2 LENGTH-SCALES FOR INCLINED STABLE BOUNDARY LAYERS

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OUTLINE

- *1) Modeled SABL over-diffused; 2) Turbulence for Ri < 1*

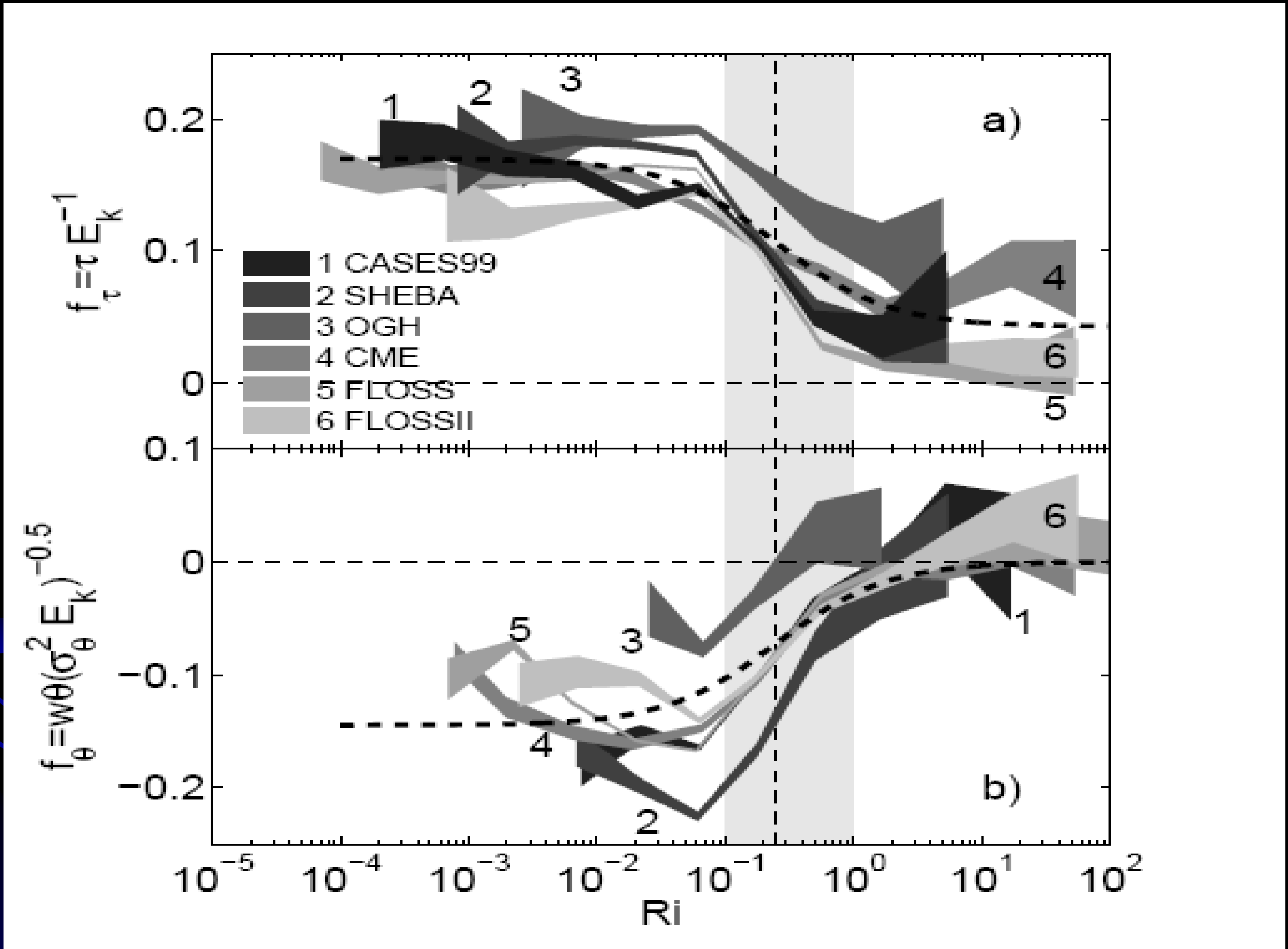
  & Ri > 1

- MODIFIED PRANDTL MODEL :

- MONIN-OBUKHOV LENGTH vs. ***LLJ*** HEIGHT

- ADJUSTING *“z-less”* MIXING LENGTH FOR ***SABL***

- Mauritsen et al. JAS 2007 ← →|



- *z-less Mix. length modified*

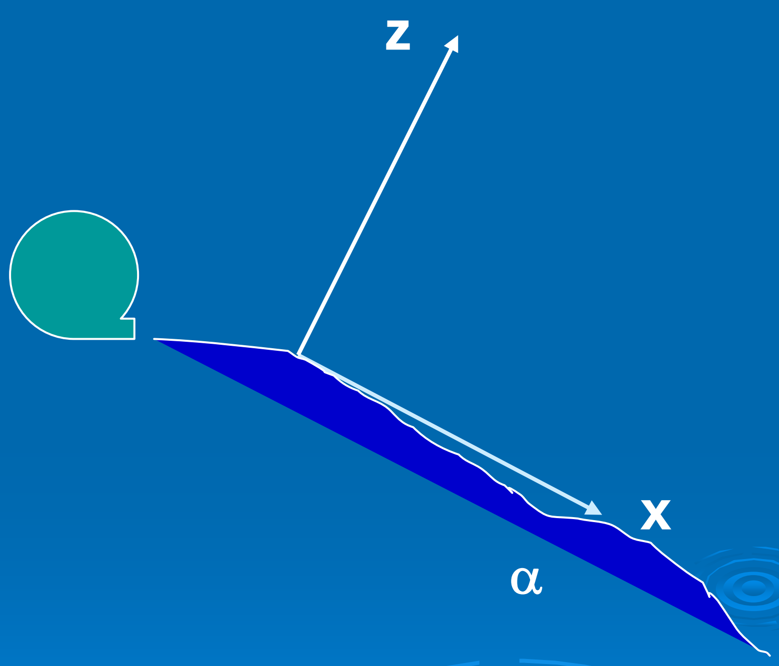
- **PRANDTL MODEL**

- ***L*** *$_{\text{MONIN-OBUKHOV}}$ modified*

(if a sfc. layer exists)

PRANDTL MODEL

- Analytical model for the "pure" katabatic flow
- Balance between the negative buoyancy & turbulent diffusion
- $\alpha < 0$



PRANDTL MODEL – *cont'd*

- Simplified N-S eqns.
- Heat (pot. temperature) eqn.

