

4.21 ON THE NUMERICAL MODELING OF 3D - ATMOSPHERIC BOUNDARY LAYER FLOW

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INTRODUCTION

The paper deals with a problem of flow and pollution dispersion in the atmospheric boundary layer. The ABL is significantly influenced by the surface over which the wind flows, its orography and roughness, by the free stream wind and also by the vertical temperature gradient which is associated with the atmospheric thermal stratification. The ABL has a very close relation to a human activity and the prediction of wind field over complex terrain plays an important role in many engineering applications such as an evaluation of environmental impact by pollutant dispersion.

MATHEMATICAL MODELS

The flow is assumed to be steady, incompressible, turbulent and indifferently stratified. The two different mathematical and numerical methods has used for numerical simulations.

The Full RANS model

The governing equations of the first model can be re-casted in the conservative, non dimensional and vector form. The artificial compressibility method is used for the numerical analysis

$$W_t + F_x + G_y + H_z = (K \cdot R)_x + (K \cdot S)_y + (K \cdot T)_z + f_v \quad (1)$$

$$F = (u, u^2 + p, uv, uw, uC)^T, \quad G = (v, vu, v^2 + p, vw, vC)^T, \quad H = (w, wu, wv, w^2 + p, wC)^T, \\ R = (0, u_x, v_x, w_x, \frac{1}{\sigma_C} C_x)^T, \quad S = (0, u_y, v_y, w_y, \frac{1}{\sigma_C} C_y)^T, \quad T = (0, u_z, v_z, w_z, \frac{1}{\sigma_C} C_z)^T.$$

$W = (\frac{p}{\sigma_C}, u, v, w, C)^T$ abbreviates the vector of unknown variables respectively, the pressure, the three velocity components and the concentration of passive pollutant, f_v denote the volume force, σ_C is the turbulent Prandtl's number and finally K represents the turbulent diffusion coefficient, see equation (7).

Boussinesq equations

The RANS equations are simplified by the so called Boussinesq approximation according to which the mean turbulent quantities appearing in the RANS equations are decomposed into the background "synoptic-scale" field denoted by subscript $_0$ and the topography induced "meso-scale" perturbation denoted by $''$. This decomposition is applied to the density $\rho = \rho_0 + \rho''$ and the pressure $p = p_0 + p''$. Then the governing equations can be re-casted in the non-conservative and dimensional form

$$(\rho_0 u)_x + (\rho_0 v)_y + (\rho_0 w)_z = 0 \quad (2)$$

$$V_t + uV_x + vV_y + wV_z = -\frac{\nabla p''}{\rho_0} + \frac{1}{\rho_0} \{[\rho_0 KV_x]_x + [\rho_0 KV_y]_y + [\rho_0 KV_z]_z\} + f_v \quad (3)$$

where the velocity vector is denoted by $V = (u, v, w)^T$ and f_v is the volume force. The transport equations for the passive pollutant C

$$C_t + (V \cdot \nabla)C = \frac{1}{\rho_0} \nabla \left[\frac{K}{\sigma_C} \nabla C \right] \quad (4)$$

where σ_C denote the turbulent Prandtl's numbers.

Turbulence and canopy layer models

The force vector \vec{f}_v includes the specific aerodynamic force corresponding to the drag induced by the vegetation.

$$\vec{f}_v = col(-c_d a |V| u, -c_d a |V| v, -c_d a |V| w) \quad (5)$$

In the above expression the $c_d(z)$ denotes the bulk drag coefficient of trees (as a function of vertical coordinate z). The characteristic area of canopy $a(z)$ could be evaluated as a product of leaf area density $a^*[m^2 \cdot m^{-3}]$ and the local canopy height $h[m]$, i.e. $a(z) = a^* h$. In our work we deal with total resistance parameter $r_h(z) = c_d a$. The vertical profile of this parameter has been set-up in the following way:

$$r_h(z) = \begin{cases} r \frac{z/h}{0.75} & \text{for } 0 \leq z/h \leq 0.75 \\ r \frac{1-z/h}{1-0.75} & \text{for } 0.75 \leq z/h \leq 1.0 \end{cases} \quad (6)$$

