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Analytical Modelling of Dispersion for Bayesian Source Term Estimation

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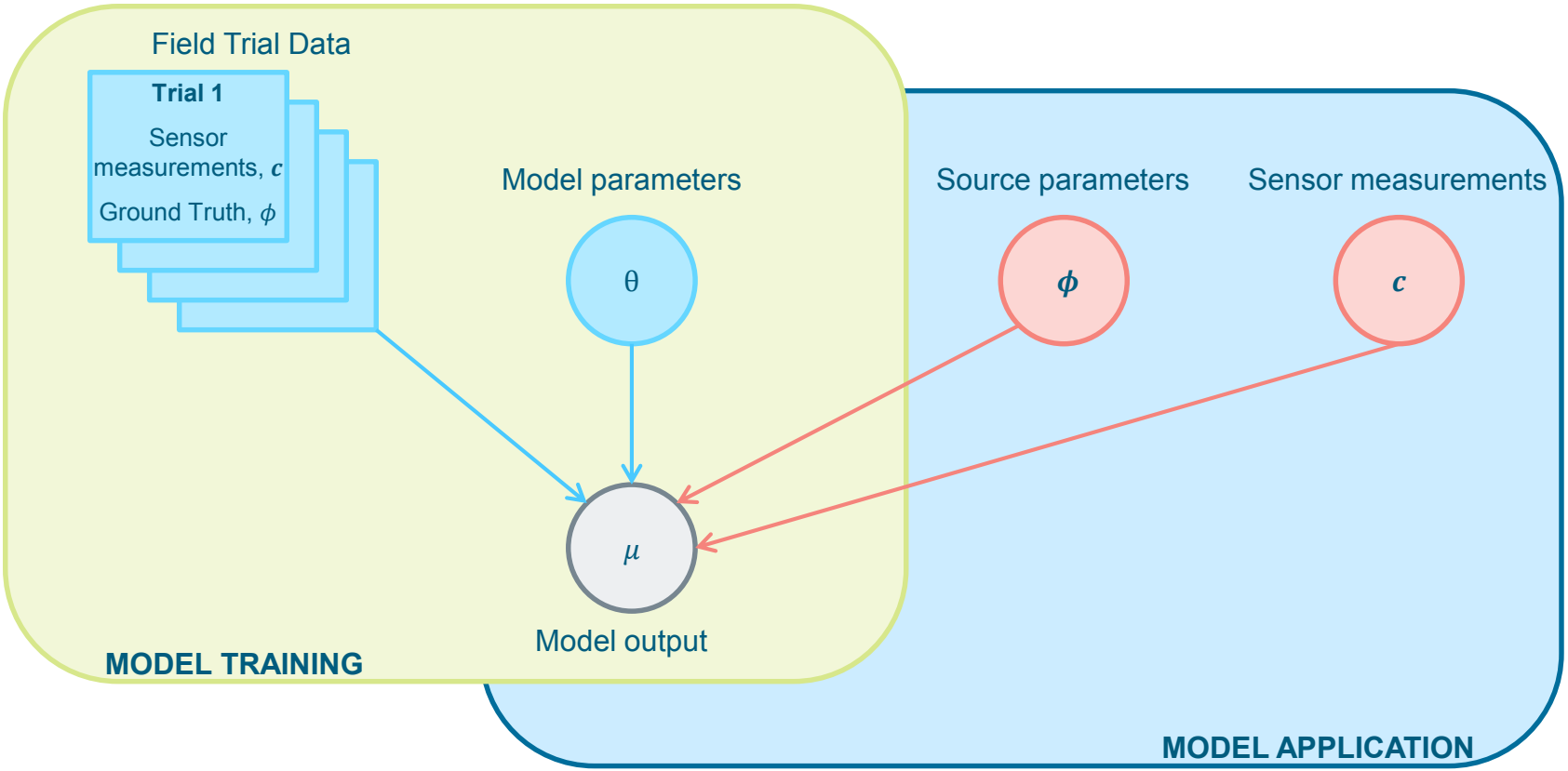
Introduction and outline

- Motivation
- Analytic model and features
- Optimising the model on field trial data
- Experimental procedure
- Results
  - JU2003
  - FFT07
- Summary and next steps

Motivation

- Decision Support Tool
- Dispersion model is typically evaluated millions of times in Source Term Estimation (STE)
- Can a rapidly evaluated analytic model be used to minimise the use of a complex dispersion model?
- Our approach is to use field trial data to determine model parameters.

