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**IMPROVEMENTS TO AN OPERATIONAL INVERSION METHOD FOR ESTIMATING  
VOLCANIC ASH SOURCE PARAMETERS USING SATELLITE RETRIEVALS**

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**Abstract:** Uncertain eruption source parameters can lead to large errors in ash cloud predictions from atmospheric dispersion models. An inversion method, which uses satellite observations to better inform the source term, has recently undergone improvements allowing better use of (potentially few) observations and a significant speed up in run-time. Here we demonstrate the performance of the method using recent volcanic eruptions as test cases and show that the determined source term results in an improved ash cloud forecast.

**Key words:** *volcanic ash, inversion technique, satellite observations, atmospheric dispersion, NAME*

## **INTRODUCTION**

The London Volcanic Ash Advisory Centre (VAAC), hosted by the Met Office, is responsible for issuing advisories on the transport of volcanic ash clouds in the North-East Atlantic region. These forecasts are produced using the atmospheric dispersion model NAME (Jones et al., 2007) and source term parameters based on the observed eruption height. Errors and uncertainties in the estimated source term have a large influence on the accuracy of the ash cloud forecasts.

Following the prolonged eruption of the Icelandic volcano Eyjafjallajökull in 2010, an inversion technique was developed employing satellite observations to better inform the source parameters. The technique uses satellite retrievals of ash column loadings and of regions of clear sky and a probabilistic approach which considers both the uncertainty in the initial best-guess source parameters (the *a priori*) and the uncertainty in the satellite retrievals. The process yields the source term parameters (the *a posteriori*) which optimally fit the predicted ash cloud to the satellite observations within their uncertainty, whilst simultaneously fitting the emissions to the *a priori* estimate of the emissions, again within their uncertainty.

Improvements to the run-time of the inversion technique have recently been made, enabling inversion calculations for prolonged eruptions and large numbers of observations to be run efficiently in an emergency response situation. This speed up has enabled us to consider an increase in the resolution (both vertically and in time) of the source term profile. In addition it has allowed the scheme to undergo further development to include correlations in the errors in the *a priori* source term profile. These correlations between the *a priori* source term components may be induced by variations in the plume rise height (from the observed value used in the *a priori*), variations in the mass release rate for a given plume rise height (from the assumed relationship between these two quantities) and variations in the vertical distribution of ash (from the assumed uniform profile). Including these correlations allows a satellite observation to influence surrounding source term components (both vertically and in time) and enables the inversion method to make better use of information from satellite observations. This is particularly important in the initial stages of an eruption when a limited number of observations are available.

## **THE INVERSION SCHEME**

For a given vector of  $n$  source terms,  $\mathbf{e}$ , an atmospheric dispersion model, such as NAME, can be used to give a vector of model predictions,  $\mathbf{o}_m$ , for  $k$  observations of ash column loading,

$$\mathbf{M}\mathbf{e} = \mathbf{o}_m$$

where  $\mathbf{M}$  is the transport matrix relating the source term to the observations. The inversion technique uses a probabilistic approach to find the time and height varying source term,  $\mathbf{e}$ , which optimally fits both dispersion model ash cloud predictions to the satellite observations,  $\mathbf{o}_a$ , and the emission source term to the *a priori* estimate of the emissions. Bayes theorem states

$$P(\mathbf{e} | \mathbf{o}_a) \propto P(\mathbf{o}_a | \mathbf{e}) P(\mathbf{e})$$

where  $P(\mathbf{o}_a | \mathbf{e})$  is the probability density of obtaining the satellite observations given a source term  $\mathbf{e}$ ,  $P(\mathbf{e})$  is the probability density of the *a priori* source term and  $P(\mathbf{e} | \mathbf{o}_a)$  is the probability density of the source term given the observations. The probability distributions for the satellite retrievals and the *a priori* source are assumed to be Gaussian:

$$P(\mathbf{o}_a | \mathbf{e}) \propto \exp \left[ -\frac{1}{2} (\mathbf{M}\mathbf{e} - \mathbf{o}_a)^T \mathbf{R}^{-1} (\mathbf{M}\mathbf{e} - \mathbf{o}_a) \right]$$

$$P(\mathbf{e}) \propto \exp \left[ -\frac{1}{2} (\mathbf{e} - \mathbf{e}_{ap})^T \mathbf{B}^{-1} (\mathbf{e} - \mathbf{e}_{ap}) \right]$$

