A mixed spectral–similarity method for the estimation of diffusion parameters

O. ISNARD[1], R. MARTENS[2], M. MONFORT[1], H. SCHNADT[3]

1: Institut de Protection et de Sûreté Nucléaire, 60-68 av. du Général Leclerc, BP 6 - 92265 Fontenay aux Roses Cedex, France
2: Gesellschaft für Anlagen- und Reaktorsicherheit, Schwertnergasse 1, 50667 Cologne, Germany
3: Technischer Überwachungsverein Rheinland / Berlin-Brandenburg, e.V., 51101 Cologne, Germany

1 Introduction

Within the framework of nuclear emergency, the early estimate of radiological consequences in the environment is a key element towards the protection of neighbouring population and the organization crisis. Thus it is necessary to have atmospheric dispersion models adapted to these early estimates, the results being used to evaluate real or potential doses.
The possible risk of a nuclear accident that yield a release in the atmosphere (nuclear power plant, transport of radioactive materials) near a state border requires an harmonization of the protection measures the concern countries should take towards the public and the environment. The evaluation of dispersion models used in this case is necessary to settle this harmonization. The French–German Commission for Safety Problems of Nuclear Installation thus investigated on operational models (Gaussian puff dispersion in both case used in France, where the diffusion parameters are based on the transfer time, and in Germany, where the diffusion parameters are based on the distance. It was shown that there were some meteorological situations where the measures to be taken yielded significative differences. The establishment of a new common puff-dispersion model (CRABOL *et al.*, 1999) was then decided.

2 Dispersion parameters modelisation

The use of Gaussian dispersion algorithms require the estimation of horizontal and vertical growth of each puff. The horizontal and vertical growth of puff are generally expressed in terms of standard deviations of concentration in lateral (σ_y) and vertical (σ_z) directions.

The modelisation of σ_y uses the separation of the diffusion process suggested by MONIN and YAGLOM (1975):

$$\sigma_y^2 = \sigma_{yr}^2 + \sigma_{yc}^2 \qquad (1)$$

where σ_{yc} represents the absolute diffusion (i.e. the diffusion of the center of mass) and σ_{yr} represents the relative dispersion in the puff. We know (BATCHELOR, 1952) that the relative diffusion is an accelerative process. As two particles in a puff separate, the range of eddy sizes contributing to the relative velocity difference will increase. Using the classical Taylor formula for absolute dispersion at small travel time show that the whole spectrum of turbulence contributes to dispersion. At large travel time, only the lowest frequencies contribute to the expansion. This suggest that a turbulence filter related to the actual puff size act in relative diffusion. This turbulence filter will progressively incorporate larger fraction of the spectral energy in the puff expansion process. Following SMITH and HAY (1961) suggestions about the relative displacement of the particles within a puff we can write:

$$\frac{d}{dt}\sigma_{yr}^2 = \frac{2}{3}\int_0^t \overline{v'(t)v'(t+\tau)}\,d\tau \tag{2}$$

The overbar denotes an averaging over all marked fluid particles, v' represents the Lagrangian velocity fluctuation. We can also write the Lagrangian correlation expressed in equation (2) as the spatial auto–correlation function $R(s)$:

$$\overline{v'(t)v'(t+\tau)} = R\left(\frac{U}{\beta}(t-\tau)\right) \tag{3}$$

Where U is a velocity scale and β is the Lagrangian to Eulerian time scale. Thus, replacing the expression of the auto–correlation function by its Fourier formulation, (2) become:

$$\frac{d}{dt}\sigma_{yr}^2 = \frac{4}{3}\frac{\beta}{U}\int_0^\infty \int_0^{Ut/\beta} E(k)\,\mathrm{sinc}(ks)\left(1-e^{-\sigma_{yr}^2 k^2}\right) ds dk \tag{4}$$

The Taylor frozen hypothesis allow us to use,

$$k = \frac{2\pi n}{U},\;\; E(k) = \frac{U}{2\pi}F_E(n),\;\; \text{and}\;\; s = \frac{U\tau}{\beta}$$

Finally we get:

$$\frac{d}{dt}\sigma_{yr}^2 = \frac{4}{3}\int_0^\infty \int_0^t F_E(n)\,\mathrm{sinc}\left(\frac{2\pi n\tau}{\beta}\right)\left(1-e^{-(2\pi n t_r)^2}\right) d\tau\,\mathrm{dn} \tag{5}$$

Where t_r is a time scale ($t_r = \sigma_{yr}/U$) and $F_E(n)$ the 3-D Eulerian turbulence spectra with the hypothesis of a stationary isotropic turbulence. We can see from (5) that the function $(1-\exp(-(2\pi n\tau)))$ is the filter related to the puff size.

