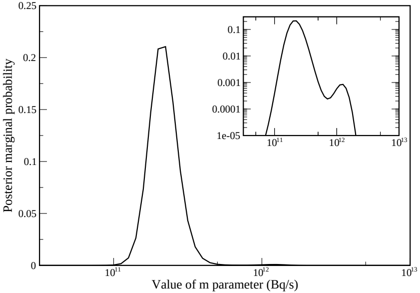
Maximum likelihood principle (3/3)

- Values screening of likelihood as a function of $(r_1, \ldots, r_{N_d}, m)$.
- Marginals of the hyper-parameters.

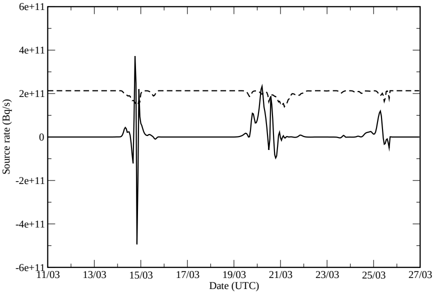


Maximum likelihood principle (3/3)





First results and new assumption



Limits of Gaussian statistics

- Negative values for the reconstructed source appear.
  - To force the retrieved source term to be positive: semi-Gaussian statistics for the background.
  - Same cost function to minimize, but with positivity constraint. No more analytical solution. Use of numerical minimization (L-BFGS).

Positivity assumption on the source term

Cost function

Retrieval of the cesium-137 source term, under semi-Gaussian constraints.

$$\mathscr{J} = \frac{1}{2}(\mu - \mathbf{H}\sigma)^{\mathrm{T}}\mathbf{R}^{-1}(\mu - \mathbf{H}\sigma) + \frac{1}{2}\sigma^{\mathrm{T}}\mathbf{B}^{-1}\sigma\,, \tag{10}$$

under the assumption $\sigma \geq 0$.

Estimation of hyper-parameters

Estimation of the hyper-parameters θ mathematically challenging (sampling of a high-dimensional truncated Normal distribution).

$$p(\mu|\theta) = \frac{e^{-\frac{1}{2}\mu^{\mathrm{T}}\left(\mathbf{HBH}^{\mathrm{T}}+\mathbf{R}\right)^{-1}\mu}}{\sqrt{(2\pi)^d|\mathbf{HBH}^{\mathrm{T}}+\mathbf{R}|}} \times \int_{\sigma\geq 0} \frac{e^{-\frac{1}{2}(\sigma-\sigma_{\mathrm{BLUE}})^{\mathrm{T}}\mathbf{P}_{\mathrm{BLUE}}^{-1}(\sigma-\sigma_{\mathrm{BLUE}})}}{\sqrt{(\pi/2)^N|\mathbf{P}_{\mathrm{BLUE}}|}}\mathrm{d}\sigma\,, \tag{11}$$

Desroziers's scheme: now only an approximation

The Desroziers iterative scheme is still used:

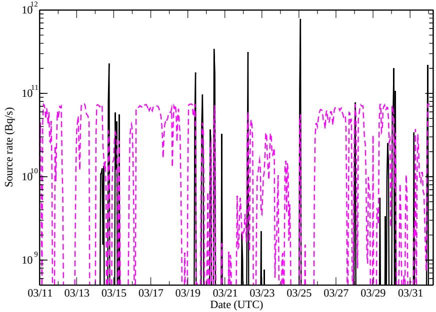
$$m^2 = \frac{\sigma_a^{\mathrm{T}}\sigma_a}{N - \mathrm{Tr}(\mathbf{P}_{\mathrm{BLUE}}\mathbf{B}^{-1})} \tag{12}$$

$$r_i^2 = \frac{(\mathbf{H}_i\sigma_a - \mu_i)^{\mathrm{T}}(\mathbf{H}_i\sigma_a - \mu_i)}{d_i - \mathrm{Tr}(\mathbf{H}_i\mathbf{P}_{\mathrm{BLUE}}\mathbf{H}_i^{\mathrm{T}}\mathbf{R}_i^{-1})} \tag{13}$$

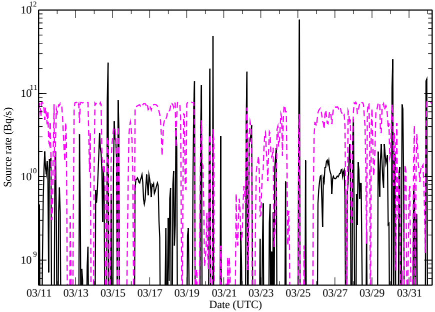
but now it is only an approximation of the maximization of the likelihood. When the number of observations is high enough, the two methods yield similar results.