- Assume:
  - **steady-state** [later: semi-stationarity (***U*** & θ, not ***V***)]
  - **Boussinesq**
  - **hydrostatic balance**
  - **linearity** (no advection) but for finite amplitude perturb.
  - *simple* ***K***-theory [later: ***K = K(z)***]
  - $\gamma = d\theta/dz$ = constant, $\theta^{tot} = \theta + \gamma z$, $(\Theta = \theta_0 + \theta^{tot})$

PRANDTL: CLASSICAL ANALYTICAL MODEL

- Scale analysis: ***fcos(α)V*** small in ***x***-eqn. for ***U***
- Stationarity for ***U*** & θ after t > **1-1.5*T***, ***T = 2π/(Nsin(α))***,

U:

$$0 = \frac{g}{\theta_0}\sin(\alpha)\theta + \Pr\frac{d}{dz}\left(K\frac{dU}{dz}\right)$$

θ:

$$0 = -\gamma\sin(\alpha)U + \frac{d}{dz}\left(K\frac{d\theta}{dz}\right)$$

MIUU: Modified 'z-less' Mixing length

- *Replace:*

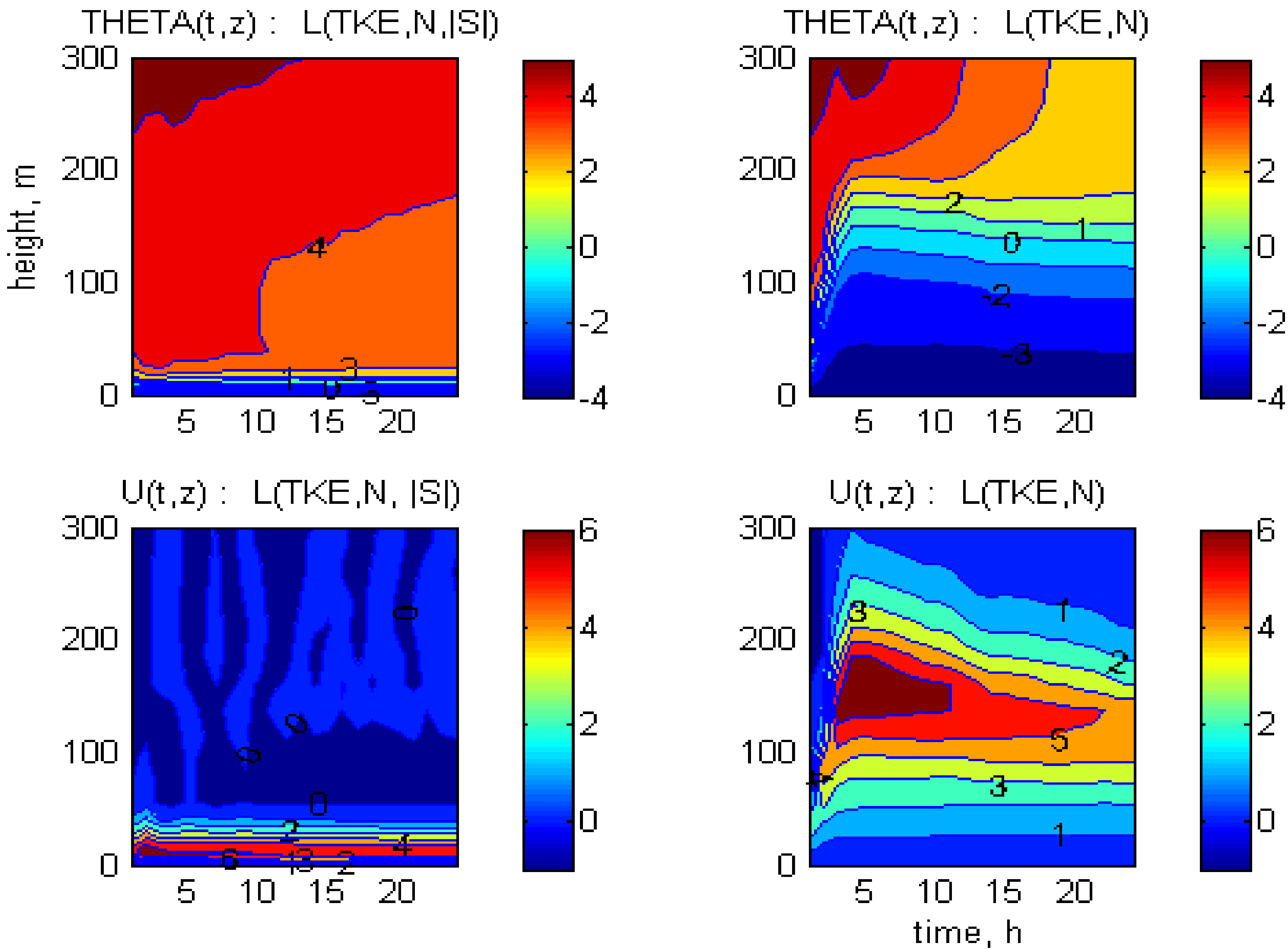
$$l_{STAB} = a\frac{(TKE)^{1/2}}{N}$$

- *By:*

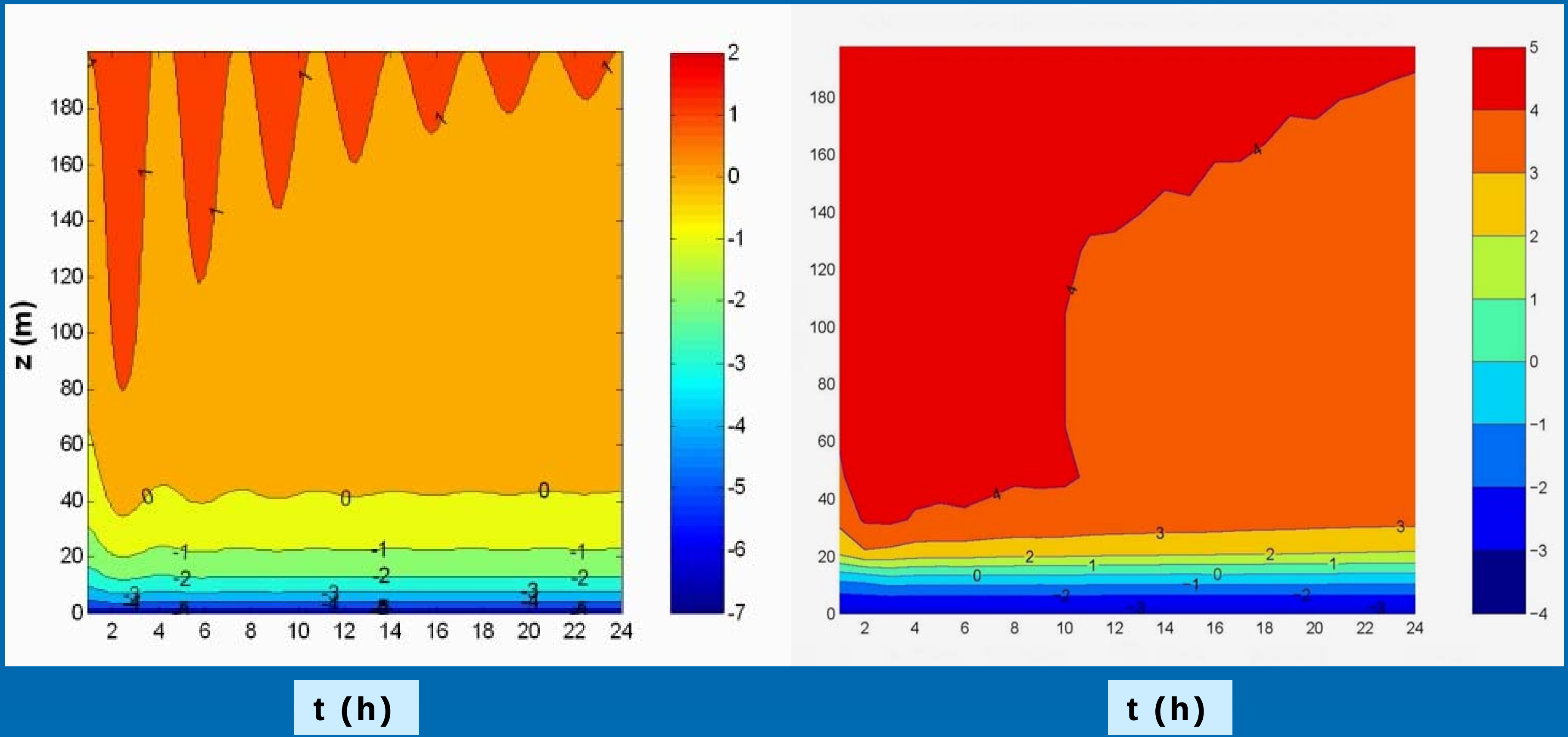
$$l_{STAB} = \min[a\frac{(TKE)^{1/2}}{N}, b\frac{(TKE)^{1/2}}{|S|}]$$

- *… and find '**b**' ('**a**' known, in MIUU model: **a** ≈ 0.5)*

Same start| **L**: Correct VSABL | **R**: Over-Diffusive SABL



(a) $\theta^{tot}_{num}(°C)$ (b) $\theta^{tot}_{MIUU}(°C)$



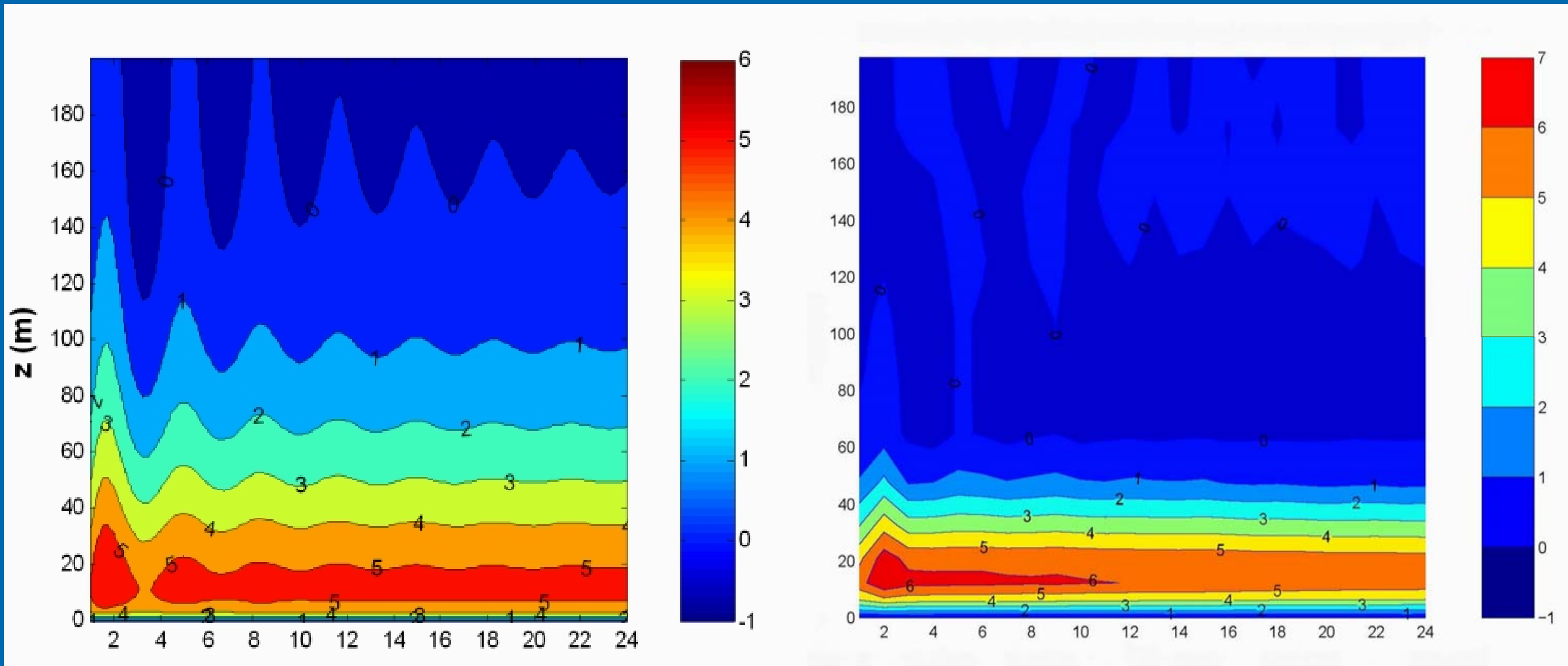
(a) simple numerical *(oscil. with T)* **(b) MIUU model**