The evolution of σ_{yc} can be calculated from statistical considerations. If we assume that the lateral outlines of an instantaneous plume can be represented by the trajectories of two particles released simultaneously, then noting y_i the particle deviation due to the fluctuation v'_i we have:

$$\frac{d}{dt}\overline{y_i^2} = 2\int_0^t \overline{v'_i(t)v'_i(t+\tau)}\,d\tau \tag{6}$$

This formula implies that the lateral wind velocity component has been modified by three filter functions. A high–pass filter with a cutoff frequency of t_s^{-1} (sampling time scale) is applied to reduce v_i to v'_i. Then the wind velocity components with scales smaller than the size of the instantaneous plume width are filtered out through averaging of $v'_i(t)$ and $v'_i(t+\tau)$. Integrating from 0 to t acts as a low–pass filter with a cutoff frequency of t^{-1}. Since the transfer functions for these filtering process are available (GIFFORD, 1968 ; SMITH and HAY, 1961), the variance σ_{yc} can be constructed with these functions, i.e,

$$\sigma_{yc}^2 = t^2 \int_0^\infty F_{ev}(n)\,\mathrm{sinc}^2\left(\frac{\pi n t}{\beta}\right)\left(1-\mathrm{sinc}^2(\pi n t_s)\right) e^{-(2\pi n t_r)^2}\,dn \tag{7}$$

where $F_{ev}(n)$ is the monodimensional lateral wind velocity spectrum. The term $t^2\mathrm{sinc}^2(\pi n t_s/\beta)$ represents the integration over the past history of the plume, and effectively damps the contribution of wind components with periods smaller than $t/2\beta$. Temporal averaging results in the transfer function $(1-\mathrm{sinc}^2(\pi n t_s))$, which reduces the contribution of wind components with period larger than $t_s/2$. The separation of relative diffusion from single–particle diffusion results from the filter transfer function $\exp(-(2\pi n t_r)^2)$ given by SMITH and HAY (1961), which reduce the contribution of wind components with scales smaller than $t_r/2$.
If $F_E(n)$ and $F_e(n)$ are specified then equations (5) and (7) can be numerically integrated and the instantaneous puff dimension is available at each time step.

In the vertical direction, we use the well known empirical formula of HANNA and STRIMAITIS (1990):

$$\sigma_z = \frac{\sigma_w t}{\sqrt{1+\dfrac{t}{2T_{Lw}}}} \tag{8}$$

Where σ_w is the vertical deviation of the wind, t is the transfer time and T_{Lw} is the vertical Lagrangian time scale. Variations of these last two parameters with z is done with the parameterization proposed by CARRUTHERS (1992). This parameterization is suitable in the Atmospheric Boundary Layer (i.e $z \leqslant h$, where h is the boundary layer depth). There are three sets of formulae depending on whether the meteorological conditions correspond to unstable conditions ($h/L \leqslant -0.3$, L is the Monin-Obukhov length), near neutral flow ($-0.3 < h/L \leqslant 1$) and stable conditions ($h/L > 1$). We also define two terms corresponding to the contributions from convectively driven and mechanically driven turbulence respectively:

$$\begin{cases} T_{wC}(z) = 2.1\left(\dfrac{z}{h}\right)^{1/3}\left(1-0.8\dfrac{z}{h}\right) \\ T_{wN}(z) = 1-0.8\dfrac{z}{h} \end{cases} \tag{9}$$

For unstable conditions the mixed layer velocity scale is given by:

$$w_*^3 = \frac{h u_*^3}{\kappa |L|} \tag{10}$$

Where u_* is the friction velocity. The vertical wind variance is given by:

$$\sigma_w^2 = 0.4 w_*^2 T_{wC}^2(z) + (1.3 u_* T_{wN}(z))^2 \tag{11}$$

For near neutral flow we have,

$$\sigma_w = 1.3 u_* T_{wN}(z) \tag{12}$$

and in stable conditions,

$$\sigma_w = 1.3u_* \left(1 - \alpha_s \frac{z}{h}\right)^{3/4} \tag{13}$$

where α_s is used to represent the changing characteristics depending on whether conditions are ideal (no waves or instabilities) or disturbed. We also assume that the vertical length scale $\Gamma_w(z)$ is determined both by local shear and the blocking effect of the surface, and that it is limited by the boundary layer depth so that over flat terrain:

$$\Gamma_w(z) = \left(\frac{0.6}{z+z_0} + \frac{1}{\sigma_w}\frac{\partial U(z)}{\partial z} + \frac{4}{h}\right)^{-1} \tag{14}$$