Estimation of the posterior uncertainty and some results

- The posterior uncertainty on the retrieved source term is computed through a second-order Monte-Carlo analysis.
- 20 000 inversions with perturbed observations and first guess. $\mu^p \sim \mathcal{N}(\mu, \mathbf{R})$ and $\sigma_b^p \sim \mathcal{T}\mathcal{N}(\sigma_b = \mathbf{0}, \mathbf{B})$.



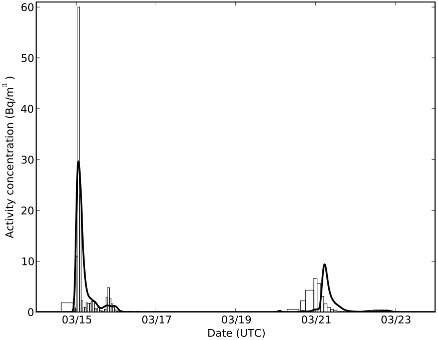
$\mathrm{M} = 12\,\mathrm{PBq}$



$\mathrm{M} = 19\,\mathrm{PBq}$

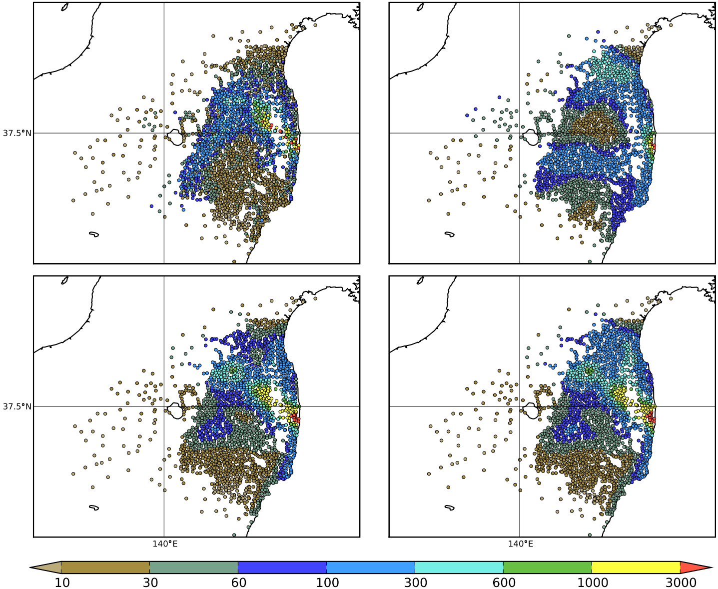
Results (2)

- Comparison between measurements at Tokyo MITRI station and corresponding reanalysed simulations for Cesium-137.



- General good agreement between observations and simulation, especially at peak times.
- The inversion is highly driven by high concentration measurements, due to the Gaussian representation of errors statistics.

results (3): Deposition map reanalysis



Conclusions

- In the Fukushima accident context, severe conditions leading to few/sparse observations make the use of rigorous mathematical methodologies mandatory to reconstruct the source term with inverse modeling methods.

- Our estimates (lower bound) : between 12 and 19 PBq (std. 15 - 20 %) for Cesium-137 and between 190 and 380 PBq (std. 5 - 10 %) for Iodine-131.

- The lack of public measurements data represents a dangerous handicap for the scientific community.

Possible improvements :

- Improve the accuracy of the dispersion model, for example the deposition parameterization. Physical parameter inverse modeling techniques could be useful (Bocquet, 2011).

- Inverse modeling methods that can handle different observation scales, for example non-Gaussian methods.

- Inverse modeling methods that can handle all the available data, including gamma dose rates (large amount of data).

Any questions?

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