An inverse modelling technique for emergency response application

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Malicious or accidental release in an urban area

What area should the first responders cordon off or evacuate?

What are the source characteristics? - uncertainty

Where will the plume spread?

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Chemical sensor

Source position

Malicious or accidental release in an urban area

What area should the first responders cordon off or evacuate?

What are the source characteristics? - uncertainty

Where will the plume spread?

The DYCE consortium

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DYnamic deployment planning for monitoring of ChEmical leaks using an ad-hoc sensor network

Chemical sensors — OWLSTONE

Communications & networking — SELEX GALILEO

Inverse modelling to estimate the source characteristics

Wind tunnel & tracer trial validation studies

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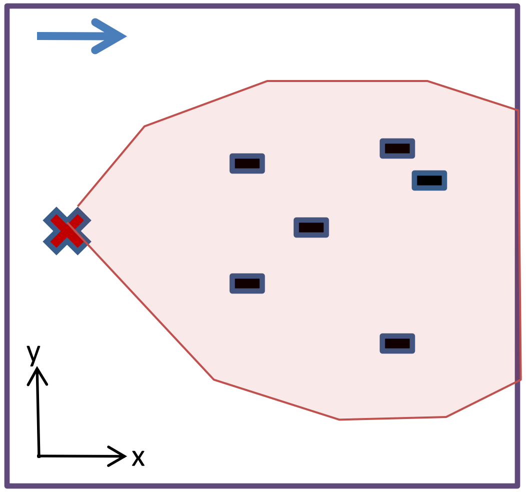
Funding — EPSRC Engineering and Physical Sciences Research Council

Technology Strategy Board Driving Innovation



Inverse modelling

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Inverse problem: extracting source characteristics from a set of concentration measurements

1. Make a first guess of the source characteristics (Q, X_s, Y_s)
2. First guess → forward model → model-predicted concentrations
3. Model-predicted concentrations vs. measured concentrations → Minimisation algorithm → `best' estimate of source characteristics.
4. `Best' estimate → forward model → predicted plume.

Forward model

Forward model → model-predicted concentrations

Gaussian plume model - well known and understood

Inputs: source strength and position, wind speed and stability

We assume

- one continuous point source
- a ground level release, i.e. $Z_s = 0$
- concentration measurements at ground level

$$C = \frac{Q}{\pi u \sigma_Y \sigma_Z} \exp\left(\frac{-(Y - Y_s)^2}{2{\sigma_Y}^2} \right)$$

Optimisation

Minimise a cost function

$$J = \frac{1}{2}\sum_{i=1}^{N}\frac{\left(C_i^o - C_i^m\right)^2}{\sigma_i^{\ 2}}$$