where  $\mathbf{e}_{ap}$  is the mean of the *a priori* probability distribution,  $\mathbf{R}$  is the error covariance matrix for the satellite retrievals and  $\mathbf{B}$  is the error covariance matrix for the *a priori* source.  $\mathbf{R}$  is assumed to be a diagonal matrix and errors in the transport model are not considered. The *a posteriori* source which maximises  $P(\mathbf{e} | \mathbf{o}_a)$  is obtained by finding the minimum of the cost function

$$J(\mathbf{e}) = (\mathbf{M}\mathbf{e} - \mathbf{o}_a)^T \mathbf{R}^{-1} (\mathbf{M}\mathbf{e} - \mathbf{o}_a) + (\mathbf{e} - \mathbf{e}_{ap})^T \mathbf{B}^{-1} (\mathbf{e} - \mathbf{e}_{ap})$$

subject to a non-negative constraint. The imposition of the non-negative constraint is a pragmatic choice necessary due to Gaussian assumptions. An alternative, but more complex, option would be to build the non-negativity constraint directly into the probabilistic assumptions (i.e., to assume a non-Gaussian distribution which is, by definition, constrained to be non-negative).

The cost function minimum is now obtained using the non-negative least squares (*NNLS*) solver (Lawson and Hanson, 1974) which has been found to give a substantial speed up in the inversion code.

### The *a priori* emissions

The inversion scheme uses an initial best guess (an *a priori*,  $\mathbf{e}_{ap}$ ) for the source term, together with an estimate of the uncertainty in the *a priori*,  $\mathbf{B}$ . The *a priori* is determined from observations of the eruptive plume heights (e.g., from radar and web cameras), an empirical relationship relating mass eruption rate to the eruptive plume height (Mastin et al., 2009) and a uniform vertical distribution assumption. The fraction of the erupted mass which survives near-source fall-out processes is assumed to be 5%. The *a priori* provides a guide to finding a more realistic solution and prevents over-fitting to uncertain satellite observations. Following recent developments, the uncertainty in the *a priori* is now estimated from errors in the plume rise height, errors in the Mastin et al. relationship or in the distal fine ash fraction, and assumptions about fluctuations in the shape of the emission profile. These errors lead naturally to correlations between the individual source terms and result in a non-diagonal  $\mathbf{B}$  matrix.

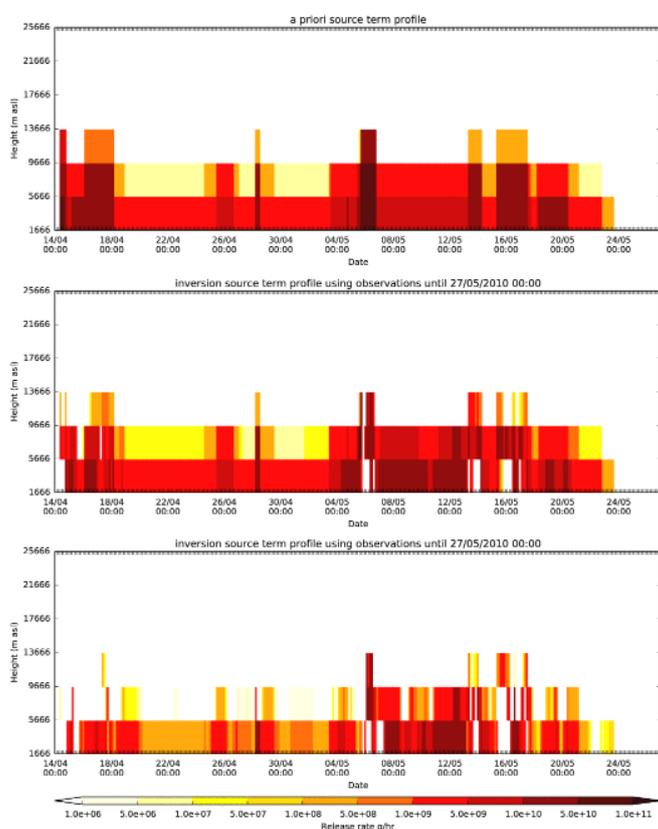
### Satellite observations

Data from the Spinning Enhanced Visible and Infrared Imager (SEVIRI) on-board the geostationary Meteosat Second Generation (MSG) satellite are used to retrieve volcanic ash physical properties using a 1-dimensional variational (1D-Var) analysis methodology (Francis et al., 2012). The retrieved properties include ash column loadings ( $\mathbf{o}_a$  in  $\text{g m}^{-2}$ ) together with an estimate of their uncertainties ( $\mathbf{R}$ ). In addition, clear sky observations, which are free from both ash and meteorological cloud, can be used in the inversion scheme.