$(f, \alpha, \gamma, Pr, C) = (1\cdot10^{-4}\ s^{-1}, -2.2°, 5\cdot10^{-3}\ Km^{-1}, 1.1, -6.5\ °C)$

K(z): h = 200 m, K_{max} = 2 m^2s^{-1} here: T = 3.4 h

(a) $U_{num}(ms^{-1})$

(b) $U_{MIUU}(ms^{-1})$



t (h)

t (h)

(a) simple numerical

(b) MIUU model

The rest as before. Max(U_{num}) ≈ 5.5 ms^{-1} & *LLJ height* ≈ 16 m

Numerical (U, θ) with elevated spurious oscillations decaying in time, ***T*** *≈ 3.4h*

Conclusion ½: MIXING LENGTH

- *(U, θ) reach **steady state** after $t > T$; V_{num} & V_{WKB} diffuse upwards*

- Simple numerical, WKB & MIUU MODEL agree for (θ_{tot}, U, V) ***only with new Mix. Length***

- ***<u>USE:</u>***

  *Very **SABL**, $Ri \to \infty$, => difficult to treat in NWP & climate models (usually over-diffusive) ⇔ a solution is offered*

▪ MONIN-OBUKHOV vs. *LLJ* HEIGHT

$$L_{MO} = \frac{-\bar{\theta}}{gk} \frac{u_*^3}{\overline{w'\theta'}}$$

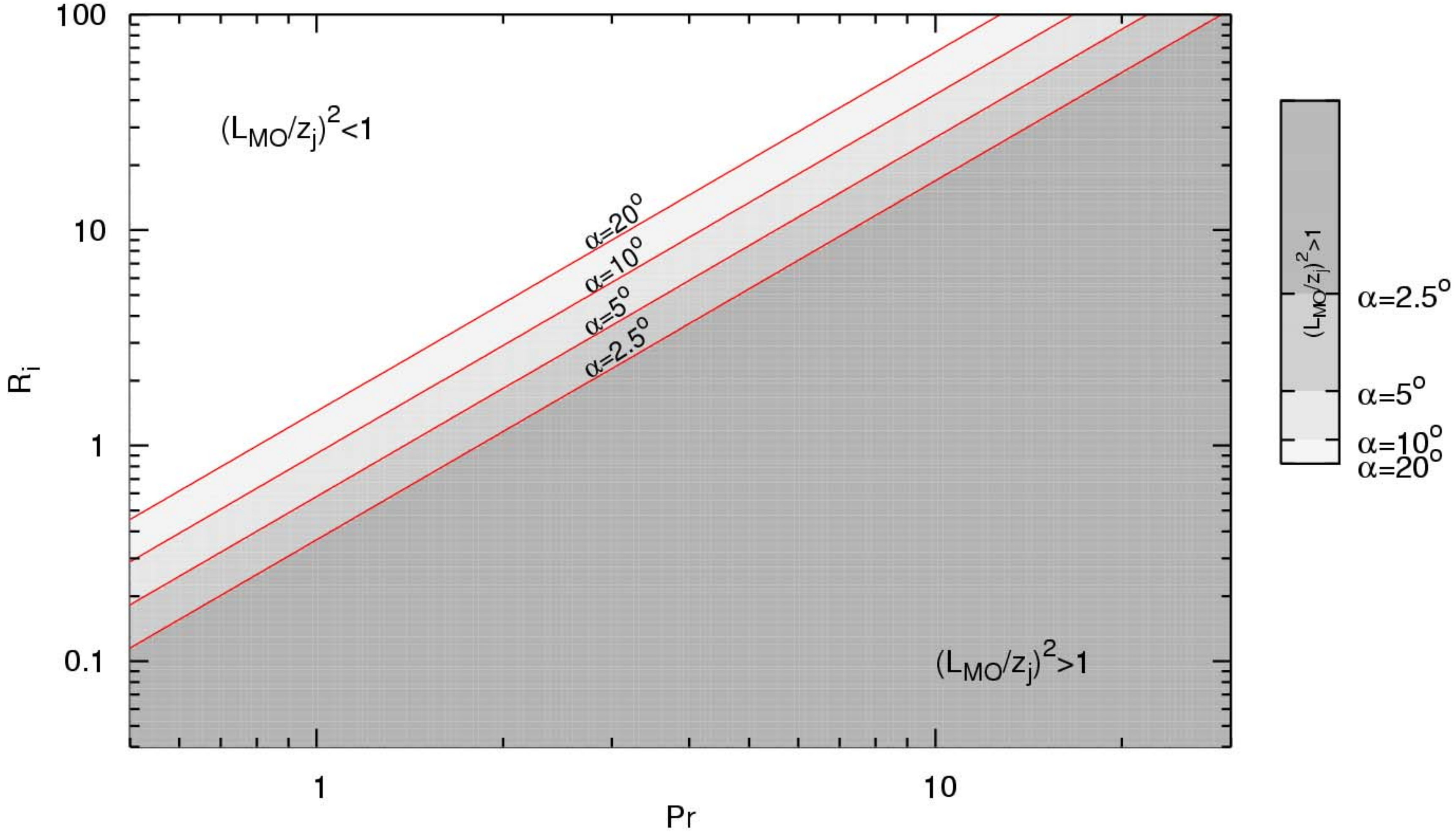
$$z_j = \frac{\pi}{4} \left(\frac{4K^2 \Pr}{N^2 \sin^2(\alpha)} \right)^{1/4}$$

$$\left({L_{MO}} \big/ {z_j} \right)^2 = \frac{8}{(k\pi)^2} |\sin(\alpha)| \left(\frac{\Pr^5}{Ri^3} \right)^{1/2}$$

