

**Monin-Obukhov similarity in the urban
inertial sublayer under unstable
conditions**

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INTRODUCTION

Monin-Obukhov similarity theory has not been verified in the urban inertial sublayer under wide range of atmospheric stability conditions, because of the lack of observations.

Observations of turbulence in Tokyo (Moriwaki and Kanda, 2006) and in Hamburg (Gryning et al., 2007) conducted under $-L > 100$ m did not provide the data under very unstable conditions. The average values of ϕ_h and ϕ_m calculated from those data were close to the empirical form presented by Businger et al. (1971).

The present paper discusses the applicability of Monin-Obukhov similarity theory for heat and momentum in urban areas under the wide range of unstable conditions for $-L > 20$ m ($-2.6 \leq z'/L < 0$) where $z' = z - d$, d is the zero-plane displacement.

Turbulence was measured at a tower in Himeji city.

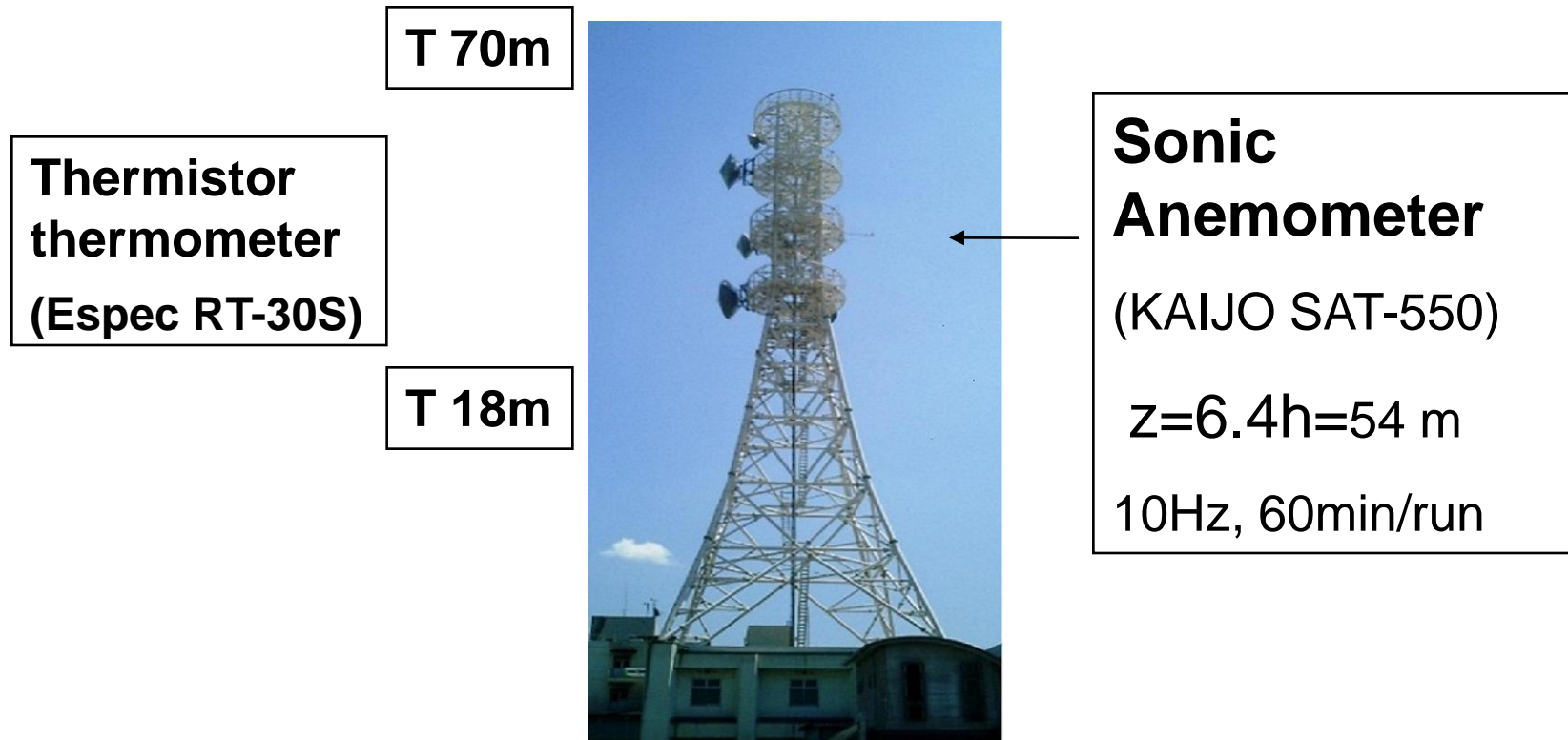


Fig. 1 A tower in Himeji city and layout of instruments.

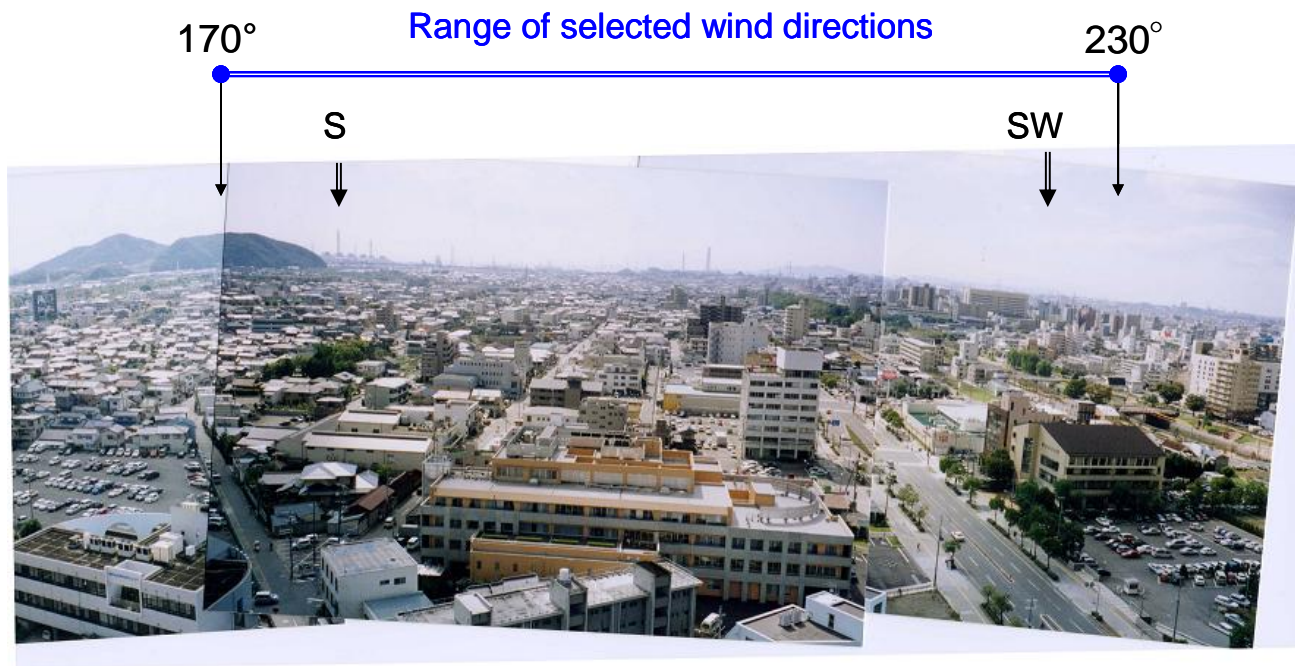


Fig. 2 Selected Wind directions: (170° – 230°)

Table 1. Summary of observed turbulence

	u (m s^{-1})	L (m)	z'/L	u_* (m s^{-1})	H (W m^{-2})
ave.	3.4	-260	-0.62	0.35	134
m ax.	5.9	-20	-0.03	0.62	419
m in.	1.0	-1769	-2.59	0.12	2.64
n	80				

Observation period: April–August, 2006 and June–July, 2007

Building height and area density were investigated upstream of the tower.