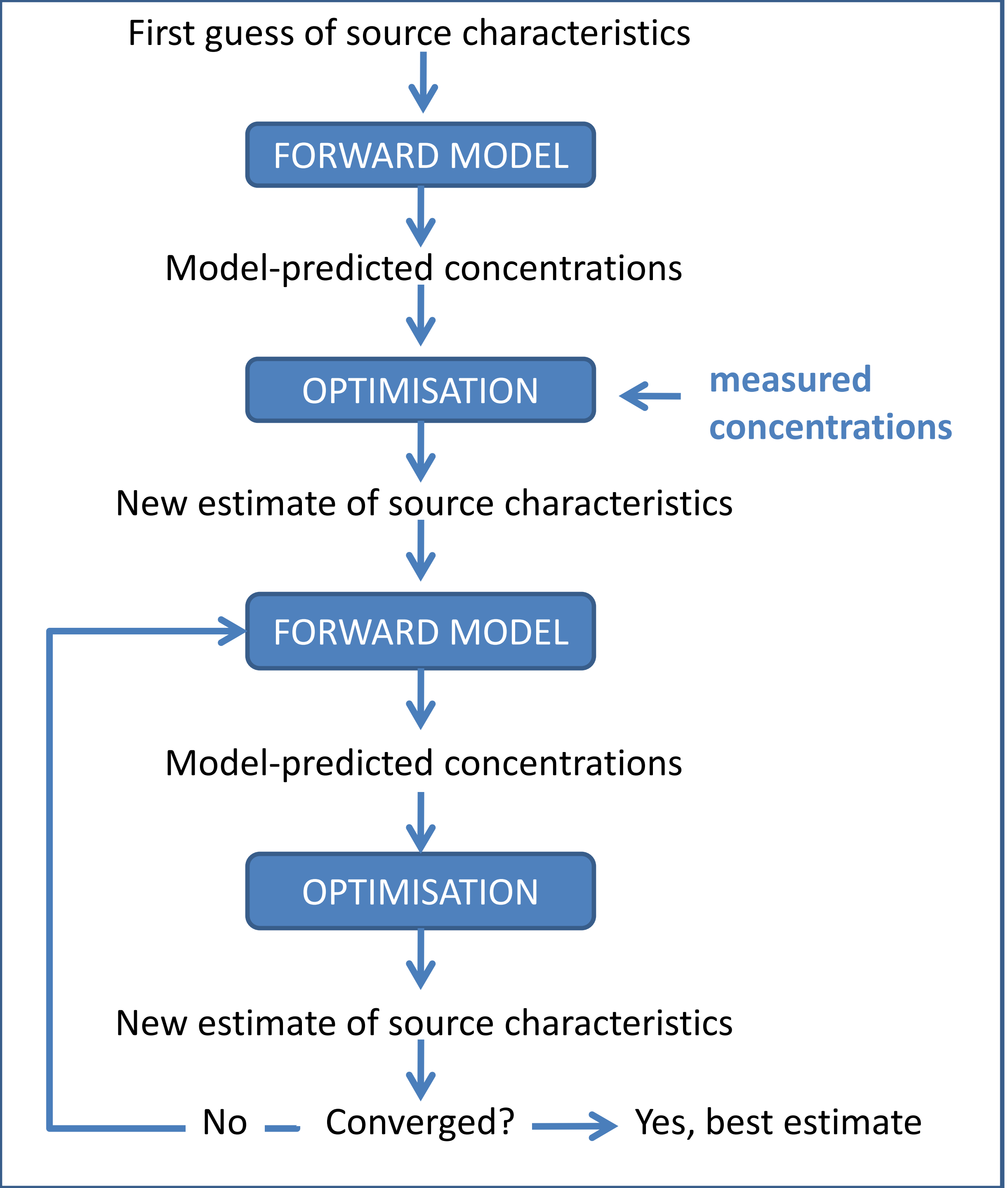
Concentration measurements C^o

Model-predicted concentrations C^m

Measures the discrepancy between the measured and model-predicted concentrations

Minimise *J,* which is the same as finding the values of the source characteristics for which the gradient of *J* is zero. This is your `best' estimate of the source characteristics.

Least squares fit plus error weighting which leads to an uncertainty estimate of the source characteristics.



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Need a rapid algorithm

Time is important in emergency situations

Estimate of uncertainty associated with the `best' estimate from second derivative of the forward model w.r.t the source characteristics

Sources of error

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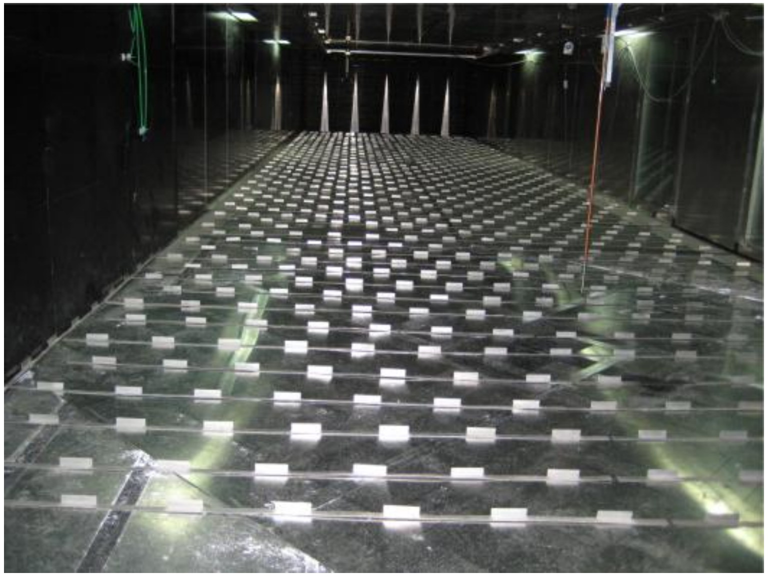
- **Measurement error** the accuracy of the concentration measurement from the sensor
  may be known

- **Model error** how good is the model at representing reality?
  can only estimate

- **Sampling error** this is dependent on the averaging time of the data due to the natural variability of the concentrations
  likely to dominate