- if α small, e.g. < 0.1 rad & Pr ≈ *Ri* ~ 1, =>$L_{MO} < \boldsymbol{z_j}$, => classical Monin-Obukhov OK; note ***N*** is external param.

For an existing inclined surface layer

- For $(L_{MO}/z_j) > 1$, L_{MO} is less relevant scale for sfc. fluxes



- **CONCLUSION** 2/2: A modified $\boldsymbol{L_{MO}}$ includes the ***LLJ***

height:

- $\boldsymbol{L_{mod} = min(L_{MO}, c\, z_j)}$, c ~ 0.8

- Or at least:

$$L_{MOD}^{-1} = aL_{MO}^{-1} + bz_j^{-1}$$

- For NWP & climate models

- It tells where/when classic L_{MO} wrong in **sloped very SABL**

APPENDIX

- If discussion desires, continue …
- Otherwise, be seated quietly…
- & Let the others continue …

MODIFIED PRANDTL: [f & $K(z)$]

$$\frac{\partial U}{\partial t} = \frac{g}{\theta_0}\sin(\alpha)\theta + f\cos(\alpha)\; V + \Pr\frac{\partial}{\partial z}\left(K\frac{\partial U}{\partial z}\right)$$

$$\frac{\partial V}{\partial t} = -f\cos(\alpha)U + \Pr\frac{\partial}{\partial z}\left(K\frac{\partial V}{\partial z}\right)$$

$$\frac{\partial \theta}{\partial t} = -\gamma\sin(\alpha)U + \frac{\partial}{\partial z}\left(K\frac{\partial \theta}{\partial z}\right)$$

B. C.

$$\theta(z=0) = C, \quad U(z=0) = V(z=0) = 0$$

$$\theta(z\to\infty) = U(z\to\infty) = V(z\to\infty) = 0$$

- PRANDTL: WKB SOLUTIONS FOR (θ, ***U***, V) ; ***K = K(z)***

$$\theta_{WKB} = C\exp(-\sigma_{WKB}(z))\cos(\sigma_{WKB}(z)),$$

$$U_{WKB} = \frac{C\sigma_0^{\,2}}{\gamma\sin(\alpha)}\exp(-\sigma_{WKB}(z))\sin(\sigma_{WKB}(z)),$$

$$V_{WKB} \approx \frac{-Cf\cot(\alpha)}{\Pr\gamma}\left[1-erf\left(\frac{I(z)}{2\sqrt{t\Pr}}\right)-\exp(-\sigma_{WKB}(z))\cos(\sigma_{WKB}(z))\right],$$

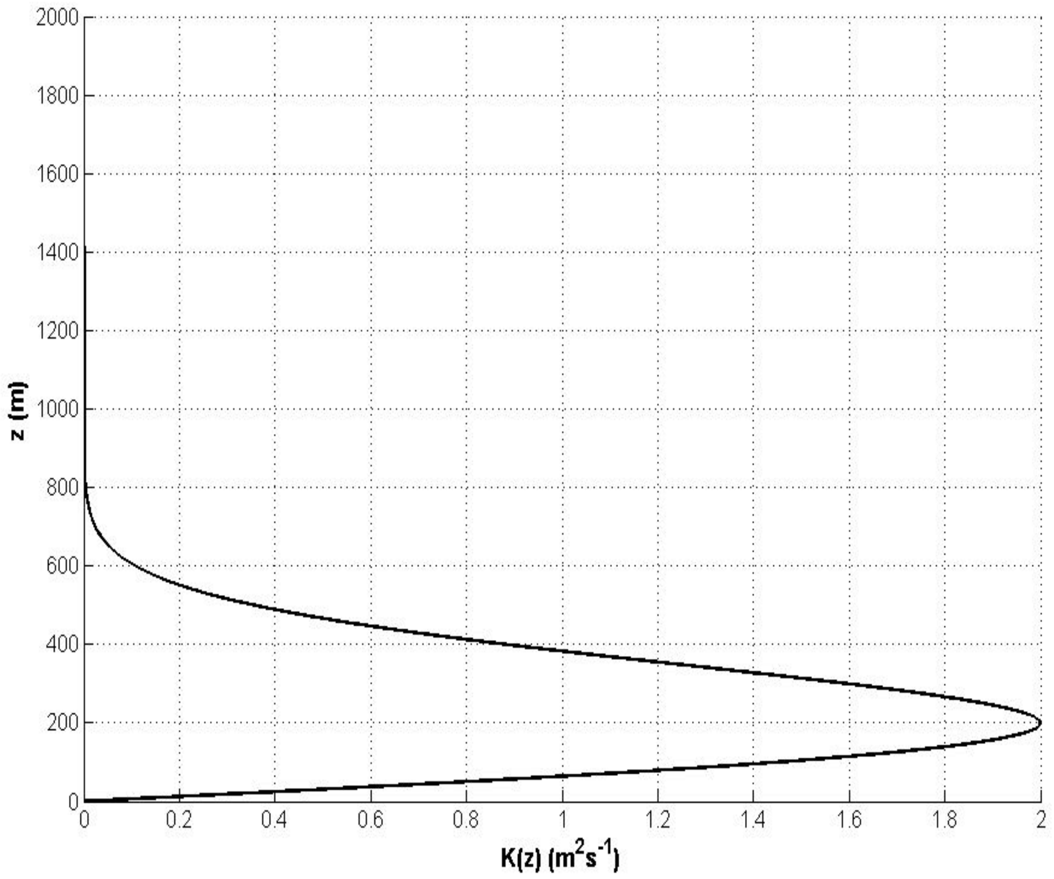
$$\sigma_{WKB}(z) = \frac{\sigma_0}{\sqrt{2}}I(z),$$

