THE URBAN WIND PROFILE

Sven-Erik Gryning[1], Ekaterina Batchvarova[1,2] and Burghard Brümmer[3]
[1]Risø National Laboratory, Technical University of Denmark, Roskilde, Denmark
[2]National Institute of Meteorology and Hydrology, Sofia, Bulgaria
[3]Meteorological Institute, University of Hamburg, Hamburg, Germany

INTRODUCTION

Analysis of profiles of meteorological measurements from a 160 m high mast at the National Test Site for wind turbines at Høvsøre (rural, Denmark) and at a 250 m high TV tower at Hamburg (urban, Germany) shows that the wind profile based on surface-layer theory and Monin-Obukhov scaling is valid up to a height of 50 to 80 m. At higher levels deviations from the measurements progressively occur.

A parameterization of the wind profile for the entire boundary layer is formulated, with emphasis on the lowest 200 - 300 m and considering only wind speeds above 3 m s^{-1} at 10 m height. The friction velocity is taken to decrease linearly through the boundary layer. The wind profile length scale is composed of three component length scales. In the surface layer the first length scale is taken to increase linearly with height with a stability correction following Monin-Obukhov similarity. Above the surface layer the second length scale (L_{MBL}) becomes independent of height but not of stability, and at the top of the boundary layer the third length scale is assumed to be negligible. A simple model for the combined length scale that controls the wind profile and its stability dependence is formulated by inverse summation. The wind profile for a number of stability classes, based on the Monin-Obukhov stability scale, L, is illustrated in Figs 1 and 2.

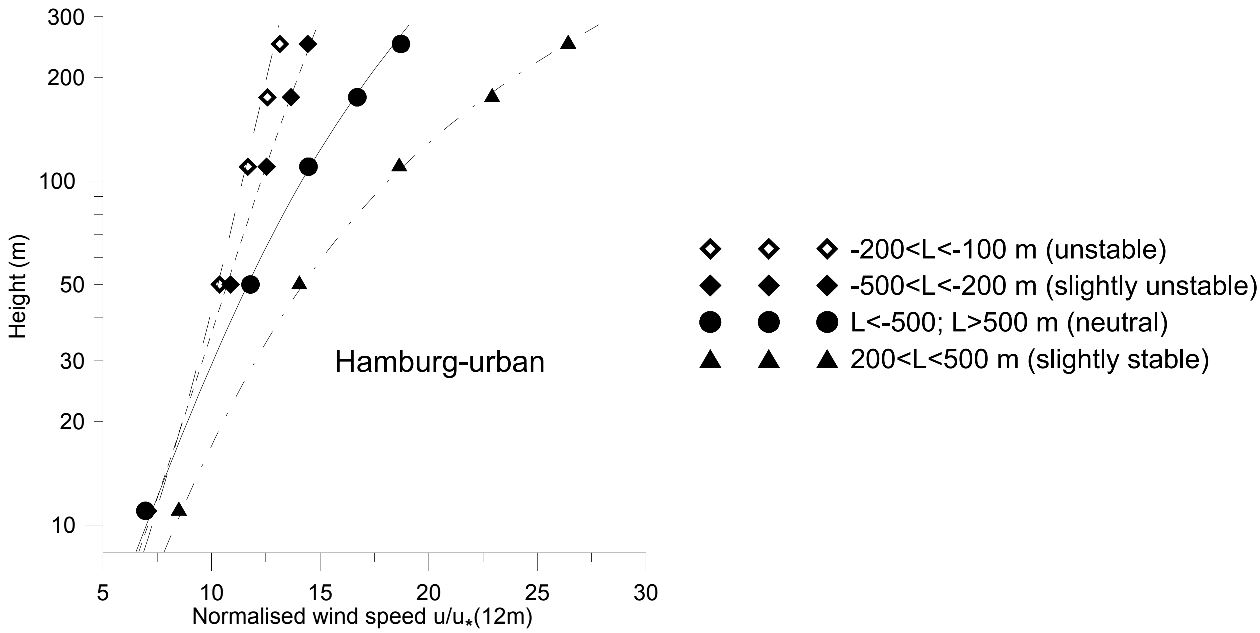

Fig. 1; Normalized wind profiles as function of stability for the urban sector for Hamburg according to Eqs. (7), (12) and (17). Lines represent model results and symbols measurements in the assigned stability classes. Details are given in Gryning et al. (2007).