Then, the Lagrangian time scale is defined by:

$$T_{Lw} = \frac{\Gamma_w(z)}{\sigma_w(z)} \tag{15}$$

3 Results

The model performance has been evaluated against several experiments (MONFORT *et al.*, 2001). We present here the performance of the model against concentrations from dispersion experiments carried out in the town of Lillestrøm, Norway in 1987 (HAUGSBAKK and TØNNESEN, 1989). These experiments were carried out in a flat residential area where the roughness length was about $0.5\,m$. SF_6 was released from a $36\,m$ mast and sonic anemometer measurements were processed along the mast.

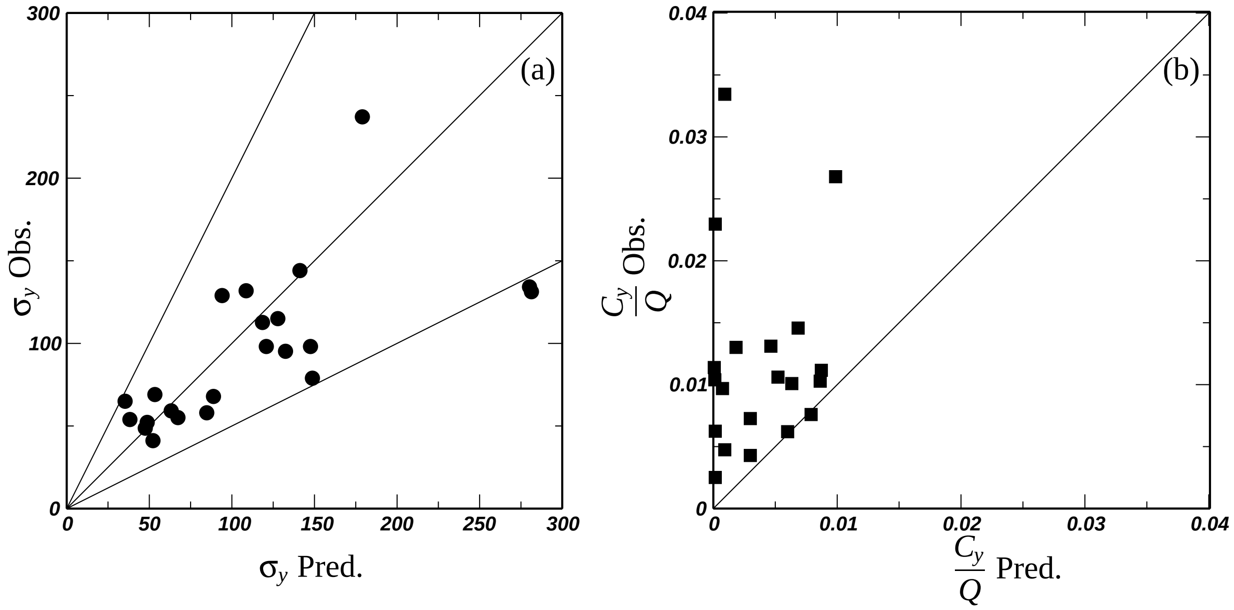

Figure 1: Scatter plots of the observed (Obs.) and predicted (Pred.): (a) σ_y, (b) C_y/Q (Cross wind integrated concentration normalized by emission)

The simulated results for the Lillestrøm experiment show, regarding the difficult dispersion regime, a good correspondence for the horizontal dispersion parameter (σ_y). Figure 1(a) shows the comparison of the predicted and observed σ_y and only two predicted values differ slightly from the observation by more than a factor of 2 (experiments 5 and 6 for the last arc distance, where the wind velocity was light i.e $< 1\,m/s$). We can see on figure 1(b) that the integrated cross-wind concentration is underpredicted by the model. The differences between the observed and predicted values are, mainly within a factor of 2.

4 Conclusion

In the present paper, a mixed-similarity approach for the estimation of dispersion parameters is presented which aims at using a common approach between two countries in case of a nuclear emergency. This parameterization can provide reasonable results against dispersion experiments, specially under light wind.

5 References

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