CLASSIC PAPERS ON DISPERSION IN THE SURFACE LAYER

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Abstract: This paper reviews two papers by A.P. van Ulden and F. Nieuwstadt that laid the foundations of currently used dispersion models for surface releases. Nieuwstadt and van Ulden (1978) showed, for the first time, that the numerical solution of the diffusion equation, based on the eddy diffusivity for heat, provides estimates of cross-wind integrated concentrations that compare remarkably well with SO_2 concentrations measured during the Prairie Grass experiment (Barad, 1958). The solution also showed that the concentration distribution in the vertical is not Gaussian during neutral and unstable conditions; the exponent, which is 2 for the Gaussian profile, is close to 1 during neutral conditions and approaches 0.5 during very unstable conditions. In a companion paper, van Ulden (1978) provided an approximate analytical solution to the diffusion equation that also produced results that agreed well with the Prairie Grass data. Results from this paper led to the formulation of dispersion models for surface releases that are included in currently used regulatory models such as AERMOD and ADMS.

Key words: *Surface layer dispersion, van Ulden, Nieuwstadt, Diffusion equation*

INTRODUCTION

There is renewed interest in dispersion in the surface layer because of recent concerns of the impacts of vehicle emissions on the health of people living close to roads. This motivates our review of the contributions made by Nieuwstadt and van Ulden (1978), and van Ulden (1978) to our understanding of dispersion in the surface boundary layer. We first present the major results from these papers and then show how they have led to the formulations that are used in currently used regulatory models such as AERMOD and ADMS.

NIEUWSTADT AND VAN ULDEN (1978)

Nieuwstadt and van Ulden (1978) showed for the first time that the numerical solution of the diffusion equation provides realistic estimates of concentration distributions associated with a near ground-level release. They solved the equation for the cross-wind integrated concentration, C,

$$U\frac{\partial C}{\partial x}=\frac{\partial}{\partial z}\left(K\frac{\partial C}{\partial z}\right), \tag{1}$$

where the eddy diffusivity, K, corresponded to that for the surface heat flux over a horizontally homogeneous surface, expressed in terms of the surface friction velocity, the Monin-Obukhov length, and the roughness length (Businger et al., 1971). The predicted surface and vertical profiles of cross-wind integrated concentrations compare well with SO_2 concentrations measured during the Prairie Grass experiment (Barad, 1958). Figure 1 reproduces their results for surface concentrations assuming a deposition velocity of $0.07u_*$, suggested by Gryning et al. (1983).

Their paper extended earlier results that indicated the concentration distribution in the vertical is not Gaussian during neutral and unstable conditions; the exponent, which is 2 for the Gaussian profile, is close to 1 during neutral conditions and approaches 0.5 during very unstable conditions. If we express the cross-wind integrated concentration as $C(x,z)=C(x,0)\left(-\beta z^{p}\right)$, the variation of p with stability predicted by the numerical model is shown in Figure 2. Notice that p, which is taken to be 2 in Gaussian models, varies both with stability as well as distance from the source. This result has not yet found its way into regulatory models primarily because regulations focus on ground-level concentrations.

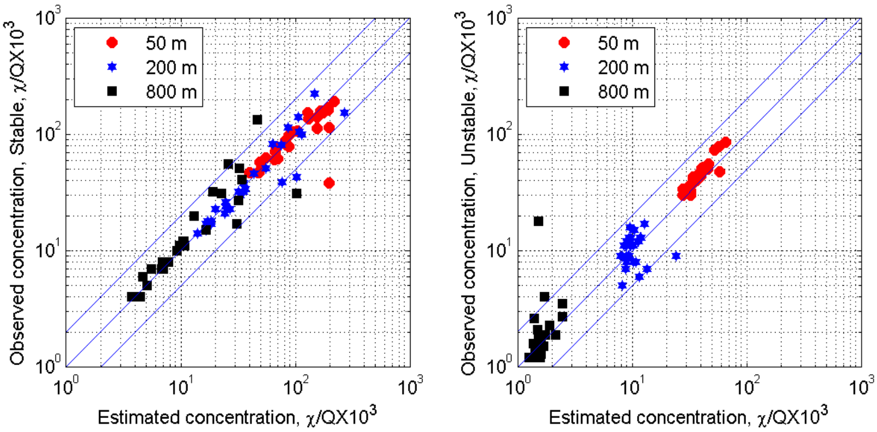

Figure 1. Comparison of numerical solution of diffusion equation with measurements from Prairie Grass