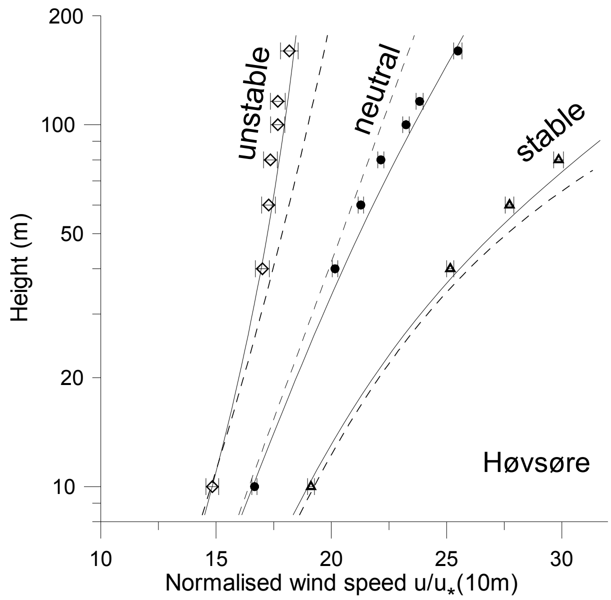

Fig. 2; Comparison between surface layer theory with Monin-Obukhov scaling for the stability according to Dyer (1974) (dashed lines) and the wind profile expressions suggested here (full lines). The stability ranges of L are: unstable ($-50 > L > -100$ m - diamond), neutral ($L < -500$ or $L >$ 500 m – filled circle) and stable (50<L< 200 m - triangle).

THEORY

The starting point for the analysis is the general expression for the wind profile for the homogeneous, stationary atmospheric boundary layer (*Panofsky* 1973):

$$\frac{du}{dz} = \frac{u_*}{\kappa l} \qquad (1)$$

where u_* is the local friction velocity, κ the von Karman constant and l is the local length scale. In the surface layer the local friction velocity can be considered constant

$$u_* = u_{*0} \ , \qquad (2)$$

where u_{*0} is the friction velocity near the ground. Above the surface layer the friction velocity diminishes and becomes small at the top of the boundary layer where it is is here approximated as

$$u_*(z) = u_{*0}\left(1 - z/z_i\right) \ , \qquad (3)$$

where z_i is the boundary-layer depth. The length scale, l, is composed of three terms, and for simplicity is modelled by inverse summation

$$\frac{1}{l} = \underset{\text{I}}{\frac{1}{L_{SL}}} + \underset{\text{II}}{\frac{1}{L_{MBL}}} + \underset{\text{III}}{\frac{1}{L_{UBL}}} \ . \qquad (4)$$

where L_{SL} represents the length scale in the surface layer, L_{MBL} in the middle of the boundary layer and L_{UBL} the upper part of the boundary layer. The formulations of the component length scales will be discussed below.

The neutral wind profile for the entire boundary layer

The length scale $L_{SL,N}$ in the neutral surface layer (SL) is proportional to height

$$L_{SL,N} = z \qquad (5)$$

where subscript N denotes neutral conditions. Above the surface layer, in the middle of the boundary layer (MBL), the length scale $L_{MBL,N}$ is no longer proportional to height but becomes near constant where $L_{MBL,N}$ depends on atmospheric properties such as baroclinicity, Brunt-Vaisala frequency, stationarity. Little is known about the behaviour of the length scale $L_{UBL,N}$ in the upper part and near the top of the boundary layer (UBL). Here we assume that the top acts as a lid and for simplicity the length scale depends linearly on the distance to the top of the boundary layer,

$$L_{UBL,N} = \left(z_i - z\right) \;. \qquad (6)$$

The resulting length scale l, is modelled by inverse summation as in Eq. (4).
Inserting Eq. (3) and (4) into Eq. (1) and integrating along z between the roughness length z_0 and height z yields for the neutral wind profile:

$$u(z) = \frac{u_{*0}}{\kappa}\left(\ln\left(\frac{z}{z_0}\right) + \frac{z}{L_{MBL,N}} - \frac{z}{z_i}\left(\frac{z}{2\,L_{MBL,N}}\right)\right) \;. \qquad (7)$$