## RESULTS

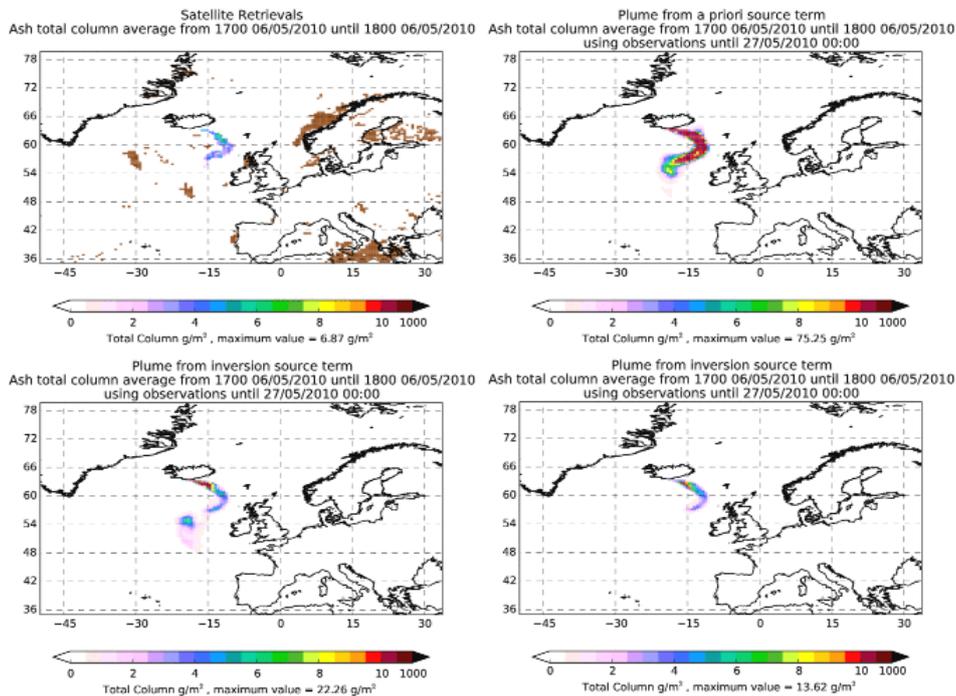
The inversion method has been used to study the recent eruptions of two Icelandic eruptions: Grímsvötn in 2011 and Eyjafjallajökull in 2010. Here we present the results from the Eyjafjallajökull eruption in 2010 and briefly discuss results from the 2011 Grímsvötn eruption.

The eruption of Eyjafjallajökull (63.63° N, 19.62° W) started at approximately 09:00 UTC on 14/04/2010 and continued for nearly 40 days. Reports of the eruption plume height, based on radar observations, were obtained throughout the event from the Icelandic Meteorological Office (IMO). Satellite retrievals of ash column loadings and clear sky regions from SEVIRI data provide observations from the start of the eruption until 23:00 UTC on 29/05/2010, thereby allowing observations of ash remaining in the atmosphere past the end of the eruption to be included in the inversion. The observations are processed to select those which coincide (in time and in space) with the model predicted plume and which provide useful information to the inversion system. After processing, a total of 40047 observations of volcanic ash column loadings are available. Including clear sky increases the total number of column load observations to 689492. Following the recent improvements, the inversion calculation of the entire Eyjafjallajökull eruption takes just over 5 minutes using ash only observations and just over 13 minutes using both ash and clear sky observations. Figure 1 compares the *a priori* source term profile with the *a posteriori* source term profiles determined using ash-only observations and both ash and clear sky observations. All source term profiles show only the distal fine ash fraction which survives near-source fall-out processes. The use of clear sky observations removes more ash from the *a posteriori* source.



**Figure 1.** The distal fine ash source term profiles for the 2010 eruption of Eyjafjallajökull: the *a priori* (top), the *a posteriori* determined using ash only observations (middle) and the *a posteriori* determined using both ash and clear sky observations (bottom). All source term profiles have a height resolution of 4 km (y axis) and a time resolution of 3 hours (x axis).

Figure 2 shows the column loadings observed by satellite and predicted by NAME between 17:00 and 18:00 UTC on 06/05/2010. In comparison to the *a priori* ash cloud, the *a posteriori* ash clouds have a reduction of ash within the cloud which agrees better with the observed ash cloud. Using clear sky observations removes the small region of ash at approximately 54°N, 20°W seen in the ash-only *a posteriori* cloud.