Closure of both systems (1) and (2)-(4) of governing equations is achieved by a simple algebraic turbulence model designed for ABL flow. The diffusion coefficient K takes the following form in the dimensional case.

$$K = \nu + \nu_T \quad \text{where} \quad \nu_T = l^2 \sqrt{(u_z)^2 + (v_z)^2} \quad (7)$$

ν_T is the turbulent viscosity, ν is the laminar viscosity and l refers to the Blackadar's mixing length computed from.

$$l = \frac{\kappa(z+z_0)}{1 + \kappa \frac{(z+z_0)}{l_\infty}}, \quad l_\infty = \frac{27|V_g|10^{-5}}{\lambda} \quad (8)$$

where κ is the von Karman constant, λ denotes the Coriolis parameter, z_0 the roughness length, l_∞ denotes the mixing length for $z \rightarrow \infty$ and V_G is the geostrophic wind velocity at the upper boundary of domain.

NUMERICAL METHODS

We have solved the governing systems of equations under stationary boundary conditions for $t \rightarrow \infty$ to obtain the expected steady-state solution. The structured non-orthogonal grids made of hexahedral control cells are used.

The finite volume method (cell centred) together with multi-stage explicit Runge–Kutta time integration scheme have applied to the first system (Bodnár T., et. Al, 2000) after its integration over each computational cell. The semi-implicit finite difference scheme has used for the second model (2)-(4). The special combination of different asymmetric space discretization at time level n and $n+1$ leads to the numerical scheme that is centered and second order both in space and time. Both models (1) and (2)-(4) use the following boundary conditions – Inlet: $u = U_0(z/L)^\alpha, v = w = C = 0$; Outlet: $u_x = v_x = w_x = C_x = 0$; Wall: the no-slip condition for the velocity components, $\frac{\partial C}{\partial n} = 0$; Top face: $u = U_0, v = 0, \frac{\partial w}{\partial z} = \frac{\partial C}{\partial z} = 0$; Side faces: periodic or non-periodic. A stationary and constant intensity area source of pollutant concentration must be defined in the computational domain.

THE REAL CASE 1

This practical problem is related to the flow over a surface coal field located in the North Bohemia. In the future, this field is supposed to be partially covered by a high forest stand. The computational domain is 1000 m long and 300 m high and is discretized by 1000x40 cells. The other parameters are: the mean free stream velocity $U = 10\text{ m/s}$, $Re = 2 \cdot 10^8$, the roughness parameter $z_0 = 0.1\text{ m}$ and the power law exponent 2/9 are used to the inlet velocity profile. In order to trace the effect of the forest stand on the ABL flow, the following cases have been computed: CASE–1 the flow over flat topography with homogeneous roughness, CASE–2 the flow over simplified theoretical topography (mainly here we tested the effect of different stand's height and its drag) and CASE–3 the flow over real topography of coal field. In the CASE–2, the homogeneous forest stand is supposed to be 90 m long and it starts 5 m after the sudden step at ground (fig. 1). The following studies have been performed: 1) the effect of different stand's heights of 0, 5, 10, 15 m for the drag coefficient $r = 0.19$ (fig. 2) and 2) the effect of different drag coefficients $r = 0.0, 0.04, 0.19, 0.95$ for the stand's height of 10 m (fig. 2). The forest stand is displayed as a gray strip in the figure 2.

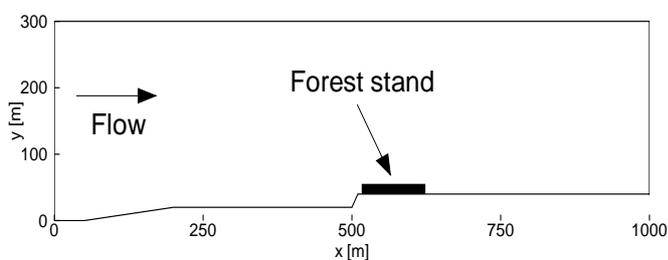


Figure 1. The computational domain in the CASE - 2

One can clearly see from figure 2 the increasing flow deceleration with the increasing stand's height on the lee side of the stand's block. However, this tendency is not so evident inside the forest. In the figure 3 the deceleration is monotonously dependent on the drag coefficient both inside and also behind the forest stand.