Stability correction

The effects of atmospheric stability are derived as a correction to the wind profile in neutral conditions. In the surface layer the influence of atmospheric stability on the length scale L_{SL} layer is expressed as

$$L_{SL} = L_{SL,N}\, f_{SL}^{-1}\left(z/L\right) \qquad (8)$$

where $L_{SL,N}$ is the length scale for neutral conditions and the function f_{SL} accounts for the stability correction. Here L is the Obukhov length scale. In a similar way the stability correction to the length scale for the middle part of the boundary layer can be written $L_{MBL} = L_{MBL,N}\, f_{MBL}^{-1}$ and for the upper part $L_{UBL} = L_{UBL,N}\, f_{UBL}^{-1}$. The functions f_{MBL} and f_{UBL} reflect the stability correction in the middle and upper parts of the boundary layer, respectively. The function f_{MBL} and its parameterization are discussed in the next section. For simplicity $f_{UBL} = 1$ is used. Inverse summation of the length scales yields:

$$\frac{1}{l} = \frac{f_{SL}\left(z/L\right)}{L_{SL,N}} + \frac{1}{L_{MBL}} + \frac{1}{L_{UBL}} \;. \qquad (9)$$

Stable conditions

For atmospheric stable conditions the usual functional form of the stability correction reads:

$$f_{SL} = \left(1 + bz/L\right) \qquad (10)$$

Then

$$\frac{1}{l} = \frac{1}{z}\left(1 + \frac{b\,z}{L}\right) + \frac{1}{L_{MBL}} + \frac{1}{L_{UBL}} \qquad (11)$$

where the terms in parenthesis represent the correction to the logarithmic wind profile in the surface layer. The corresponding wind profile for stable conditions reads

$$\frac{u(z)}{u_{*0}} = \frac{1}{\kappa}\left(\ln\left(\frac{z}{z_0}\right) + \frac{b\,z}{L}\left(1 - \frac{z}{2\,z_i}\right) + \frac{z}{L_{MBL}} - \frac{z}{z_i}\left(\frac{z}{2\,L_{MBL}}\right)\right) \qquad (12)$$

Unstable conditions

For atmospheric unstable conditions the usual functional form of the stability correction reads:

$$f_{SL} = \left(1 + a\,z/L\right)^p \quad , \qquad (13)$$

Here $p = -1/3$ is used in accordance with the theoretical limit for convective conditions (*Carl et al.* 1973) and $a = -12$. Then

$$\frac{1}{l} = \frac{1}{z}\left(1 - \frac{12\,z}{L}\right)^{-1/3} + \frac{1}{L_{MBL}} + \frac{1}{L_{UBL}} \qquad (14)$$

where the terms in the parenthesis reflect the stability dependence on the wind profile. The gradient of the wind profile can be formulated and integration yields

$$\frac{u(z)}{u_{*0}} = \frac{1}{\kappa}\left(\ln(\frac{z}{z_0}) - \psi\left(\frac{z}{L}\right) + \frac{z}{z_i}\left(1 + \frac{\left(1 - 12\,z/L\right)^{2/3} - 1}{8\,z/L}\right) + \frac{z}{L_{MBL}} - \frac{z}{z_i}\left(\frac{z}{2\,L_{MBL}}\right)\right) \qquad (15)$$

where the stability correction for the surface boundary layer is

$$\psi\left(\frac{z}{L}\right) = \frac{3}{2}\ln\left(\frac{1 + x + x^2}{3}\right) - \sqrt{3}\arctan\left(\frac{1 + 2x}{\sqrt{3}}\right) + \frac{\pi}{\sqrt{3}} \qquad (16)$$

and $x = \left(1 - 12\,z/L\right)^{1/3}$. The full expression, Eq. (15), is rather unattractive and for simplification the influence of the surface boundary layer stability in the third term of the right-hand side is included in L_{MBL}. The justification is discussed in the *Gryning et al.*, 2007). This reduces Eq. (15) to:

$$\frac{u(z)}{u_{*0}} = \frac{1}{\kappa}\left(\ln(\frac{z}{z_0}) - \psi\left(\frac{z}{L}\right) + \frac{z}{L_{MBL}} - \frac{z}{z_i}\left(\frac{z}{2\,L_{MBL}}\right)\right) \qquad (17)$$

which is the proposed wind profile for unstable conditions.

