Point Source Reconstruction : Analyzing Localization Features of a Weighted Least-Squares Technique

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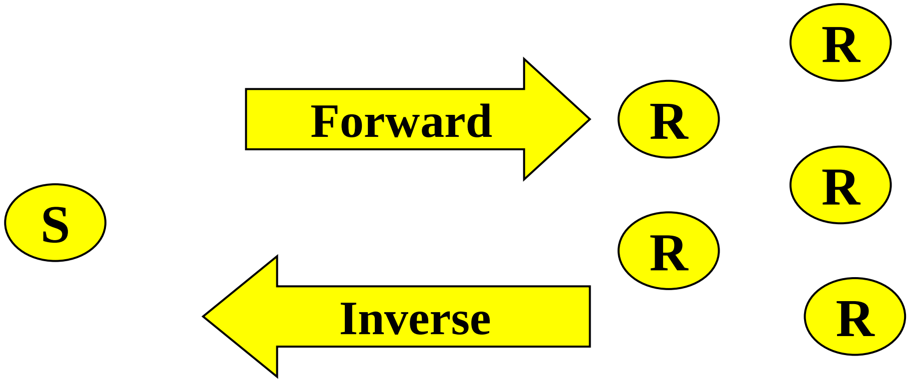
Motivation



- Importance in emergency actions and national security.
- Necessity of finding optimal solution to inverse problems.
- Localization features of the inverse solution.
- Resolution of the source estimates.

Objective

- To propose an inversion technique.
- To discuss optimal localization properties of the inverse solution.
- Evaluation using Fusion Field Trials 2007 dataset.

The presentation is focused
for
inversion of a continuous point release.

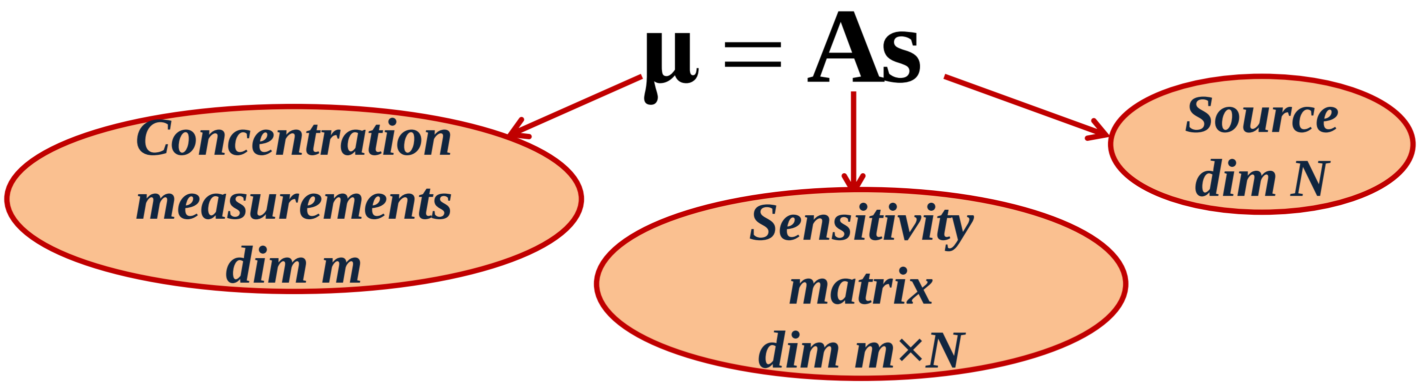
- Point source reconstruction : **Parametric estimation**
- **Unknowns**: location & strength

Approach

- **Source–receptor relationship**

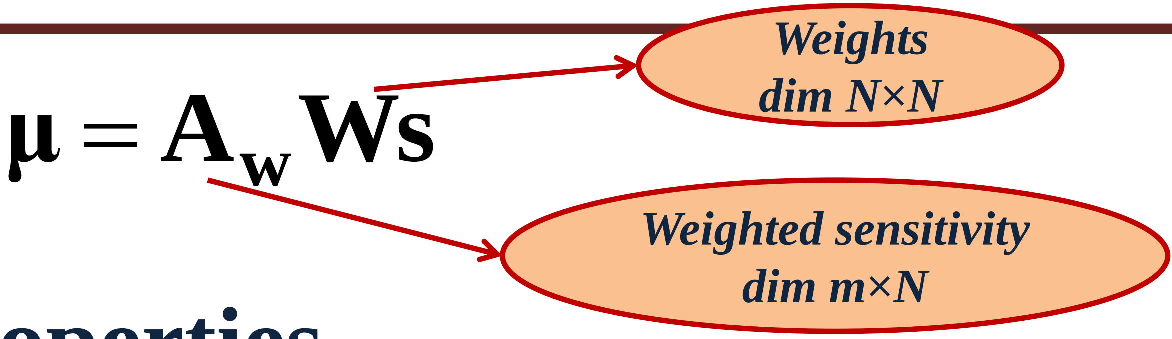
A discrete version of the source is retrieved in finite dimension.

