LAGRANGIAN TIME SCALES OF THE TURBULENCE ABOVE TWO-DIMENSIONAL CANOPIES

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1. Introduction
2. Motivations and goals
3. Theoretical framework
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1. INTRODUCTION – Physical phenomenon

\[ \bar{u}(z) = \frac{u_*}{k} \ln \left( \frac{z}{z_0} \right) \]

\[ \bar{u}(z) = \frac{u_*}{k} \ln \left( \frac{z-d_0}{z_0} \right) \]

- Mixing Layer
- Constant Flux Layer
- Roughness Sublayer
- Urban Canopy Layer

\[ u_* = \sqrt{\frac{\tau}{\rho}} \quad \text{friction velocity} \]

\[ k = 0.4 \quad \text{Von Kàrmàn constant} \]

\[ z_0 \quad \text{aerodynamic roughness length} \]

\[ d_0 \quad \text{displacement height} \]

Lagrangian time scales of the turbulence above two-dimensional canopies
1. INTRODUCTION – Physical phenomenon

**Two-dimensional urban street canyon** → Flow regimes classified by Oke, 1987

**ASPECT RATIO:** \( AR = \frac{W}{H} \)

- **SKIMMING FLOW**
  - \( AR < 1.5 \)

- **WAKE INTERFERENCE REGIME**
  - \( 1.5 \leq AR \leq 2.5 \)

- **ISOLATED ROUGHNESS FLOW**
  - \( AR > 2.5 \)

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2. MOTIVATIONS

Lagrangian time scales are fundamental for the development of Lagrangian stochastic models (LSM), which are the most suitable tool for predicting pollutant concentration.

In LSM the particle’s trajectory can be statistically calculated as (Thomson 1987, JFM):

\[ d u_i = a_i dt + b_{ij} d \xi_j \]

- \( a_i \) = particle acceleration along \( i \)-direction
- \( b_{ij} \) = random forcing caused by the fluctuating pressure gradients and molecular diffusion

\[ b_{ij} = \sqrt{C_0 \varepsilon} \delta_{ij} \quad C_0 = \frac{2\sigma^2}{T_L \varepsilon} \]

- \( C_0 \) = Kolmogorov constant (2 ÷ 7)
- \( \varepsilon \) = dissipation rate of the Turbulent Kinetic Energy
- \( \delta_{ij} \) = Kronecker delta

Convective boundary layer ➔ Hanna 1981, JAM; Degrazia et al. 2000, AE


Urban canopies ➔ ???
2. GOALS

- Experimental investigation of 2D urban canopy flow
- Experimental estimation of both streamwise and vertical components of the Lagrangian time scales over complex terrain
- Analysis of the dependence of the Lagrangian time scales on the Aspect Ratio
- Comparison between experimental data and parametric law (Raupach 1989, AFM)
- Eulerian investigation of flow
- Estimation of eddy diffusivity of momentum with different theoretical formulations

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3. THEORETICAL BACKGROUND – Lagrangian approach

Lagrangian average velocity: \[ \langle U \rangle (x_0, \tau) = \frac{1}{M_{x_0}} \sum_{k \mid x_0} U^{(k)} (x_0, \tau) \] (1)

Standard deviation of the j-th component of the velocity: \[ \sigma_j^L (x_0, \tau) = \sqrt{\frac{1}{M_{x_0}} \sum_{k \mid x_0} \left[ U_j^{(k)} (x_0, \tau) - \langle U_j \rangle (x_0, \tau) \right]^2} \] (2)

Auto-correlation coefficient: \[ \rho_j^L (x_0, \tau) = \frac{1}{M_{x_0}} \sum_{k \mid x_0} \frac{\left[ U_j^{(k)} (x_0, \tau) - \langle U_j \rangle (x_0, \tau) \right] \left[ U_j^{(k)} (x_0, 0) - \langle U_j \rangle (x_0, 0) \right]}{\sigma_j^L (x_0, \tau) \sigma_j^L (x_0, 0)} \] (3)

Lagrangian time scale of the j-th velocity component: \[ T_j^L (x_0) = \int_0^\infty \rho_j^L (x_0, \tau) d\tau \] (4)

The Lagrangian time scale can be rigorously defined only for homogeneous, isotropic turbulence. For inhomogeneous turbulence, \( T_j^L \) are not the Lagrangian time scales but local decorrelation time scales, which could be interpreted as a measure of the persistence of motion along the j-th direction.