Concept



The model

- Gaussian puff model used within an optimisation framework:
  - It is not steady state
  - The release is of finite duration
  - Reflections from the ground and boundary layer top are included
  - There are non-zero initial values for σ_x, σ_y and σ_z

$$\frac{\sigma_x}{\sigma_u} = \frac{\sigma_y}{\sigma_v} = \frac{\sigma_z}{\sigma_w} = \begin{cases} \alpha + t & t \leq 2\tau \\ [(\alpha + 2\tau)(\alpha + t)]^{\frac{1}{2}} & t > 2\tau \end{cases}$$

τ - Lagrangian interval time scale; α - initial value for σ_x/σ_u, σ_y/σ_v and σ_z/σ_w;
σ_u, σ_v and σ_w - standard deviations of the turbulent velocity fluctuations in x, y and z directions.
Common value, σ_{uvw}, assumed for σ_u, σ_v and σ_w.

Approach: Optimisation (training)

- We define a cost function, $\mathcal{L}$, that measures the difference between the predictions and the observations at a set of locations and times.

$$\mathcal{L} = \mathcal{L}(\boldsymbol{c}(t, \theta, \phi, \nu), \boldsymbol{m}(t), \{\boldsymbol{u}\}),$$

$\boldsymbol{c}(t, \theta, \phi, \nu)$ - predictions at a set of sensor locations at time t for model parameters θ, ϕ and ν;

$\boldsymbol{m}(t)$ - measured concentration at the sensors at time t

$\{\boldsymbol{u}\}$ - set of wind measurements.

Parameters

ϕ - the source term parameters

θ - the dispersion model parameters

ν - meteorological parameters

Optimise with respect to θ and ν (ϕ is known)

Approach: Application to STE (test)

- For an optimised model, the application to STE is by optimising $\mathcal{L}(\boldsymbol{c}(t,\theta,\phi,\nu),\boldsymbol{m}(t),\{\boldsymbol{u}\})$, on data, with respect to:
  - ϕ, the source term parameters,
  - ν, the meteorological parameters
- θ, the model parameters, are assumed known
- The optimised values, ϕ^*, are then compared with the true source term values.

Experimental procedure (1)

- Training
  - maximum likelihood and least squares cost function
- Evaluation
  - Fractional Bias (FB)
  - Normalised Mean Square Error (NMSE)
  - Factor of 2 (FAC2)

Chang, J.C. and Hanna, S.R. (2004)

Assessment measure	Rural	Urban
|FB|	<0.3	<0.67
NMSE	<3	<6
FAC2	>0.5	>0.3

Experimental procedure (2)

- Paired comparison
- Threshold schemes

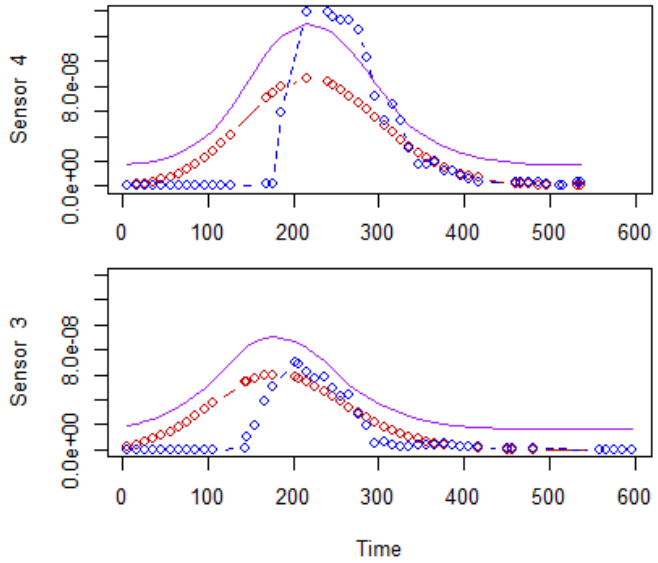
T1	*Both* the observation and the prediction are greater than a threshold
T2	*Either* the observation or the prediction is greater than a threshold