For non-parametric source, m<<N



$$\boldsymbol{\mu} = \mathbf{A}\mathbf{s} \tag{1}$$

Weight matrix



$$\mathbf{\mu} = \mathbf{A_w W s} \tag{2}$$

Weights
dim N×N

Weighted sensitivity
dim m×N

where $\mathbf{A_w} = \mathbf{A W}^{-1}$

- **Properties**

⇨ Purely diagonal and $w_{j\,j} > 0, \quad for \;\; j = 1, \ldots, N$

⇨ $\sum_{j=1}^{N} w_{j\,j} = m$

⇨ Criterion for best choosing w is

$$\boxed{diag\left(\mathbf{A_w^T H_w^{-1} A_w}\right) \equiv 1} \tag{3}$$

W is computed by an iterative algorithm (Issartel, 2005).

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- Source estimate

$$\widehat{\mathbf{s}} = \overbrace{\mathbf{A}_{\mathbf{w}}^{\mathrm{T}}\mathbf{H}_{\mathbf{w}}^{-1}}^{\textit{dim}\ N\times m}\boldsymbol{\mu} \qquad \textit{where,}\ \ \overbrace{\mathbf{H}_{\mathbf{w}}}^{\textit{dim}\ m\times m} = \mathbf{A}_{\mathbf{w}}\mathbf{W}\mathbf{A}_{\mathbf{w}}^{\mathrm{T}} \qquad ④$$

- **In case of a point source of strength *q* located at $\mathbf{x}_0$**

$$\mathbf{s}(\mathbf{x}) = q\delta(\mathbf{x}-\mathbf{x}_0)$$

$$\boldsymbol{\mu} = qw(\mathbf{x}_0)\mathbf{a}_{\mathbf{w}}(\mathbf{x}_0)$$

The source estimate is,

$$\mathbf{s}(\mathbf{x}) = qw(\mathbf{x}_0)\overbrace{\mathbf{a}_{\mathbf{w}}(\mathbf{x}_0)^{\mathrm{T}}\mathbf{H}_{\mathbf{w}}^{-1}\mathbf{a}_{\mathbf{w}}(\mathbf{x})}^{<=1} \qquad ⑤$$

Maximum of the **s** coincides with the point source **location**.

*Now, **intensity** can be computed as,* $\widehat{q} = s(\mathbf{x}_0)/w(\mathbf{x}_0)$

Desired localization features

1. **Measurements should be well retrieved**

   data resolution matrix, ideally, identity m×m

2. **Source should be well retrieved in spite of :**

   information sparsity (limited number of meas.)

   N×N model resolution matrix, ideally, identity.

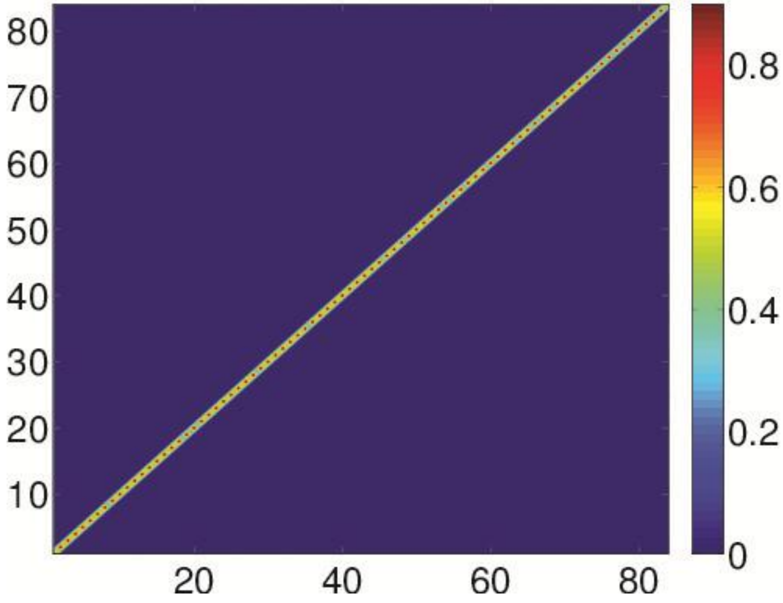
   information accuracy (errors in the measurements)

   Variance of the source, ideally, zero.