Reference time: \( t_0^{(k)} \)
Reference position: \( x_0^{(k)} \)
Time lag: \( \tau = t - t_0^{(k)} \)
Position at a generic time \( t \): \( X^{(k)}(t) \)
Velocity at a generic time \( t \): \( U^{(k)}(t) \)
Trajectories starting from \( x_0 \): \( M \)

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4. EXPERIMENTAL SETUP – Laboratory facility

**WATER-CHANNEL DIMENSIONS**
- height: 0.35 m
- width: 0.25 m
- length: 7.40 m

**GEOMETRICAL CONFIGURATIONS**

- $B = H = 20$ mm
- $W = 20$ mm, $AR = 1$ (skimming flow)
- $W = 40$ mm, $AR = 2$ (wake interference regime)

**FLOW CHARACTERISTICS**
- water depth: 0.16 m
- freestream velocity ($U$): 0.33 m s$^{-1}$
- friction velocity ($u^*_{ref}$): from 0.019 to 0.027 m s$^{-1}$
- Reynolds number ($Re = u^*_{ref} H / \nu$): from 390 to 470

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### 4. EXPERIMENTAL SETUP – Measurement technique

#### EULERIAN STATISTICS

**High Speed-CMOS-Camera**
- resolution: $1280 \times 1024$ pixels
- frame rate: 250 Hz
- sample: 40 s (10000 frames)

**LD PUMPED ALL-SOLID-STATE GREEN LASER**
- wavelength: 532 nm
- thickness: 2 mm
- power: 5 W

**Framed area**
- 0.06 m long (x-axis) 0.06 m high (z-axis)

#### RESULTS

- velocity field over a 120x120 regular array
- spatial resolution: 0.5 mm
- temporal resolution: 1/250 s

#### LAGRANGIAN STATISTICS

**High Speed-CMOS-Camera**
- resolution: $1280 \times 1024$ pixels
- frame rate: 500 Hz
- sample: 120 s (60000 frames)

**COBRA SLIM - HALOGEN WHITE LAMP**
- thickness: 20 mm
- power: 1000 W

**Framed area**
- 0.2 m long (x-axis) 0.06 m high (z-axis)

#### RESULTS

- Lagrangian time scale for layers 1 mm tick above the canopy
- trajectories n.: $\approx 200000$
- temporal resolution: 1/500 s
4. EXPERIMENTAL SETUP – Measurement technique

FEATURE TRACKING (FT) Image analysis technique that allows the identification of features basing on brightness gradients in following frames.

1. Identification and subtraction of the background from frames
2. Feature identification
3. Choice of the best feature to track
4. Comparison of brightness at each pixel in consecutive images
5. Reconstruction of velocity fields with scattered data on the x-z plane

Eulerian description of the velocity field over a regular 120x120 array

Tracking of the particle trajectories for the evaluation of the Lagrangian statistics.

Identification of trajectories long enough to calculate integral time scales

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5. RESULTS – Inflow

- average pebbles diameter: 5 mm
- turbulent boundary-layer depth $\delta$: 0.14 m
- reference friction velocity $u_{*,\text{ref}}$: 0.017 m s$^{-1}$

Streamwise mean velocity: 
$$ \bar{u} = \frac{u_{*,\text{ref}}}{k} \ln \left( \frac{z-d_0}{z_0} \right) \quad (5) $$

Dissipation rate of the Turbulent Kinetic Energy (Hinze 1975)
$$ \varepsilon = \frac{15}{4} \nu \left[ \left( \frac{\partial u'}{\partial z} \right)^2 + \left( \frac{\partial w'}{\partial z} \right)^2 \right] \quad (6) $$