Parameterization of L_{MBL}

At the top of the boundary layer the wind profile conforms to the geostrophic wind. A parametrization of L_{MBL} will be achieved by use of Rossby similarity theory that relates the wind speed at the top of the boundary layer to the friction velocity near the ground.

Owing to ambiguity in the formulation of the A and B functions an empirical fit of the dependence between $u_{*0}/f\,L_{MBL}$, $u_{*0}/f\,z_0$ and $u_{*0}/f\,L$ will be devised. The theoretical roughness dependence in neutral conditions (*Gryning et al.*, 2007)can be approximated as

$$\frac{u_{*0}}{f\,L_{MBL,N}} = -2\ln\left(\frac{u_{*0}}{f\,z_0}\right) + 55 \ . \qquad (18)$$

Figure 1 shows measurements of the dependence between $u_{*0}/f\,L_{MBL}$ normalized with its neutral value by use of Eq. (18), and $u_{*0}/f\,L$.

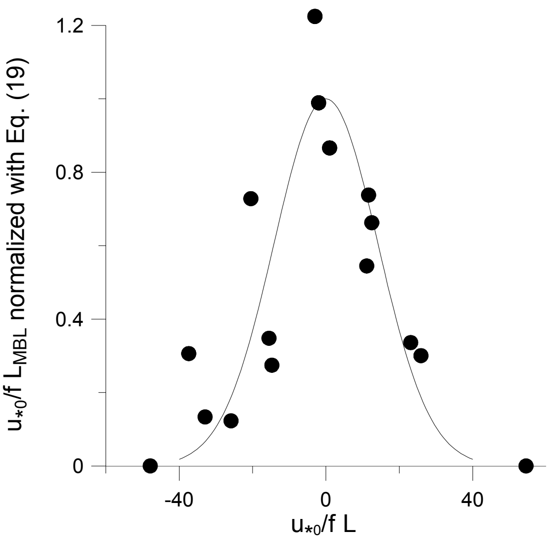

Fig. 3 ; Measurements (dots) from Høvsøre (rural) and Hamburg (urban) of the normalized length scale $u_{*0}/f\,L_{MBL}$ as a function of stability parameter $u_{*0}/f\,L$. The scaling is based on Rossby similarity theory. The full line shows a fit (Eq. (19)) to the measurements.

An empirical fit is also shown:

$$\frac{u_{*0}}{f\,L_{MBL}} = \left(-2\ln\left(\frac{u_{*0}}{f\,z_0}\right)+55\right)\exp\left(-\frac{\left(u_{*0}/f\,L\right)^2}{400}\right). \qquad (19)$$

It is noted that the data fit to Eq. (19) is better for stable than for unstable conditions. Knowing u_{*0}, z_0 and L as well as f allows L_{MBL} to be determined from Eq. (19) and the wind profile can then be estimated from Eqs. (7), (12) and (17). The height of the boundary layer can be taken from measurements, from an estimate from a validated meteorological pre-processor, e.g. *Batchvarova and Gryning* (1991), or it can be approximated by $z_i \approx 0.1\,u_{*0}/f$.

ACKNOWLEDGEMENTS

The study was supported by the Danish Energy Research Programme (EFP-2003 1363/03 0002), NATO (ESP.EAP.EV 981781) and internal funding from the Risø National Laboratory. The work is related to activities of the authors within COST728 , COST732 and COST735.

REFERENCES

Batchvarova E. Gryning S.-E., 1991: Applied Model for the Growth of the Daytime Mixed *Layer. Boundary-Layer Meteorol* **56**, 261-274

Dyer A.J. 1974: A review of flux-profile relationships. *Boundary-Layer Meteorol* **7,** 363-372

Carl D.M., Tarbell T.C., Panofsky H.A., 1973: Profiles of wind and temperature from towers over homogeneous terrain. *J Atmos Sci* **30**, 788-794

Gryning, S.-E,; Batchvarova, E., Brümmer, B., Jørgensen, H.E., Larsen, S.E., 2007: On the extension of the wind profile over homogeneous terrain beyond the surface boundary layer. *Boundary-Layer Meteorol.* (Article in press)

Panofsky H., 1973: Tower micrometeorology. *Workshop on Micrometeorology,* Ed D A Haugen. American Meteorological Society, pp 151-176