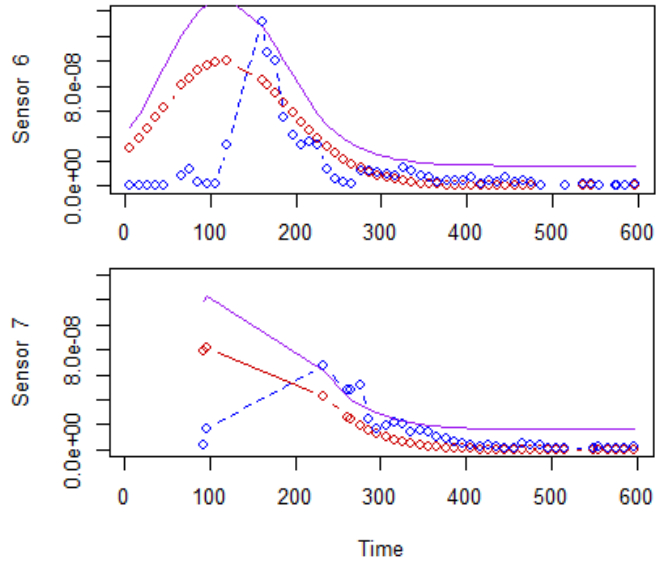
- From the final converged solution, we derive:
  - $\sigma_{x0} = \sigma_{y0} = \sigma_{z0} \triangleq \alpha\sigma_{uvw}$ - the initial size of the puff (m)
  - $\gamma \triangleq \sigma_{uvw}/|u|$ - the rate of expansion (m/m)

	σ_{y0}, σ_{z0} (m)	γ (m/m)
Day	40	0.25
Night	40	0.08

Hanna, S. and Baja, E. (2009)

Results – JU2003





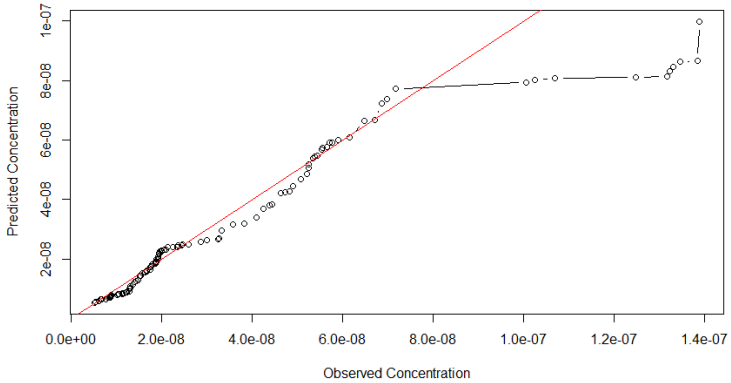
IOP3 Release 3

◊ - data

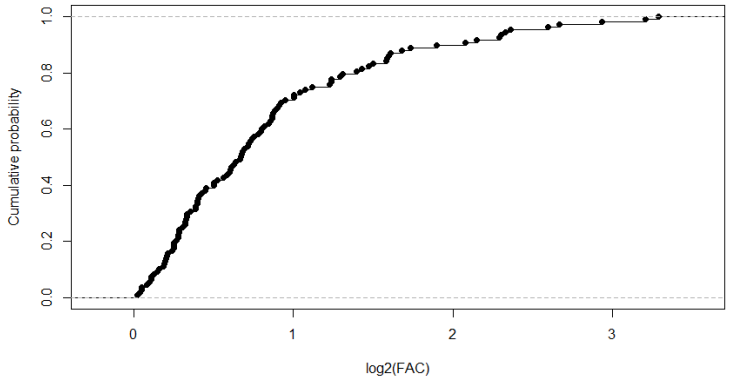
◊ - model

— - model+sd

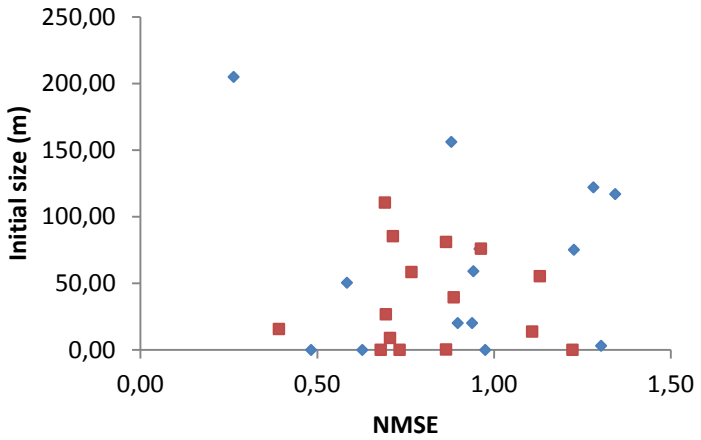
Predicted vs Observed Concentration QQ plot