**Figure 2.** The distal ash cloud between 17:00 and 18:00 UTC on 06/05/2010 observed by satellite (ash and clear sky, with clear sky shown in brown) (top left) and predicted by NAME using the *a priori* source term (top right), using the *a posteriori* source term derived using ash only observations (bottom left) and using the *a posteriori* source term derived using both ash and clear sky observations (bottom right).

The eruption of Grímsvötn ( $64.42^{\circ}$  N,  $17.33^{\circ}$  W) at 19:13 UTC on 21/05/2011 was a much shorter eruption, lasting for approximately four days. A total of 3293 useful observations of volcanic ash column load are available from SEVIRI for employing with the inversion system. Including clear sky increases the total number of observations of volcanic ash column load to 88791. The inversion calculation of the Grímsvötn eruption takes less than 13 seconds using ash-only observations and just over 1 minute using both ash and clear sky observations. The erupted mass of fine ash is considerably less in the *a posteriori* source than in the *a priori* source, and is in line with the view held at the time of the eruption that the actual ash cloud contained less ash downwind than the modelling suggested.

## VALIDATION

Webster et al. (2012) validated a scheme for forecasting peak ash concentrations against both ground-based observations and measurements from instrumentation onboard research aircraft from the 2010 eruption of Eyjafjallajökull. Table 1 compares model predictions of peak ash concentrations against these observations for both a simple uniform source term profile based on the observed eruption height (Webster et al., 2012) and the *a posteriori* source terms obtained from the inversion scheme using ash-only observations and both ash and clear sky observations. The model predicts mean ash concentrations over large volumes and time periods and hence a peak-to-mean factor is applied to estimate peak ash concentrations. In line with Webster et al. (2012) a peak-to-mean factor of 10 is used with model predicted mean ash concentrations over 25FL layers. In the comparison of modelled and observed peak ash concentrations, an uncertainty in the observations of a factor of 2 is assumed. Agreement is assessed both with and without consideration of uncertainty in the modelled peak ash concentrations. This model uncertainty is due to slight positional errors in the predicted ash cloud and is accounted for by assessing the variability in the modelled concentrations over neighbouring model output grid-boxes.

The ash cloud predictions obtained from the *a posteriori* source terms show better agreement with the independent observations than the model predictions obtained using a simple uniform source term, particularly when errors in the model predictions are not considered in the comparison. Using clear sky

observations with the inversion scheme reduces the ash in the resulting *a posteriori* source term and the model then has a tendency to under-predict peak concentrations within the ash cloud. This needs further investigation but may be caused by errors in the transport matrix  $\mathbf{M}$  (which are not currently considered in the inversion scheme) resulting in the incorrect removal of ash when deriving the *a posteriori* source term.

**Table 1.** A statistical comparison of modelled and observed peak ash concentrations using different source terms and the high resolution 25FL peak ash concentration scheme (see Webster et al. (2012) for details). Agreement is assessed both with and without consideration of uncertainty in the model predictions due to positional errors in the ash cloud. Uncertainty in the observations is included in both assessments.

Source term	No model uncertainty			With model uncertainty		
	% in agreement	% of over-predictions	% of under-predictions	% in agreement	% of over-predictions	% of under-predictions
Uniform based on eruption height	30	23	48	75	3	22
<i>a posteriori</i> (ash-only observations)	43	25	32	79	3	18
<i>a posteriori</i> (ash and clear sky observations)	36	7	57	61	1	38

## CONCLUSIONS

This inversion technique uses available observations and reported details of the eruption to determine the optimal source (the *a posteriori*) within the uncertainty of the satellite observations and of the *a priori*. The *a posteriori* source can then be used as input to the transport model to give an improved forecast of the ash cloud. The recent improvements have resulted in a significant speed up, enabling the technique to be run quickly in an operational setting. Furthermore the scheme has been extended to include cross correlations in the errors in the *a priori* source term profile which enables better use to be made of the observations, which may be few in number. Two test cases have been studied in detail: the eruption of Eyjafjallajökull in 2010 and the eruption of Grímsvötn in 2011. The *a posteriori* source gives a predicted ash cloud which is in better agreement with the observations. The use of clear sky observations can reduce further the quantity of ash within the *a posteriori* source term and this may lead to an under-estimation of ash within the predicted plume. Further investigation is required here to fully understand the reasons for this.

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