Measurements should be well retrieved.

- Data resolution matrix

$$
\begin{aligned}
\boldsymbol{\mu}^{retrieved} &= \mathbf{A}\mathbf{s}^{retrieved} \\
&= \overbrace{\mathbf{A}\mathbf{A}_{\mathbf{w}}^{\mathrm{T}}\mathbf{H}_{\mathbf{w}}^{-1}}^{\textit{dim } m\times m}\boldsymbol{\mu}^{true} \\
&= \underbrace{\mathbf{A}_{\mathbf{w}}\mathbf{W}\mathbf{A}_{\mathbf{w}}^{\mathrm{T}}}\mathbf{H}_{\mathbf{w}}^{-1}\boldsymbol{\mu}^{true} \\
&= \mathbf{H}_{\mathbf{w}}\mathbf{H}_{\mathbf{w}}^{-1}\boldsymbol{\mu}^{true} \\
&= \boldsymbol{\mu}^{true}
\end{aligned}
$$



Source should be well estimated from exact measurements

- Exact source retrieval is not feasible

$$\mathbf{s}^{retrieved} = \mathbf{A}_\mathbf{w}^\mathrm{T}\mathbf{H}_\mathbf{w}^{-1}\boldsymbol{\mu} = \underbrace{\mathbf{A}_\mathbf{w}^\mathrm{T}\mathbf{H}_\mathbf{w}^{-1}\mathbf{A}}_{\text{Projection matrix, dim } N\times N}\,\mathbf{s}^{true}$$

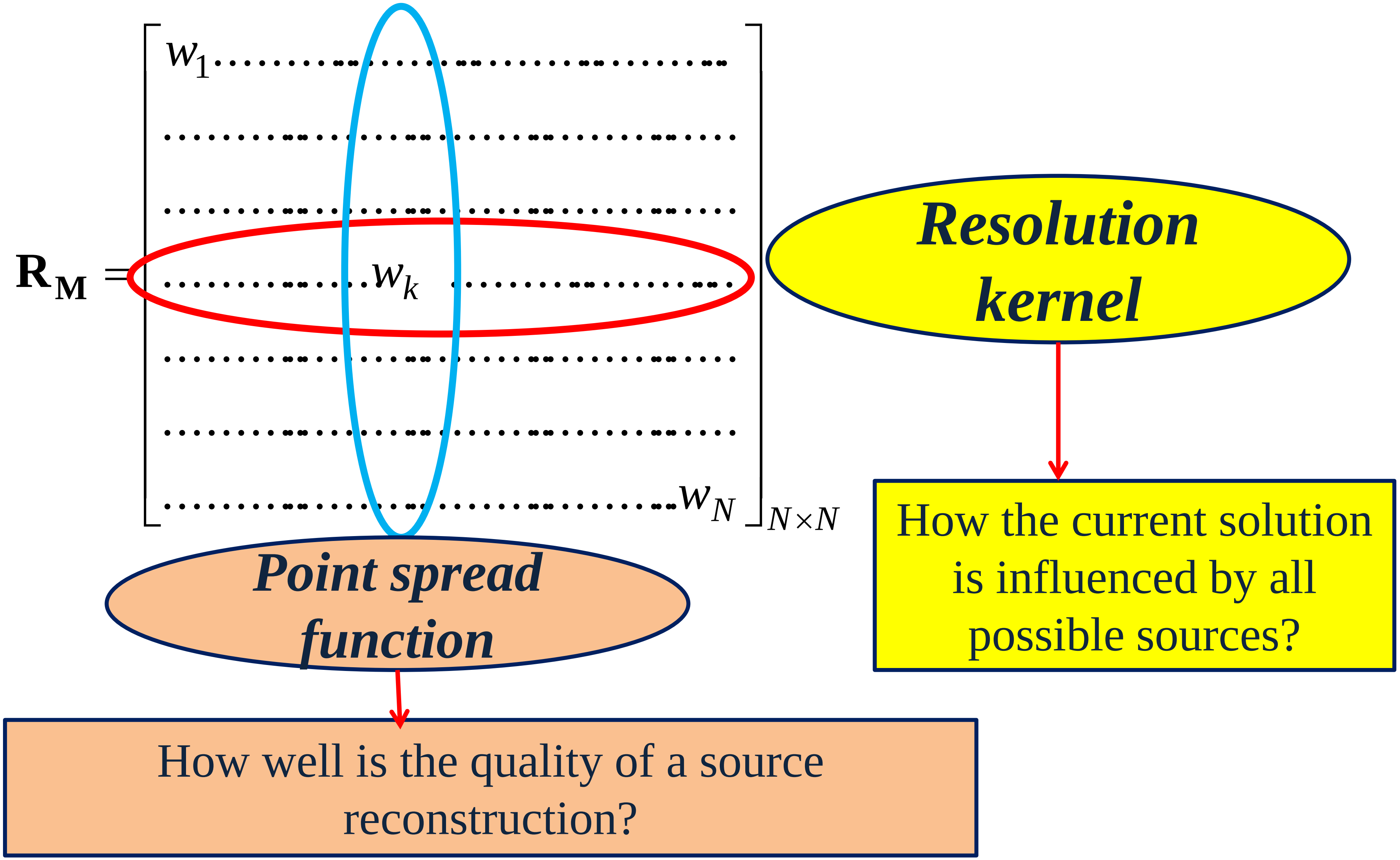
$$\mathbf{R}_\mathbf{M} = \mathbf{A}_\mathbf{w}^\mathrm{T}\,\mathbf{H}_\mathbf{w}^{-1}\mathbf{A} = \mathbf{A}_\mathbf{w}^\mathrm{T}\,\mathbf{H}_\mathbf{w}^{-1}\mathbf{A}_\mathbf{w}\mathbf{W}$$