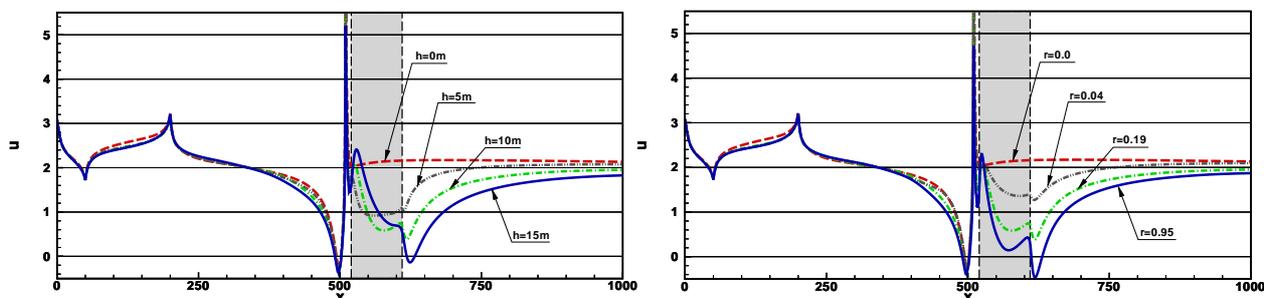


Figure 2. The near ground profile of the u -velocity component for different stands heights h . (on the left) and for different drag coefficients r (right).

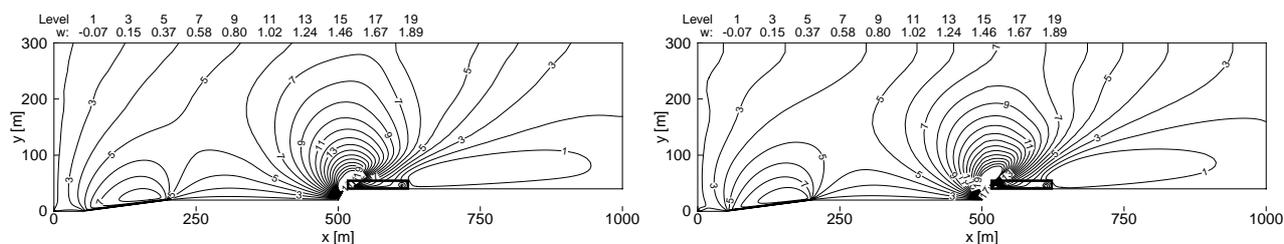


Figure 3. The wall-normal velocity component in the case of forest stand height 15m and $r=0.19$, RANS equations (left), Boussinesque equations (right).

THE REAL CASE 2

A wall modeling study of the 3D-flow over a complex relief of Prague's agglomeration have been performed and mainly the concept of a wall function approach was evaluated and the results have been compared with a no-slip wall modeling.

Wall modeling is very important. If the no-slip condition is applied, the grid must be sufficiently fine in the wall vicinity and this increases the CPU-cost. On the other hand, the wall-function approach is much less CPU time consuming since the grid is significantly coarser at the wall. The near-wall velocities are computed from an analytical expression

$$\sqrt{u^2 + v^2 + w^2} = \frac{u^*}{\kappa} \log\left(\frac{z_1}{z_0}\right) \quad (9)$$

where u^* denotes the friction velocity, and z_1 is the distance of the first inner grid node from the wall.

The computational domain is 43 km long, 35 km wide, about 1 km high .

The other parameters are: the mean free stream velocity $U = 10$ m/s , the Reynolds number $Re = U \cdot L / \nu = 6.7 \cdot 10^8$, the roughness parameter $z_0 = 1$ m and the power law exponent 0.3 and the friction velocity $u^* = 0.33$ m/s are used for the inlet velocity profile, indifferently stratified ABL is supposed. The comparison of results obtained from both wall modeling approaches can be seen on the horizontal near ground cut-planes colored by the u -velocity component, see figure 5.