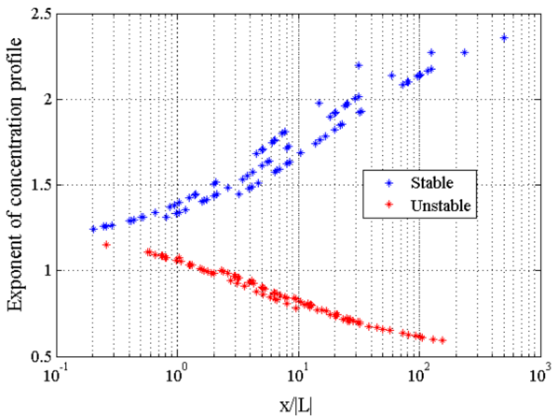

Figure 2. Variation of exponent of vertical concentration profile with stability

van Ulden (1978)

van Ulden made the diffusion solution more accessible to the modeling community by proposing an approximate analytical solution using a technique pioneered by Chaudhry and Meroney (1973). The diffusion equation does not allow an analytical solution for the Monin-Obukov (MO) similarity forms for the eddy diffusivity, K, and the mean wind, U. So the approach consists of first assuming forms that allow an analytical solution. The solution is then expressed in terms of functions of K and U, which are written in terms of their original MO variables. In effect, we have an approximate solution expressed in terms of K and U that did not allow an analytical solution. The error in the solution is estimated a posteriori when it compared with that of the numerical solution.

Following Pasquill (1974), van Ulden first expressed *U(z)* and *K(z)* as powers of z,

$$U(z) = U_1 z^m \qquad K(z) = K_1 z^n . \tag{2}$$

This allows the following analytical solution of Equation (1):

$$\frac{C}{Q} = \frac{p}{U_1 \Gamma(s)} \left(\frac{b}{x}\right)^s \exp\left(-\frac{bz^p}{x}\right), \tag{3}$$

where b, p, and s are functions of m and n, and $\Gamma(s)$ is the gamma function.

van Ulden (1978) then expresses this solution, Equation (3), in terms of the mean wind speed, $\overline{U}$, and mean plume height, $\overline{z}$, defined by

$$\overline{U} = \frac{\int_0^\infty U(z)C(z)dz}{\int_0^\infty C(z)dz} \quad \overline{z} = \frac{\int_0^\infty zC(z)dz}{\int_0^\infty C(z)dz}. \tag{4}$$

as

$$\frac{C}{Q} = \frac{S}{\overline{U}\overline{z}} \exp\left(-\left(\frac{Bz}{\overline{z}}\right)^p\right), \tag{5}$$

where S and B are functions of $p = m - n + 2$. This solution becomes useful with the accompanying equation for $\overline{z}$ obtained by differentiating the expression in Equation (4)

$$\frac{d\overline{z}}{dx} = \frac{K(q\overline{z})}{U(q\overline{z})q\overline{z}} \quad q = \left(B^p p\right)^{\frac{1}{1-p}}. \tag{6}$$

van Ulden (1978) then derives approximate relationships for $U(\overline{z})$, which when substituted in Equation (6) allows him to integrate Equation (6) to obtain useful formulas that allow computation of $\overline{z}$ given z_0 and x/L. Gryning et al. (1983) extended the solution to allow prediction of the vertical concentration distribution by deriving analytical expressions for the exponent, p, in Equation (5). They started with $m = z/U\ (dU/dz), n = z/K\ (dK/dz)$, and $p = m - n + 2$, suggested by the the power law forms of K and U. They substituted the actual MO forms in the expressions for m and n, to derive formulas for p that provide estimates that compare well with the numerical values shown in Figure 2.

This set of formulas then represent a complete analytical formulation for the cross-wind integrated concentration, which can be used to estimate concentrations associated with line sources (Venkatram et al., 2007).