dim N×N

$$diag(\mathbf{A}_\mathbf{w}^\mathrm{T}\,\mathbf{H}_\mathbf{w}^{-1}\mathbf{A}_\mathbf{w}) = 1$$

$$\boxed{diag(\mathbf{R}_\mathbf{M}) = diag(\mathbf{W})}$$

Continue : point source at x_k



Resolution kernel

How the current solution is influenced by all possible sources?

Point spread function

How well is the quality of a source reconstruction?

Source retrieval and measurement errors

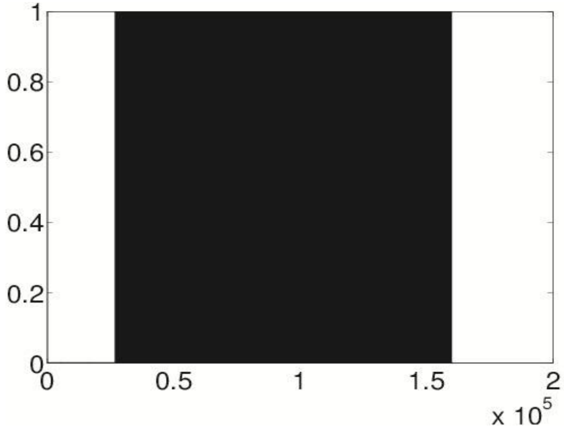
- covariance matrix of estimated source versus measurement errors cov matrix

$$\mathrm{cov}[\mathbf{s}^{retrieved}] = \mathbf{A}_\mathbf{w}^\mathbf{T}\mathbf{H}_\mathbf{w}^{-1}\ \mathrm{cov}[\boldsymbol{\mu}]\ \mathbf{H}_\mathbf{w}^{-1}\mathbf{A}_\mathbf{w}$$

- **Assumption** : measurement errors are mainly due to model and :

Least-squares cost function is written as: $J = (\boldsymbol{\mu} - \mathbf{A}_\mathbf{w}\mathbf{W}\mathbf{s})^\mathbf{T}\mathbf{H}_\mathbf{w}^{-1}(\boldsymbol{\mu} - \mathbf{A}_\mathbf{w}\mathbf{W}\mathbf{s})$

- Issartel et al. (2012) have shown that $\mathbf{H}_\mathbf{w}$ provides an optimal discrimination to the measurements.