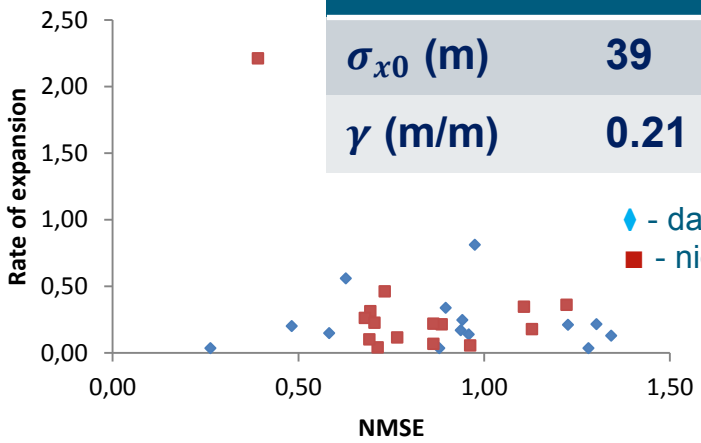


Cumulative FAC plot (T1)



Results – JU2003





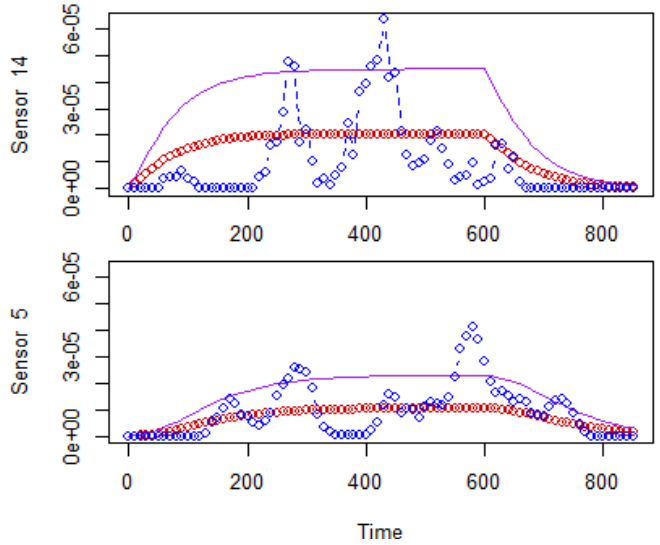
Parameter	median
σ_{x0} (m)	39
γ (m/m)	0.21

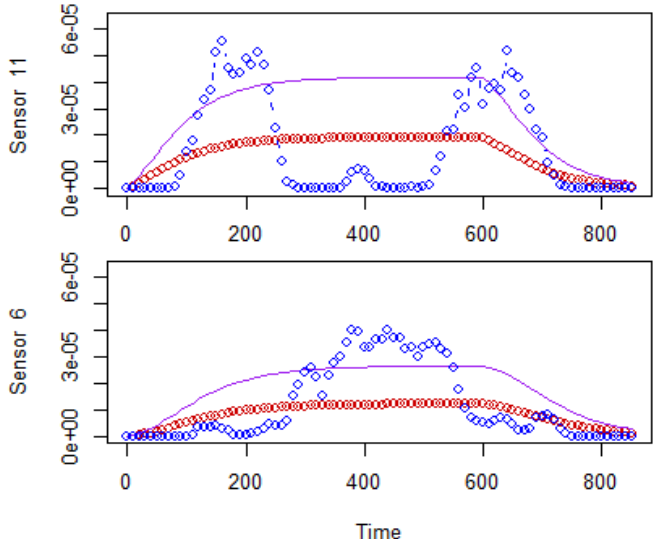
Scatter plots of initial size and rate of expansion against NMSE for JU2003 puff releases