SOME CONSEQUENCES

Although van Ulden's solution avoids cumbersome numerical computation, it still requires the solution of an implicit equation for plume spread, which cannot be readily incorporated into practical dispersion models. Briggs (1982), Venkatram (1982), Venkatram et al. (2013) and others converted the solution to explicit expressions for plume spread that could be inserted into the framework of practical dispersion models, such as AERMOD (Cimorelli et al., 2004). Equation (6), derived by van Ulden, the basis of the formulations derived by Venkatram (1982), can be recast as

$$U(\sigma_z)\frac{d\sigma_z^2}{dx} \square \tag{7}$$

This equation can be used to derive analytical expressions for σ_z in the asymptotic limits of neutral, stable, and unstable surface layers. We illustrate the application of Equation (7) to the neutral boundary layer, for which $K_z \sim u_* z$, and Equation (7) implies

$$\frac{d\sigma_z}{dx} \sim \frac{u_*}{U(\sigma_z)}. \tag{8}$$

Evaluating the effective wind speed at σ_z

$$U(\sigma_z) \sim u_* \ln\left(\frac{\sigma_z}{z_0}\right), \tag{9}$$

and integrating Equation (8) yields

$$\sigma_z \left[\frac{U}{u_*} - 1 \right] + z_0 \sim x \tag{10}$$

Because u_* is usually a small fraction of U except at small distances from the source, Equation (10) can be approximated by

$$\sigma_z U \sim u_* x \tag{11}$$

Then, cross-wind integrated concentration, relevant to long line sources of pollution such as roads, can be written as

$$\overline{C}^y \sim \frac{Q}{u_* x} \tag{12}$$

This result, which has been derived using other methods by van Ulden (1978) and Briggs (1982), implies that the concentration of an inert pollutant emitted from a line source, such as a road, falls off linearly with distance from the source.

The reformulation of the horizontal spread equations is based on results obtained by Eckman (1994), who showed that the variation of σ_y with distance, the initial linear increase followed by a smaller increase with distance (or travel time) could be explained by the increase of the wind speed with height if one assumed that σ_y is governed by the expression

$$\frac{d\sigma_y}{dx} = \frac{\sigma_v}{U} \tag{13}$$

where σ_v is the standard deviation of the horizontal velocity fluctuations, and the transport wind speed, U, is evaluated at $\bar{z}$. Using arguments similar to those used to derive expressions for σ_z we can derive asymptotic expressions for σ_y, which can then be patched together to obtain expressions for the entire range of stabilities.

Then, the plume spread equations with the empirical constants that provide the best fit between model estimates and observations become, for stable conditions:

$$\sigma_z = 0.57 \frac{u_*}{U} x \left(1 + 3 \frac{u_*}{U} \left(\frac{x}{L} \right)^{2/3} \right)^{-1} \tag{14a}$$

$$\sigma_y = 1.6 \frac{\sigma_v}{u_*} \sigma_z \left(1 + 2.5 \frac{\sigma_z}{L} \right) \tag{14b}$$

and the semi-empirical formulations for unstable conditions are

$$\sigma_z = 0.57 \frac{u_*}{U} x \left(1 + 1.5 \left(\frac{u_*}{U} \frac{x}{|L|} \right) \right) \tag{15a}$$

$$\sigma_y = 1.6 \frac{\sigma_v}{u_*} \sigma_z \left(1 + \frac{\sigma_z}{|L|} \right)^{-1/2} \tag{15b}$$

These expressions for plume spread are implicit because the wind speed, U, on the right hand side of the equation is a function of $\bar{z}$, which in turn is a function of σ_z .Figure 3 shows the performance of these equations in describing concentrations measured during the Prairie Grass experiment.

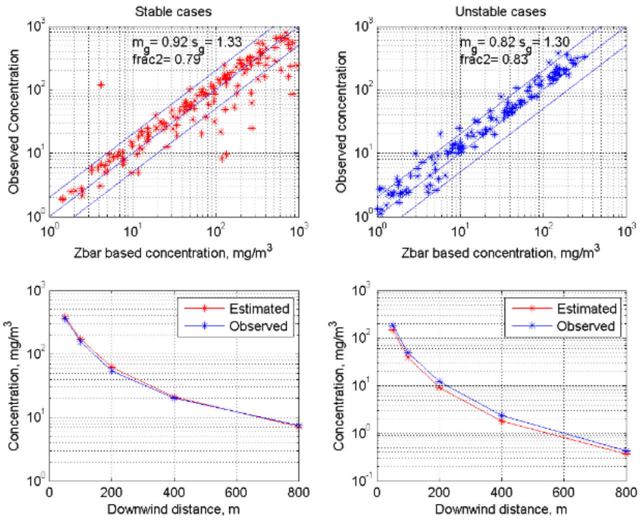

Figure 3. Comparison of maximum concentration measurements made during the Prairie Grass Experiment at 50m, 100m, 200m, 500m, and 800m from the release with model estimates based on Equations (14) and (15). Bottom panel shows the variation of the average of the maximums measured on each sampling arc.

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