**Lagrangian time scales of the turbulence above two-dimensional canopies**
5. RESULTS – Eulerian statistics

Lagrangian time scales of the turbulence above two-dimensional canopies
5. RESULTS – Lagrangian time scales

- In case of flat terrain, $T_u^L$ and $T_w^L$ greatly increase with height.
- $T_w^L$ for $z/\delta \lesssim 0.2$ (i.e. into the CFL) grows in agreement with Eq. (7a) by Raupach 1989, AFM
  \[ \frac{T_w^L u_{*,\text{ref}}}{\delta} = \frac{k}{(\sigma_w / u_*)_{\text{ref}}} \frac{z}{\delta} \]

- For $z/\delta \gtrsim 0.2$ $T_u^L$ and $T_w^L$ are nearly constant in the case of flat terrain.
- For $AR=1$, $T_u^L$ and $T_w^L$ grow almost linearly with height for the whole boundary layer.
- $T_u^L$ and $T_w^L$ for $AR=1$ $T_w^L$ for $AR=2$ grow in agreement with Eq. (7b)
  \[ \frac{T_w^L u_{*,\text{ref}}}{H} = \frac{k}{(\sigma_w / u_*)_{\text{ref}}} \frac{z - d_0}{H} \]
  where $d_0=0.9H$ and $d_0=0.77H$ for $AR=1$ and $2$, respectively (Kastner-Klein and Rotach 2004, BLM)
- For $AR=2$, $T_u^L$ is constant in the whole RSL as well as for a considerable portion of CFL. For $z > 2H$, it grows roughly linearly with height.
- For $AR=2$, $T_u^L \approx T_w^L$ for $z > 2H$.
- $T_w^L$ do not change appreciably with $AR$.
5. RESULTS – Turbulent diffusivity

- Satisfactory agreement for AR=1

- For AR=2 disparities of a factor of 2 between Eqs. 8 and 9 occur for the whole domain

- Eq. 8 and Eq. 9 grow roughly linearly with height according to Prandtl’s law, even though with different slopes.

\[ K_T = -\bar{u}'\bar{w}'d\bar{u}/dz \quad (8) \]

relies on the first-order closure for the momentum flux

\[ K_T = \sigma_w^2 T_w^L \quad (9) \]

eddy diffusivity based on Taylor’s theory

\[ K_T = k u_{*,ref}(z - d_0) \quad (10) \]
eddy diffusivity based on Prandtl’s mixing-length theory

**Lagrangian time scales of the turbulence above two-dimensional canopies**
6. CONCLUSIONS AND FURTHER WORK

✓ Experimental determination of streamwise and vertical components of the Lagrangian time scales (not previously reported in the literature)
✓ For AR=1, \( T_u^L \) and \( T_w^L \) grow linearly with height. For AR=2, \( T_u^L \) is almost constant for \( z<2H \) and \( T_w^L \) grows linearly with height
✓ Dependence of the streamwise component of the time scale \( T_u^L \) on AR, independence of the vertical component of the time scale \( T_w^L \) on AR
✓ Comparison of streamwise and vertical components of the Lagrangian time scales with theoretical prediction: \( T_u^L \) and \( T_w^L \) obey Raupach’s law (except \( T_w^L \) when AR=2)

- Analysis of the Lagrangian time scales within the cavities
- Investigation for other Aspect Ratios of the canopy
- Comparison with other theoretical laws
- Investigation above and within three-dimensional geometries, varying the building height and the planar area fraction
Thank you for your attention!
FLAT TERRAIN: Lagrangian autocorrelation functions calculated at various heights for the (a) streamwise and (b) vertical velocity component.

CANOPIES: Lagrangian autocorrelation functions calculated at various heights for the (a) streamwise and (b) vertical velocity component for AR=1.

FLAT TERRAIN: (a) Vertical profiles of the non-dimensional turbulent diffusivity and (b) of the Kolmogorov constants (Red and blue refer to $\mathcal{C}_D u$ and $\mathcal{C}_D w$, respectively).

CANOPIES: Vertical profiles of the Kolmogorov constants.