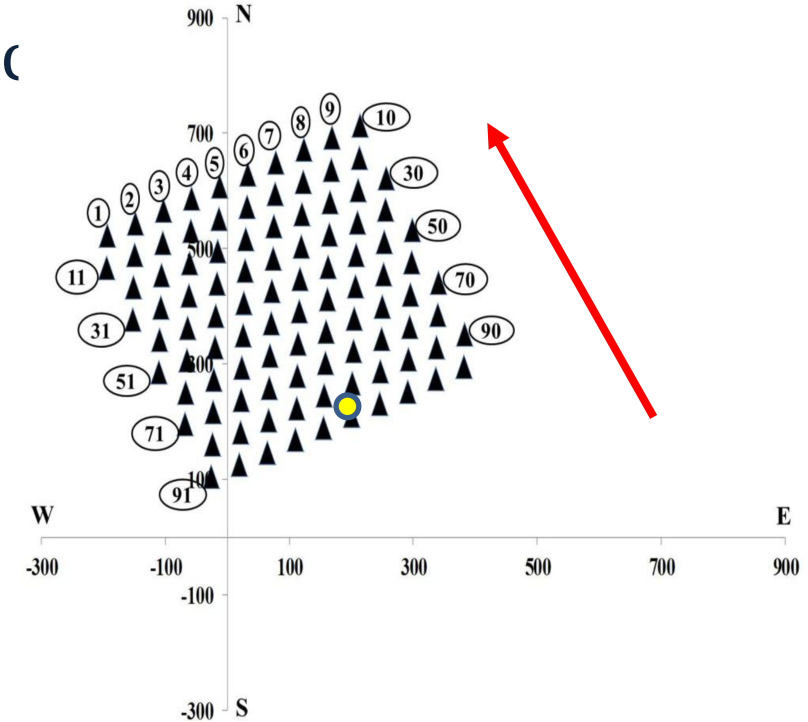


If $\mathrm{cov}[\boldsymbol{\mu}] = \mathbf{H}_\mathbf{w}$

Then $\mathrm{var}[\mathbf{s}^{retrieved}] = diag(\mathbf{A}_\mathbf{w}^\mathbf{T}\mathbf{H}_\mathbf{w}^{-1}\mathbf{A}_\mathbf{w}) = 1$

Fusion Field Trial 2007

- 10 min Propylene release in a flat terrain (Storwald, 2007)
- Source height = 2m
- 100 DGPID fast response conc
- Sampling height = 2m
- Trial 7
- North-West wind direction



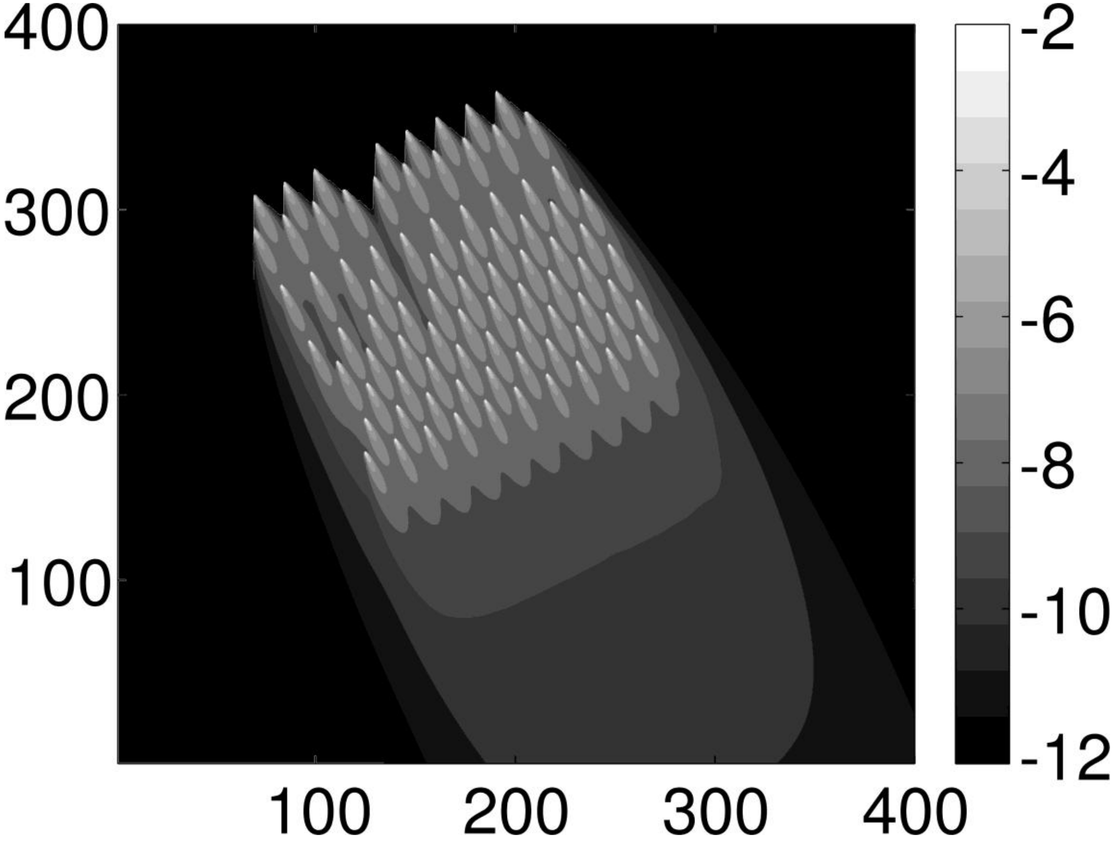
Computations

- Discretized domain, 1200 m × 1200 m.
- 400 grid points in each direction with uniform grid size 3m.
- 89 concentration measurements.
- Gaussian dispersion model (Sharan et al. 1996).
- Iterative computation of matrix **W**.
- True source (200, 200)
- True source strength = 5.53 g/s