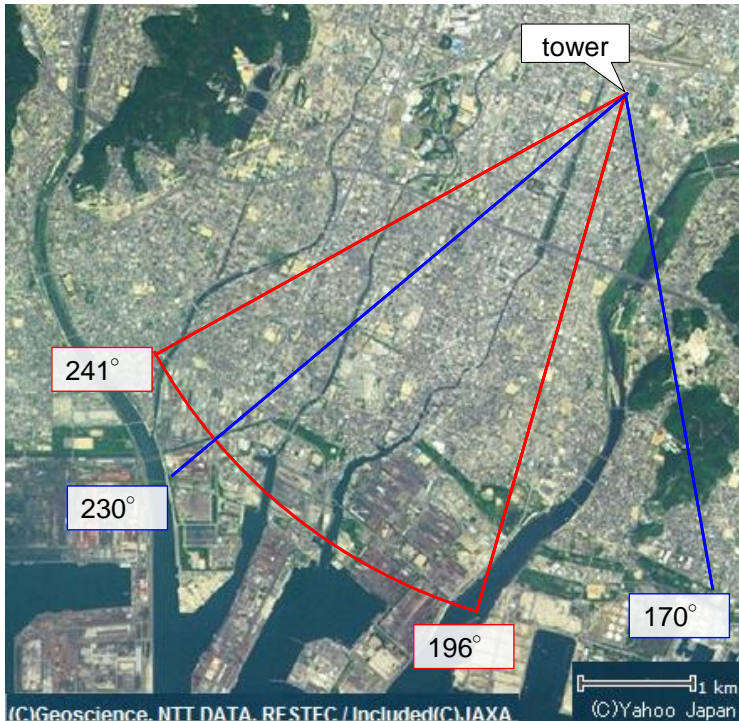


Fig. 3 A sector (red line) where the building height and area density were investigated.

Table 2. Building height and area density in the sector (196°-241°).

x(km)	Average building height h(m)	Geometrical roughness h_g (m)	Building-to-land ratio ρ (m ² /m ²)
0-1 k m	8.8	2.3	0.27
0-2 k m	9.7	3.1	0.33
0-3 k m	9.2	3.4	0.38
0-5 k m	8.5	2.8	0.32

x(km): distance from the tower to the arc

$$\mathbf{h} = \sum \mathbf{h}_i s_i / \sum s_i \quad \mathbf{h}_g = \sum \mathbf{h}_i s_i / S$$

S: total area of the sector, h_i : building height, s_i : the area occupied by the building.

We assumed that zero-plane displacement $d=h_g=2.8$ m.

The z_0 value was determined.

The z_0 value was determined using the logarithmic wind profile and values of wind velocity u and friction velocity u_* which were obtained at the tower for neutral conditions. The corresponding surface roughness $z_0=1.6$ m (sector 170° to 230° : blue line).

Scaling velocity for L

It is known that in the roughness sublayer, the momentum flux has maximum value approximately at $1.5h$ for uniform building arrangements.

According to Moriwaki and Kanda (2006), the maximum friction velocity u_{*r} was used as the scaling velocity (surface scaling) to obtain the Monin-Obukhov length, L . We estimated that $u_{*r} = 1.5u_*(z=6.4h)$ from the wind tunnel experiments (Uehara et al, 1997) and the field experiments (Oikawa and Meng, 1995).

On the other hand, the heat flux observed at $z=6.4h$ (local value) was used to estimate L , according to Moriwaki and Kanda (2006).

Calculation of heat flux by K-theory

Heat flux was calculated using equation (1)-(3).

φ_h : equation (3) given by Businger et al. (1971).

$$H = C_p \rho \overline{\theta' w'} = -C_p \rho K_h \frac{d\theta}{dz} \quad (1)$$

$$K_h = \frac{kzu_*}{\varphi_h} \quad (2) \quad k=0.35.$$

$$\varphi_h = 0.74 \{1 - 9(z'/L)\}^{-0.5} \quad (3)$$

Estimation of M-O similarity function

Using equation (4),(5) φ_h is calculated from potential temperature gradient between two height and T_* .

$$(5) \quad \varphi_h = \frac{kz}{T_*} \frac{d\theta}{dz} \quad (4) \quad T_* = -\overline{\theta' w'} / u_*$$

According to Moriwaki and Kanda (2006), Local T_* value was used in the present paper, because they mentioned that T_* did not show remarkable change with height.