$$I(z) = \int_0^z K(z)^{-1/2}\,dz,$$

$$\sigma_0^{\,4} = \frac{N^2\Pr\sin^2(\alpha)+f^2\cos^2(\alpha)}{\Pr^2}$$

- ***K(z)* profile:**

$$K(z) = K_{max}\sqrt{e}\,\frac{z}{h}\exp\left(-\frac{z^2}{2h^2}\right)$$



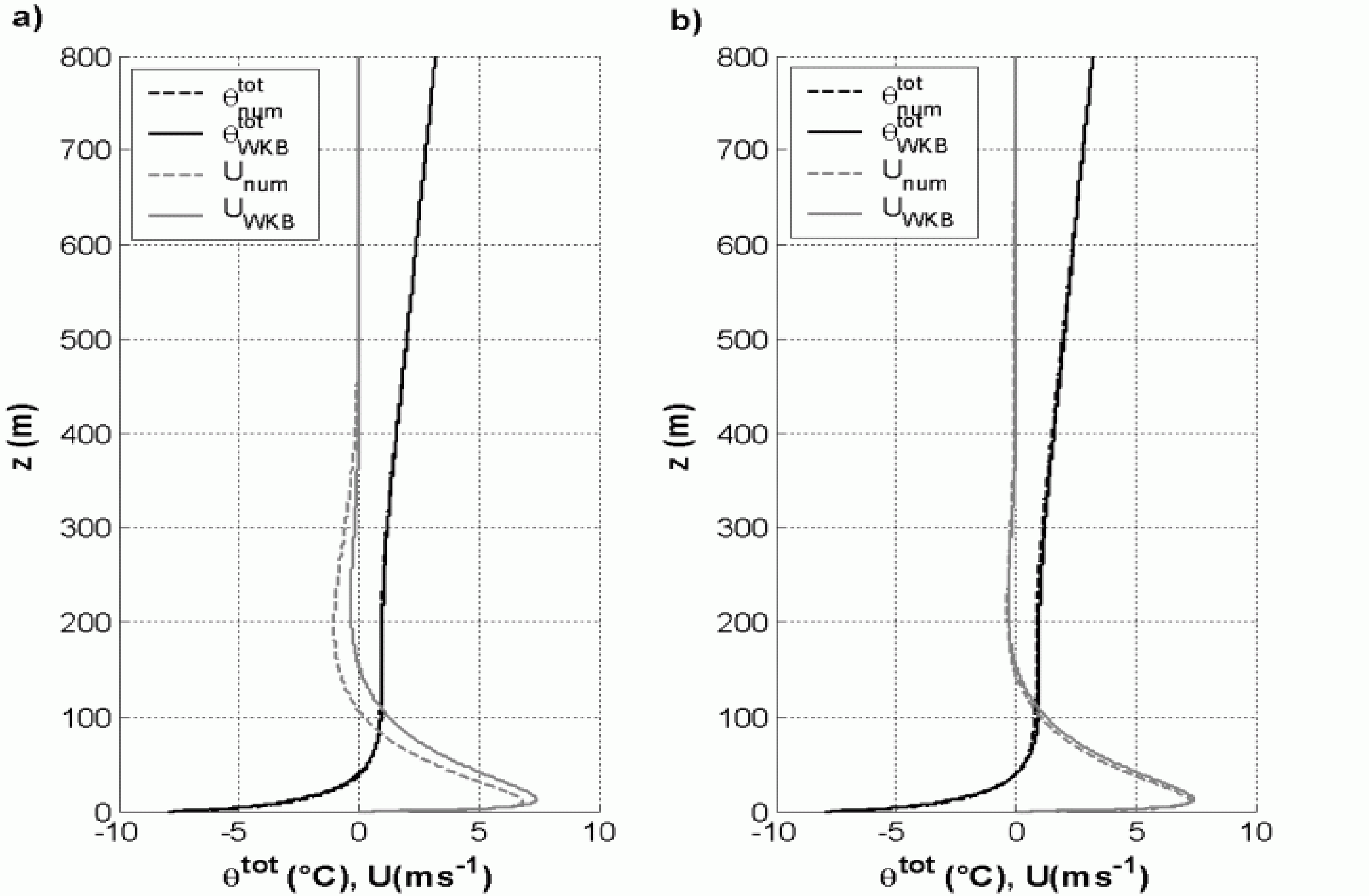
K(z) for numerical & analytical solutions, only 2 parameters!

e.g.