	T1	T2
JU2003 Puff Releases	29/29	22/29
JU2003 Extended Releases	22/24	12/24

Proportion of trials passing all assessment measure criteria

Results FFT07



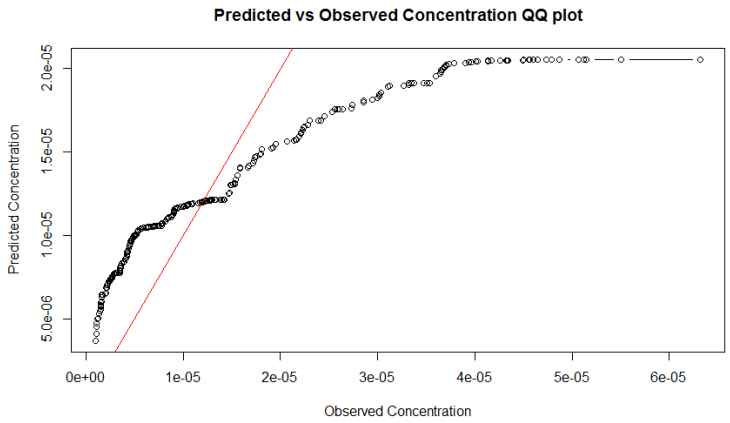


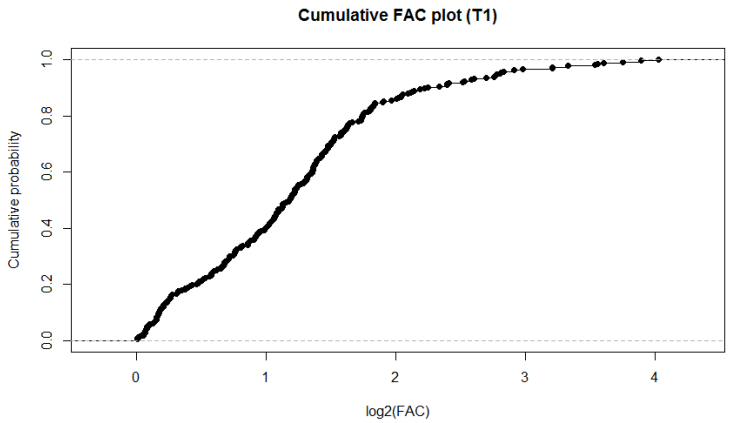
Case 53

◊ - data

◊ - model

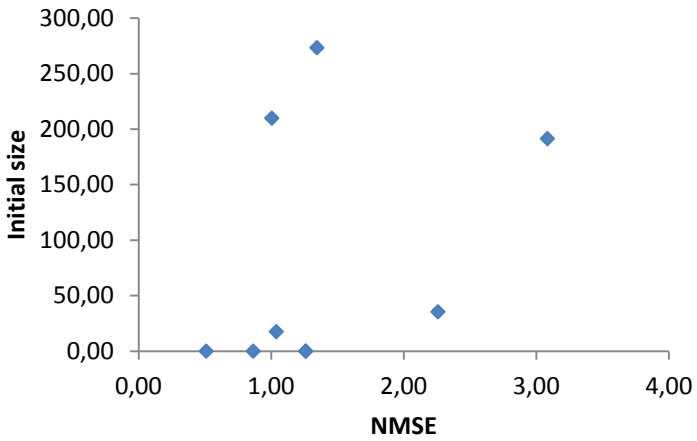
— - model+sd

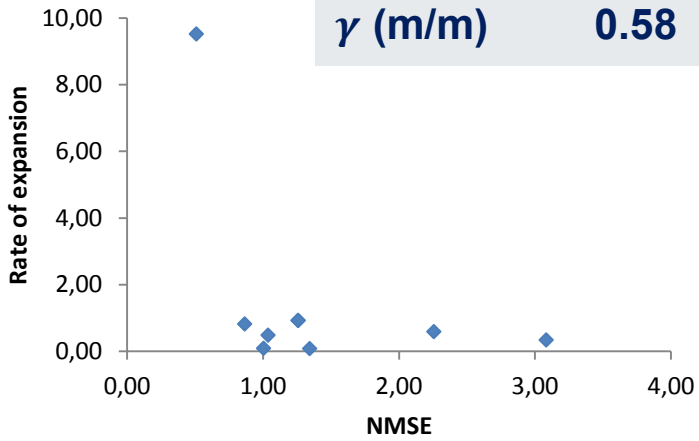




Results FFT07

Parameter	median
σ_{x0} (m)	18
γ (m/m)	0.58





Scatter plots of initial size and rate of expansion against NMSE for FFT07 extended releases

	T1	T2
FFT07 Puff Releases	0/7	0/7
FFT07 Extended Releases	5/9	0/9

Proportion of trials passing all assessment measure criteria

Summary

- Analytic model developed for initial STE study
- Assessment on JU2003 and FFT field trial data puff and extended releases
- Maximum likelihood parameter estimation with fixed and optimised variance model
- Good performance on JU2003 (in terms of acceptance test), but not for FFT07 puff releases - lack of model flexibility for given ground truth
- Derived parameters (JU2003 puff releases) consistent with proposed model parameters in Hanna and Baja (2009)
- Next stage: assess model within a Bayesian STE procedure

Thank you.

Questions?

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