Integrated momentum stability correction function Ψ_m is calculated from equation (6).

$$\frac{u}{u_*} = \frac{1}{k} \left(\ln \frac{z'}{z_0} - \psi_m \right) \quad (6)$$

$$\psi_m \equiv \int_{\zeta_0}^{\zeta} (1 - \varphi_m) \zeta^{-1} d\zeta$$

(7)

where $\zeta = z'/l$.

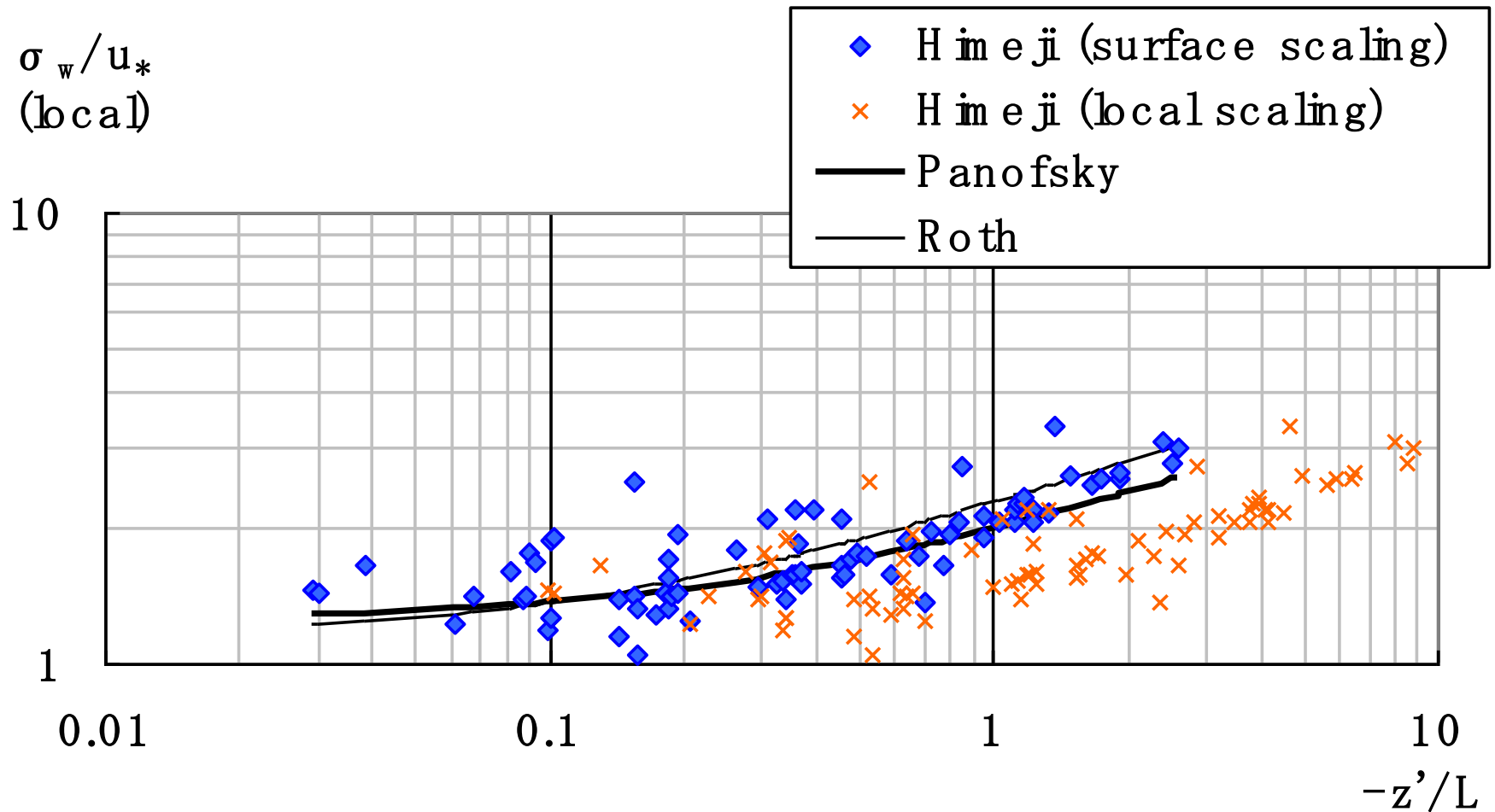


Figure 4. Intensity of turbulence in the vertical direction σ_w / u_* observed at 54m in height in Himeji.

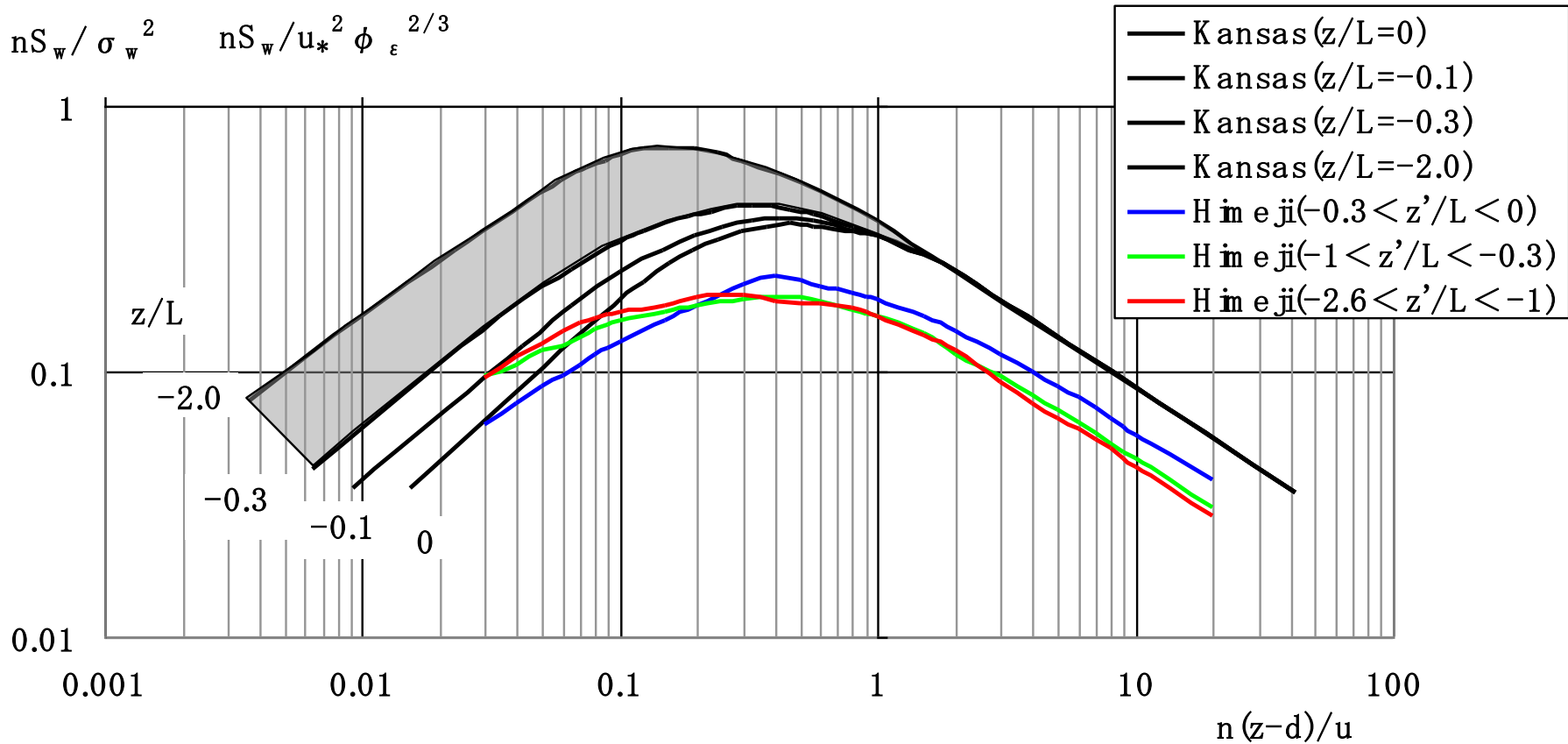


Figure 5. Normalized w spectra observed in Himeji versus Kansas.