K_{max} **= 2 m²s⁻¹,**

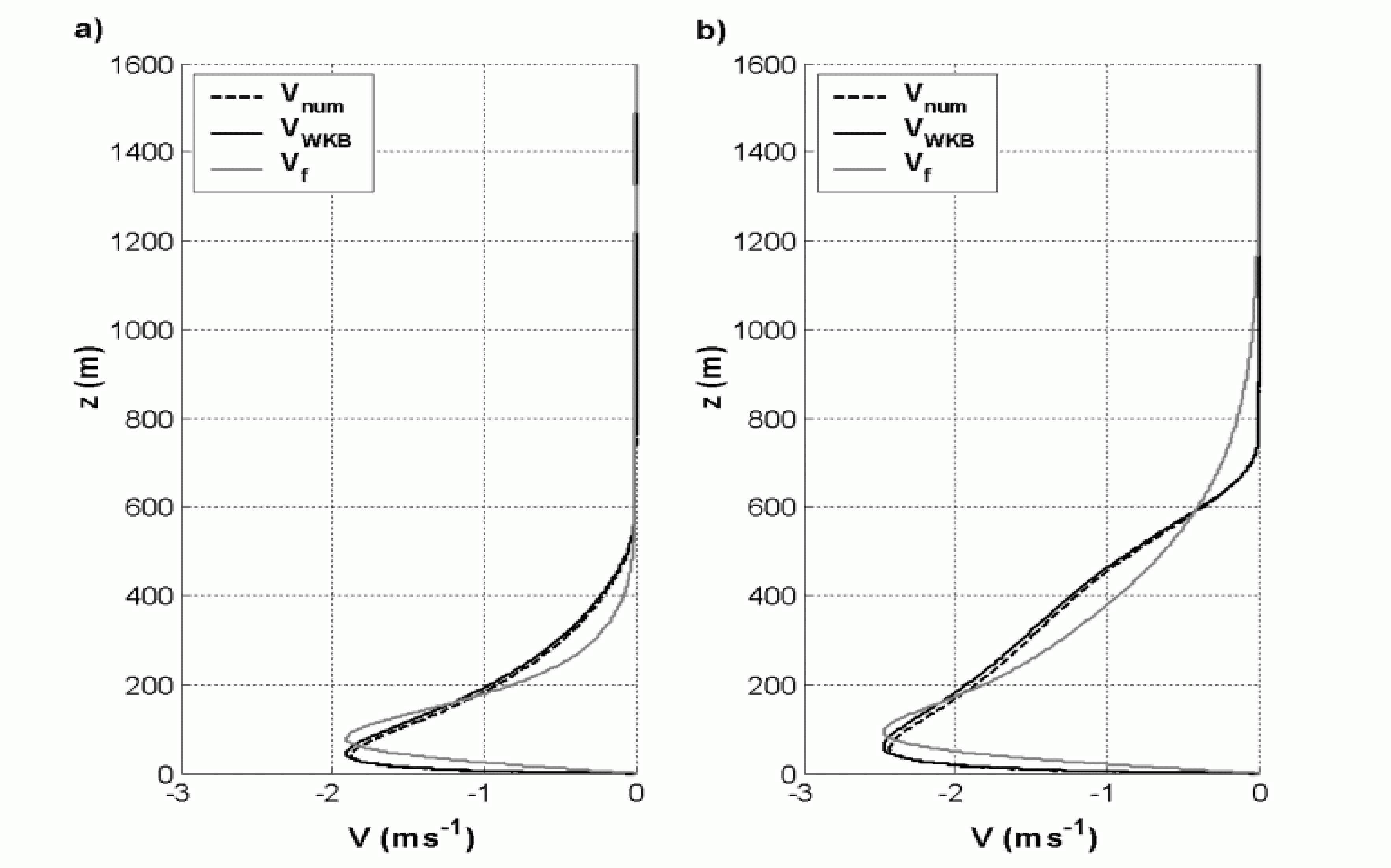
***h* = 200 m,**

(JAS 2001, QJ2001,...)



a) $t = T$ b) $t = 10T$, $T \approx 2.1$h

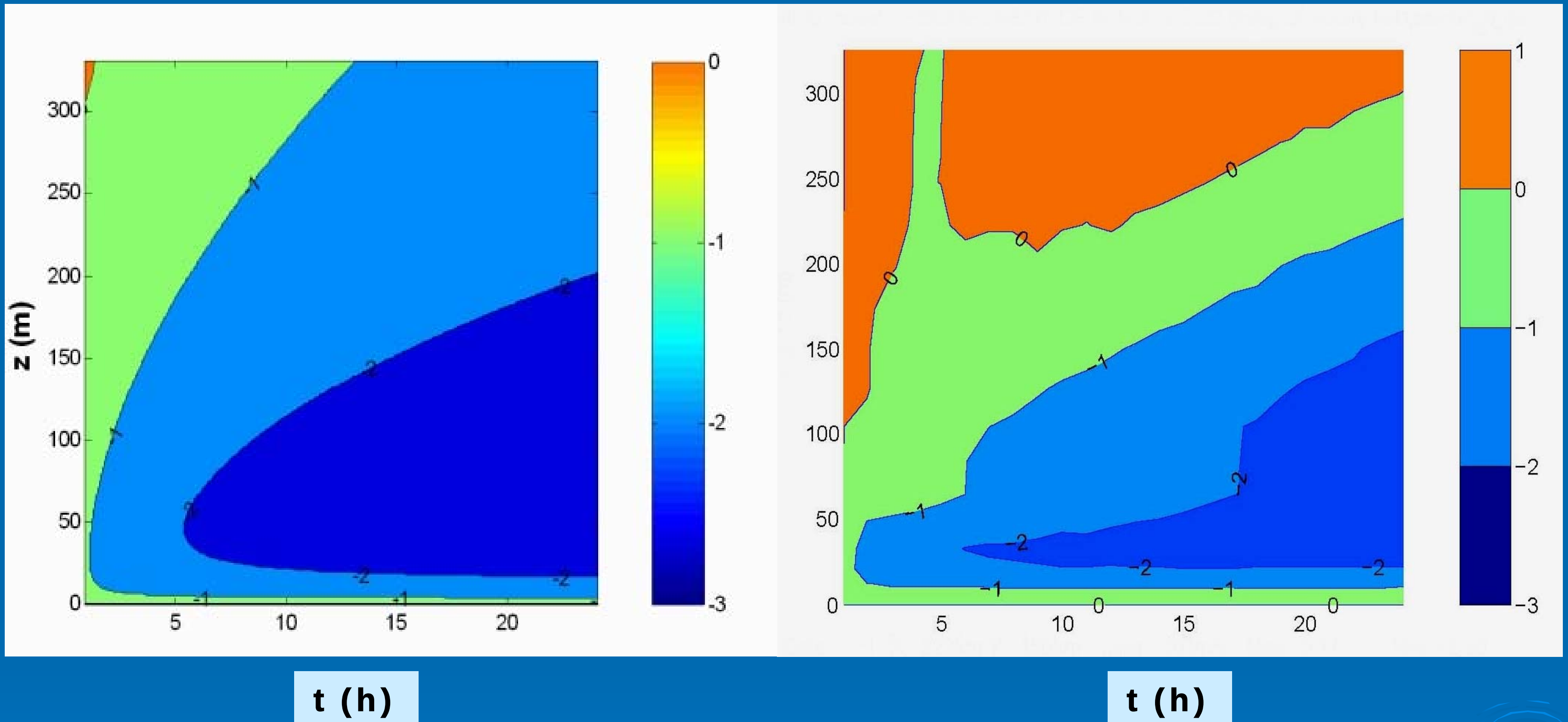
$(\alpha, \gamma, Pr, C, f)$ = -4°, 4 ·10^{-3} Km^{-1}, 1.1, - 8°C, 1.1·10^{-4}s^{-1}