Results

- A priori information apparent to monitoring network
- Well & poorly monitored regions are distinguished.

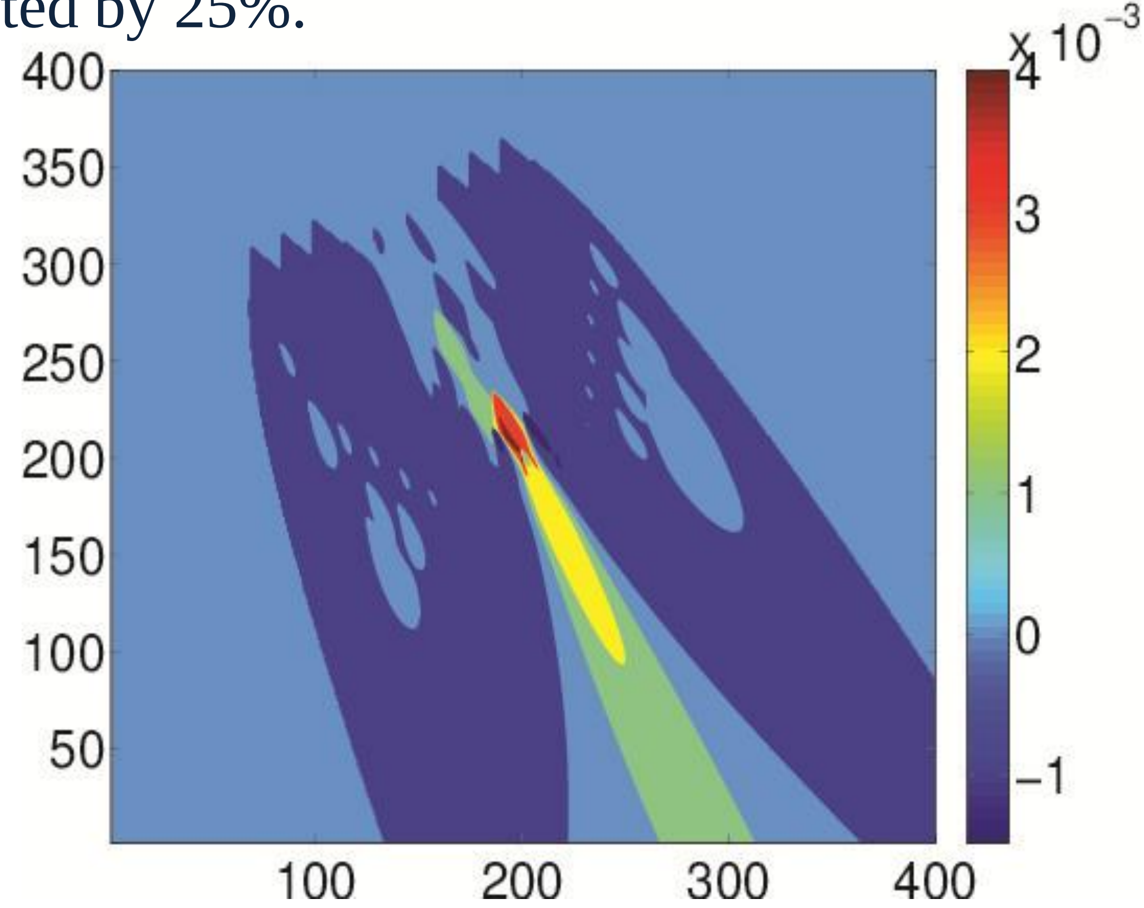
Distribution of weights



Retrieval

- Source is retrieved at a distance of 3m from true source.
- Strength is over-predicted by 25%.