Figure 4. Prague's area, geographical altitude contours.

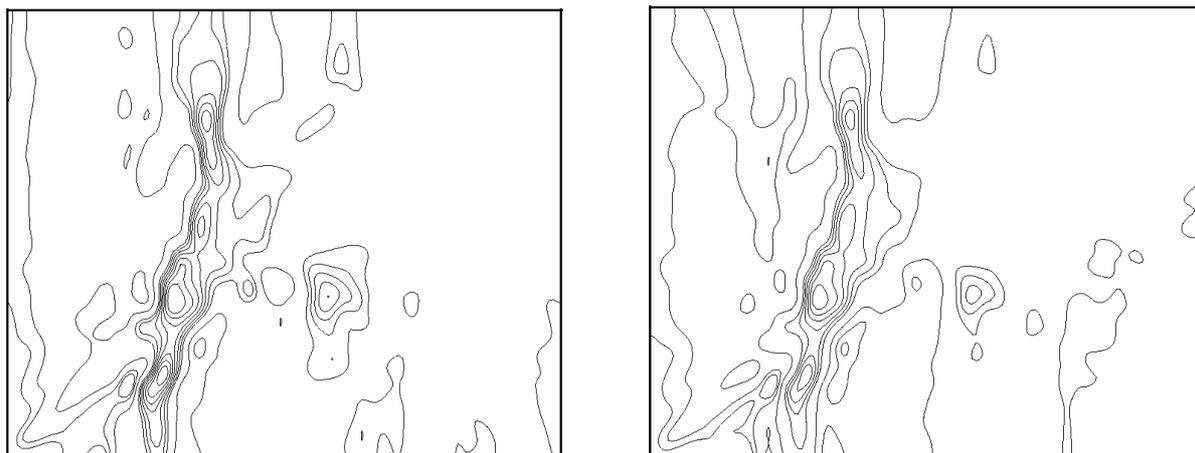


Figure 5. Comparison of u -velocity component on the horizontal cut-plane. No-slip b.c. (left) and wall function (right).

The cut-planes are at the level of 28.3 m above the ground and one can see the good agreement between the figures 7 and 8.

CONCLUSION

The performed computations shows a good applicability of our model to the real terrain simulations. The results obtained using this models are in good agreement with our expectations. The improvement of turbulence model is necessary. These models require just a few input data to provide a quite complex and detailed information about the flow structure in the domain of interest. This makes possible a complex study of the 3D atmospheric processes in deep.

ACKNOWLEDGMENT

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REFERENCES

- Bodnár T., Kozel K., Ph. Fraunié, Jaňour Z., 2000:* Numerical Simulation of Flow and Pollution Dispersion in 3D Atmospheric Boundary Layer, Computing and Visualization in Science, Vol. 3, Num. 1-2, Springer Verlag, Berlin, May 2000.
- Bodnár T., Fraunié P., Kozel K., Beneš L., 2001:* Numerical Modelling of Pollution Dispersion in Complex Terrain, Air Pollution IX. Southampton, WIT Press, 2001, pp. 85-94, ISBN 1-85312-877-5.
- Beneš L., Jaňour Z., Kozel K., Sládek I., 2001:* Mathematical Modelling and Numerical Solution of Atmospheric Boundary Layer, Proceedings in CD-ROM, ISSN 1560-7534, Vol 6, Novosibirsk, June 2001.
- Bodnár T., Beneš L., Jaňour Z., Kozel K., Sládek I., 2003:* On the Complex Atmospheric Flow Modelling Including Pollution Dispersion, conference – Wind Effects On Trees, Karlsruhe, September 2003, pp. 183-188, ISBN 3-00-011922-1.
- Wilson J.D., Flesch T.K., 1999:* Wind and remnant tree sway forest cutbl ocks. III. A windflow model to diagnose spatial variation, Agricultural and Fore st Meteorology No. 93, pp. 259–282, 1999.