(a) t = T (b) *t* = 10*T*

(Stiperski et al. QJRMS 2007; Kavčič & B.G., BLM 2007) *V_f uses K = const = K_{max} / 3*

(a) $V_{WKB}(ms^{-1})$ (b) $V_{MIUU}(ms^{-1})$



(a) WKB solution **(b) MIUU model**

The rest as before. Min(V_{WKB}) ≥ -2.75 ms^{-1}

WKB sol. has no spurious (low energy) oscillations

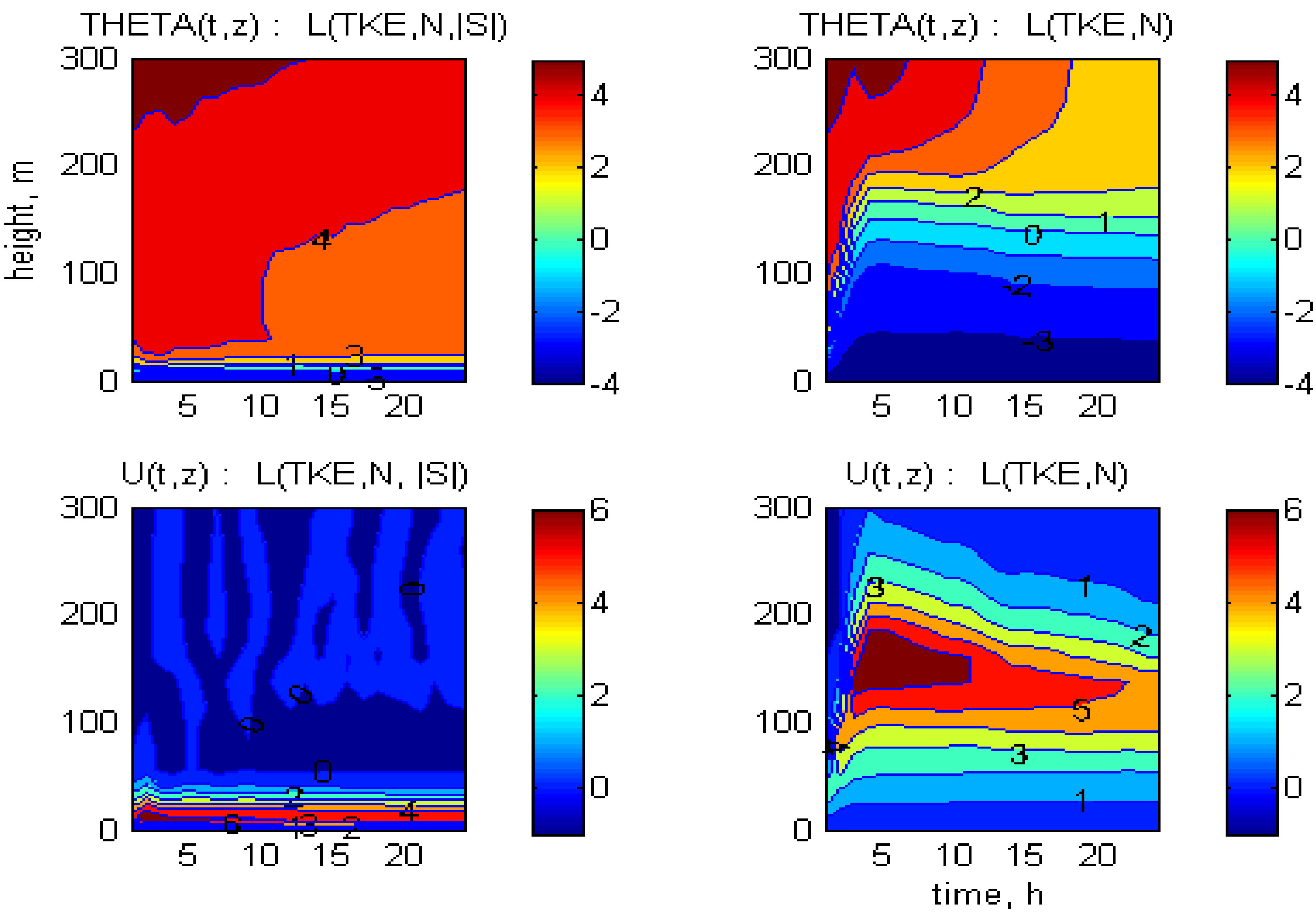
MIUU MODEL & Simulation Parameters

- nonlinear, hydrostatic, *f*-plane
- 2.5-turbulence closure level
- 5 progn. eqns: ***U***, ***V***, Θ, ***q***, ***TKE*** (e.g. *QJRMS 2004*)

➢

- $\Delta x \approx 1.9$ km = const, $\alpha = -2.2^{\circ}$, $C \approx -6.5^{\circ}C$
- 211 × 7 × 201 gridpoints (*y=const*), total(nk)=402
- $\Delta t = 13$ s, $z_{TOP} = 5.6$ km (***"sponge"*** for $z \geq 4.4$ km)
- **1 m < Δz < 30 m**, ***"staggered, terrain influenced"***
- Initialisation: ***U*** & ***V*** ≈ 0, $\gamma = 5\cdot10^{-3}$ Km^{-1}

$L_{STAB} = \min[a(TKE)^{1/2}/N,\ a/2(TKE)^{1/2}/|S|], \quad a \approx 0.5$



Mix. length: NEW, left; OLD, right