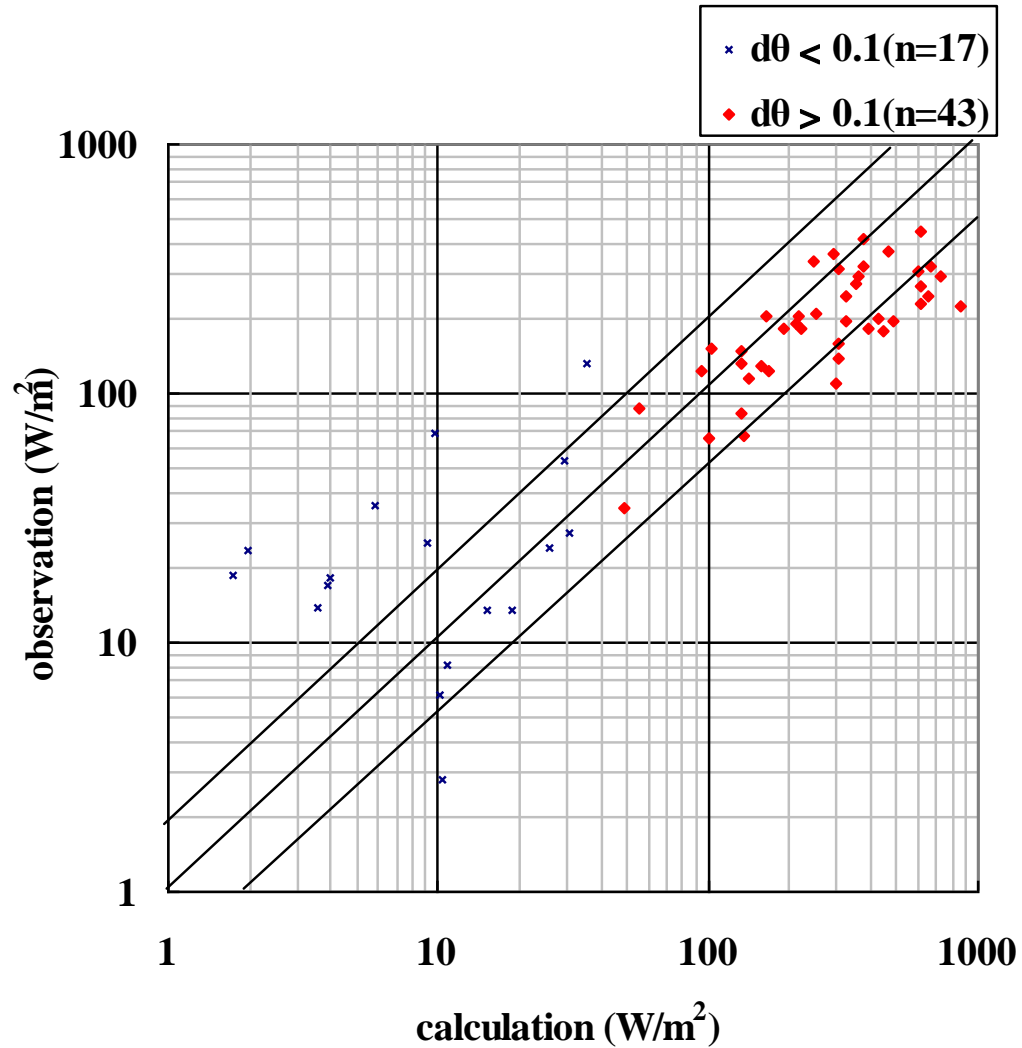


Figure 6. Comparison between calculated heat flux and observed heat flux in Himeji.