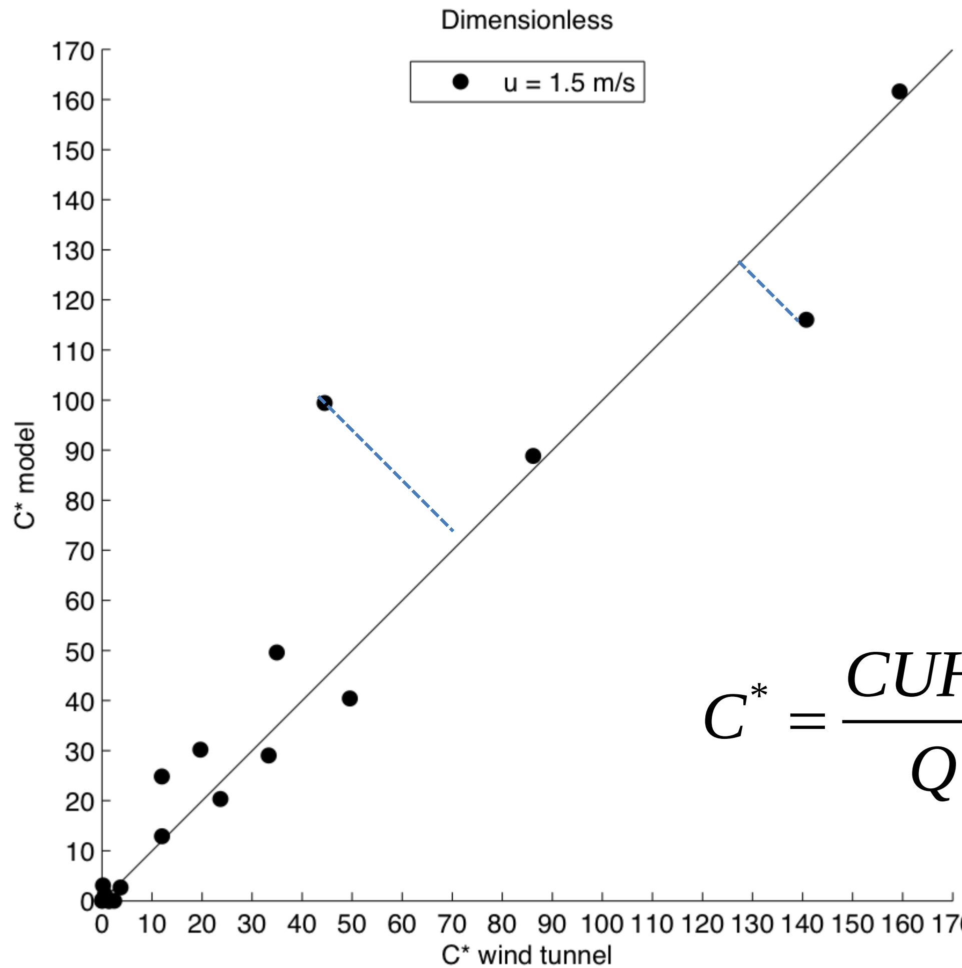
Could prevent the inverse algorithm from making a good estimate of the source characteristics

Wind tunnel data



Gaussian plume model tuned to the wind tunnel data

Difference due to model error and instrument error?



$$C^* = \frac{CUH^2}{Q}$$

Sampling error

How to quantify the sampling error associated with taking a short time average to estimate the true mean in a turbulent flow

Standard deviation of the shorter time mean estimate of the true mean concentration

$$\sigma_{\overline{C}^t} = \left(\frac{1}{n} \sum_{i=1}^{n} \left(\overline{C}_i^t - \overline{C}^T \right)^2 \right)^{\frac{1}{2}}$$

t is the shorter averaging time

T is the total time length

n is the n° of shorter averaging time samples

$\overline{C}_i^t$ = mean concentration averaged over time t

$\overline{C}^T$ = true mean concentration

Sampling error

$\frac{\sigma_{\overline{C}^t}}{\overline{\overline{C}}}$ = the uncertainty in the short time mean estimate compared to the true mean concentration