Distribution of source information



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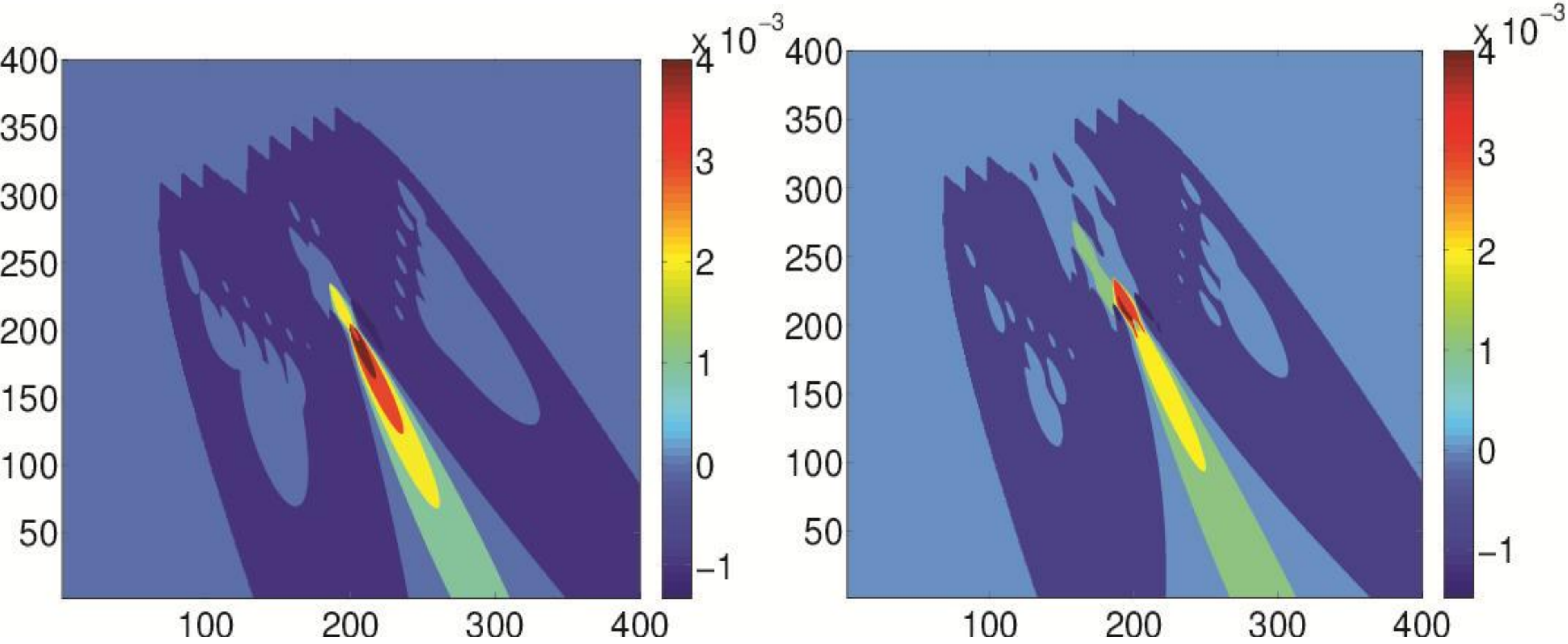

(a) Synthetic data

(b) Real data

(b) is the best resolution, achieved with limited measurements.

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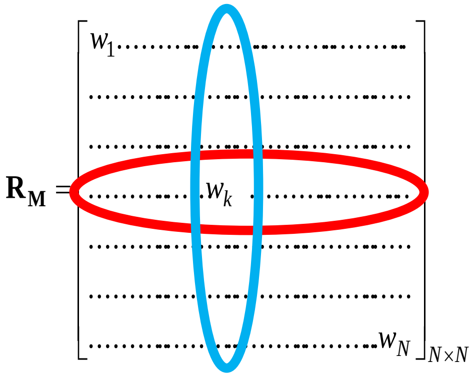
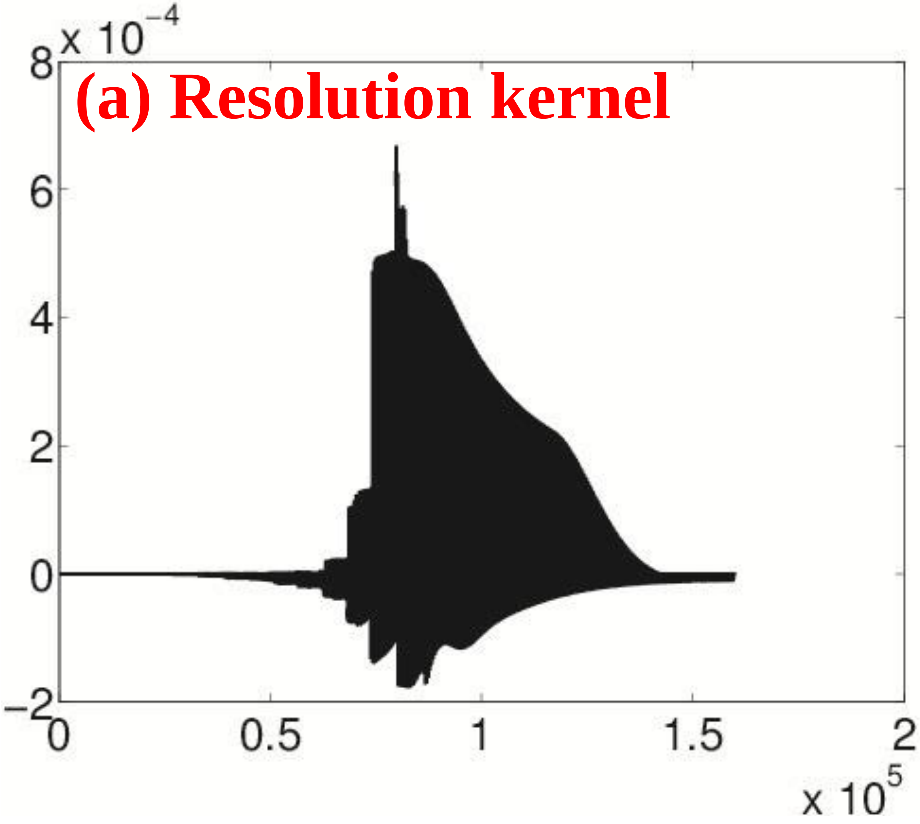
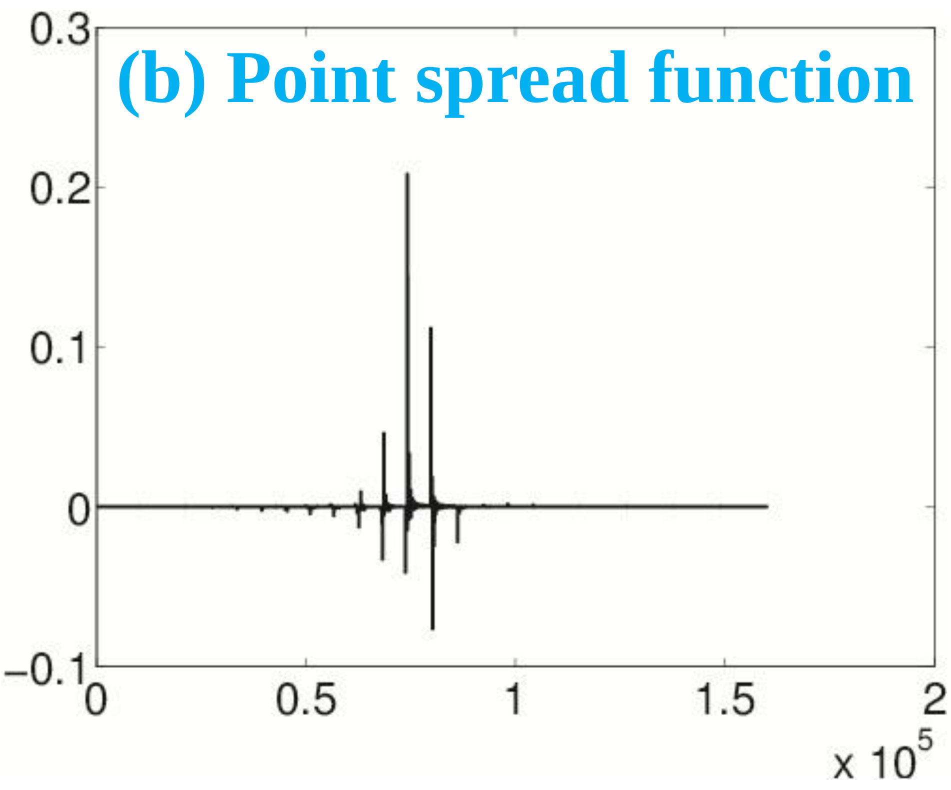
Model Resolution Matrix




(a) Resolution kernel


(b) Point spread function

Conclusions

- Inversion technique retrieve the point source close to the true source.
- Technique fullfills the proposed localization criteria.
- Source is observed to be distinctly located and lies in a highly illuminated region of the monitoring network.
- Sharpness of the resolution is maximum at the retrieved location of the point source.

Thank You for Your Kind Attention

References

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