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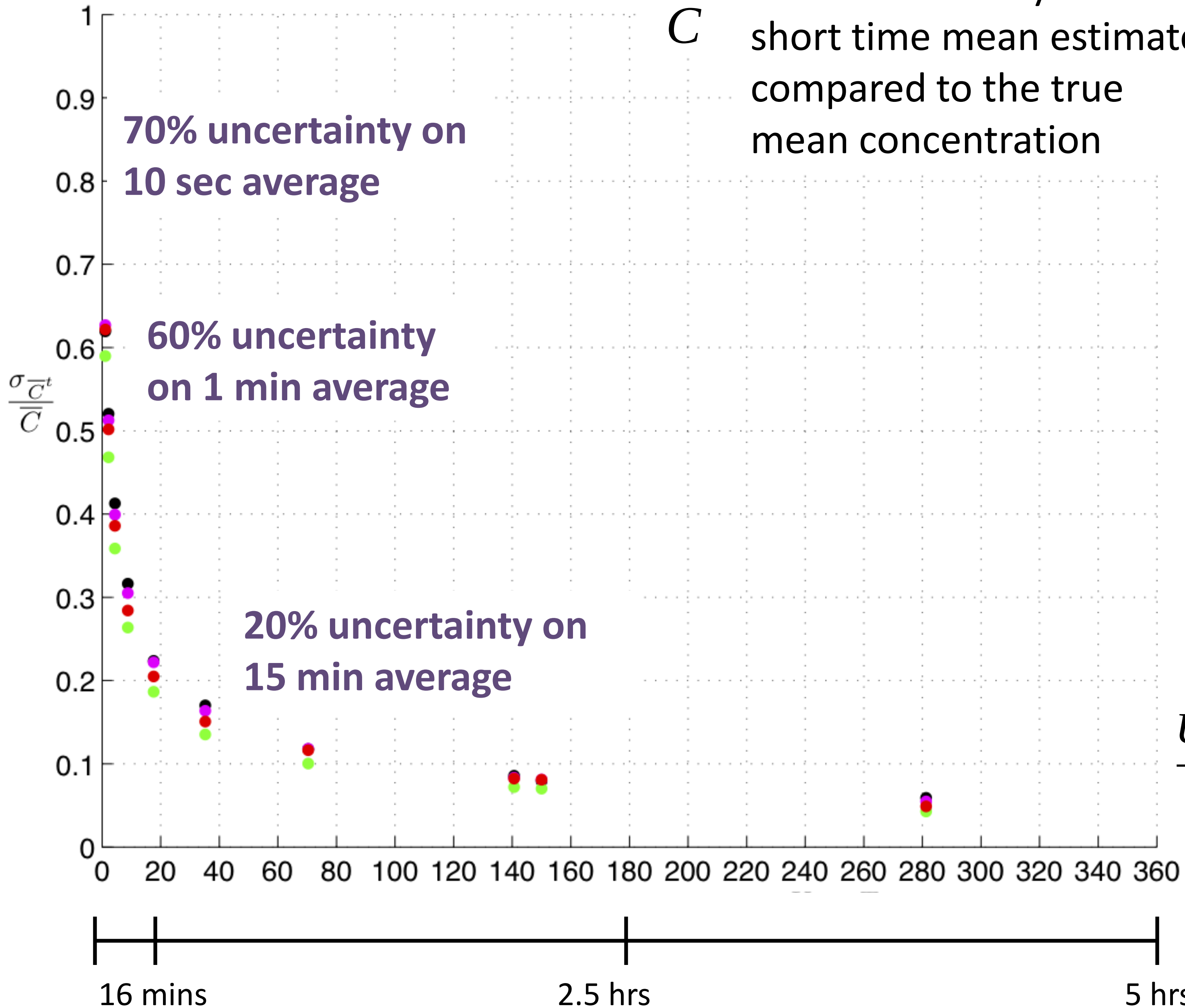


$\frac{U_{ref} T_{AV}}{H}$

Wind tunnel
U_{ref} =2.5 m/s
H = 1m

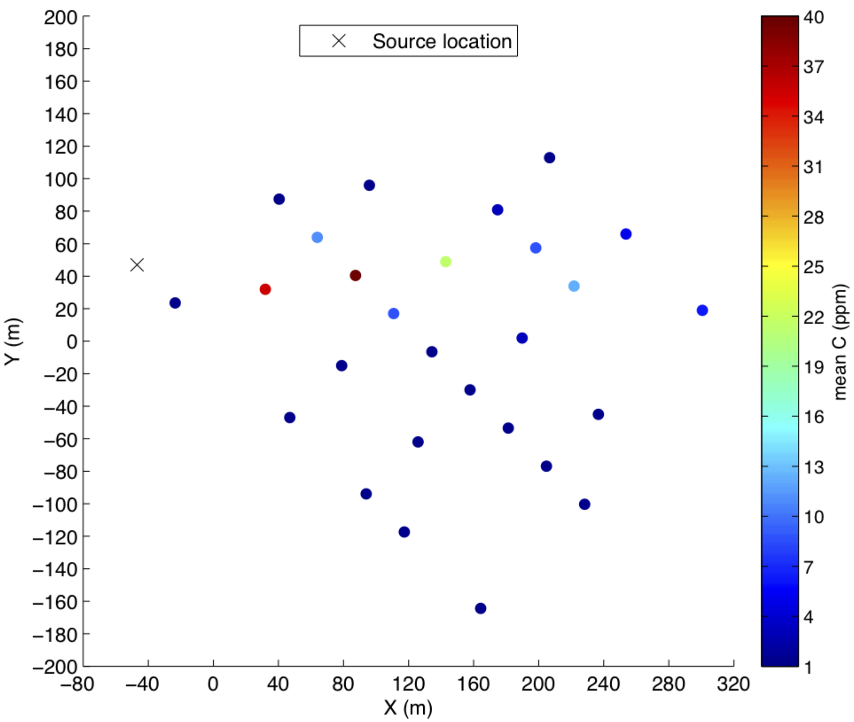
Equivalent full scale
U_{ref} =10 m/s
H = 500m

Inverse modelling - WT data

Source parameter	True value	First guess	units
Q	0.1	1	$m^3\ s^{-1}$
Xs	-47	-24	m
Ys	47	22	m

Source parameter	Estimate	Uncertainty	units
Q	0.075	0.002	$m^3\ s^{-1}$
Xs	-30.37	1.54	m
Ys	43.70	0.20	m

27 data points from wind tunnel data



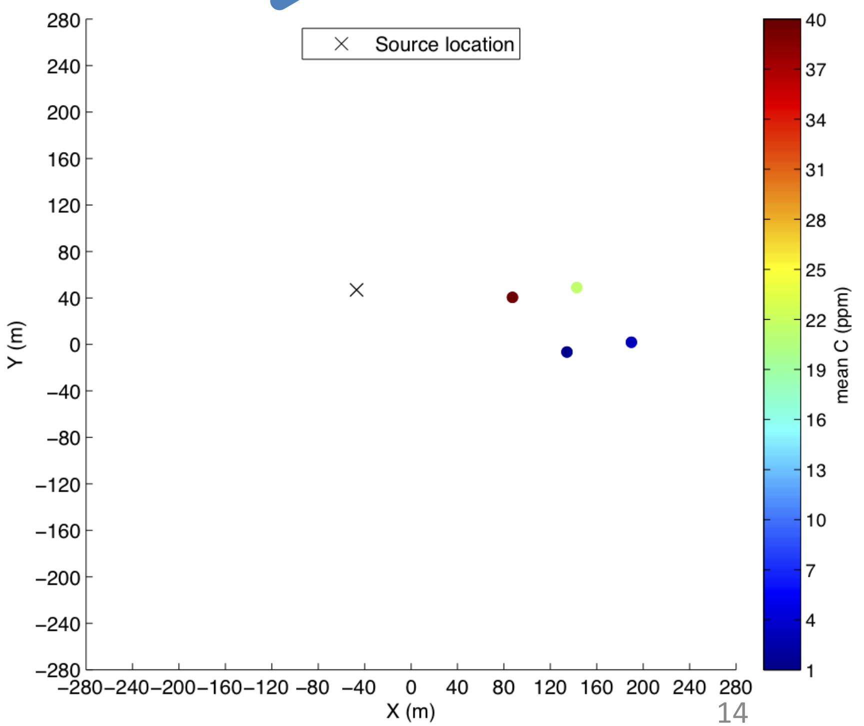
The true values of (Q, X_s, Y_s) do not lie within the uncertainty range of the estimates.

Inverse modelling - WT data

Source parameter	True value	First guess	units
Q	0.1	1	$m^3 s^{-1}$
Xs	-47	-24	m
Ys	47	22	m

Source parameter	Estimate	Uncertainty	units
Q	0.097	0.010	$m^3 s^{-1}$
Xs	-46.57	7.84	m
Ys	46.51	1.37	m

Sub set of 4 data points where the data values were accurately predicted by the Gaussian plume model



The true values of (Q, X_s, Y_s) lie within the uncertainty range of the estimates.

Conclusions

- Characterising the errors is essential for inverse modelling
  - can quantify the measurement error
  - can estimate the model error for the wind tunnel data
  - however, it is sampling error that appears to be the most important, it could potentially hamper the inverse algorithm from finding the `best' estimate.
- We have a method for estimating the uncertainty due to sampling error that can feed into the inverse algorithm – need to test it.
- Other studies we have done with synthetic data showed that measurements scattered about the plume in a square configuration lead to better estimates of the source characteristics because they contain direct information on the lateral spread of the plume.

Further work

- Test the inverse algorithm with a different forward model – the network model approach for urban dispersion.
- Use wind tunnel data collected using rectangular blocks to represent buildings in an urban area for validation.

Thank you for your attention