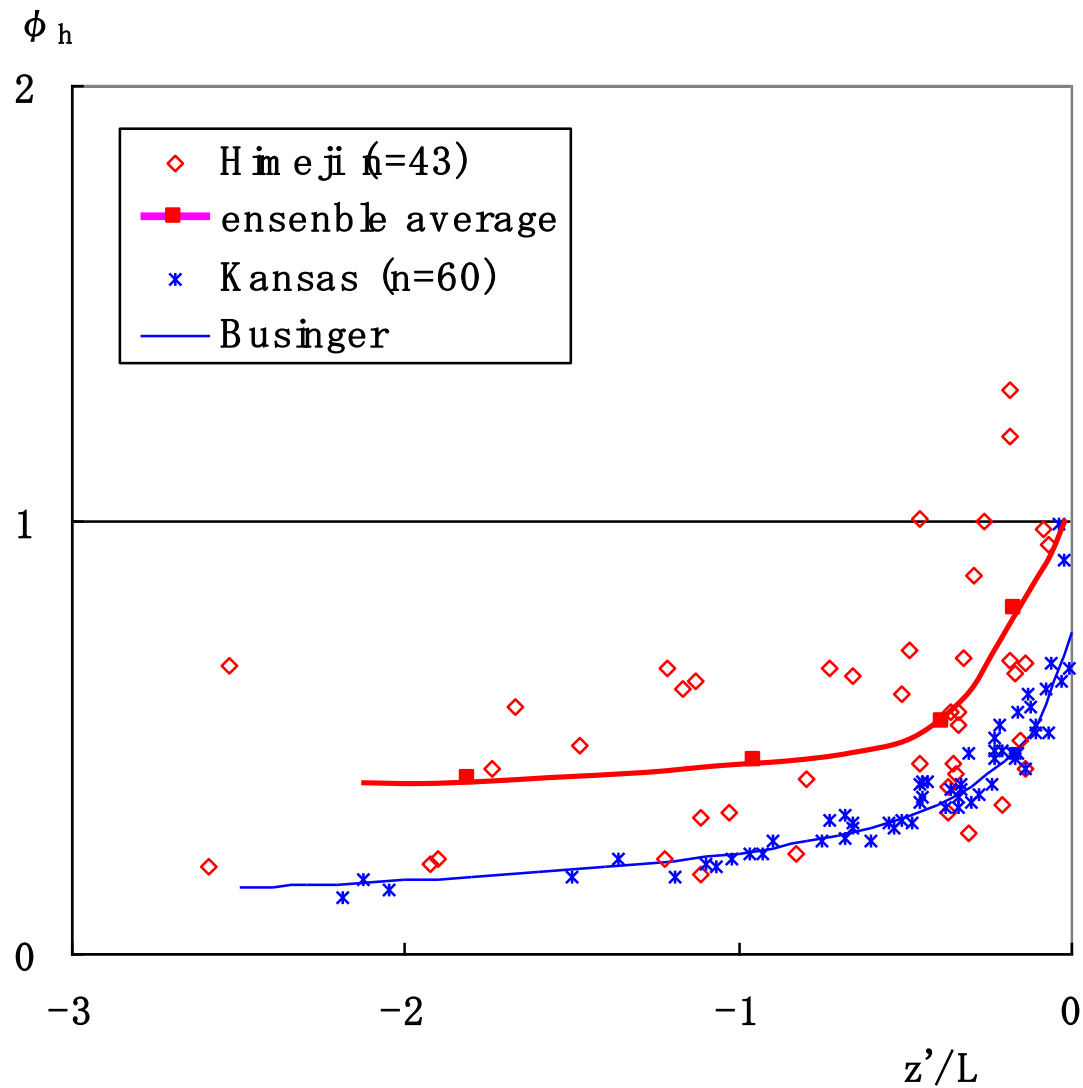


Figure 7. Comparison of calculated ϕ_h using the data of Himeji with those of Kansas and Businger's ϕ_h function.

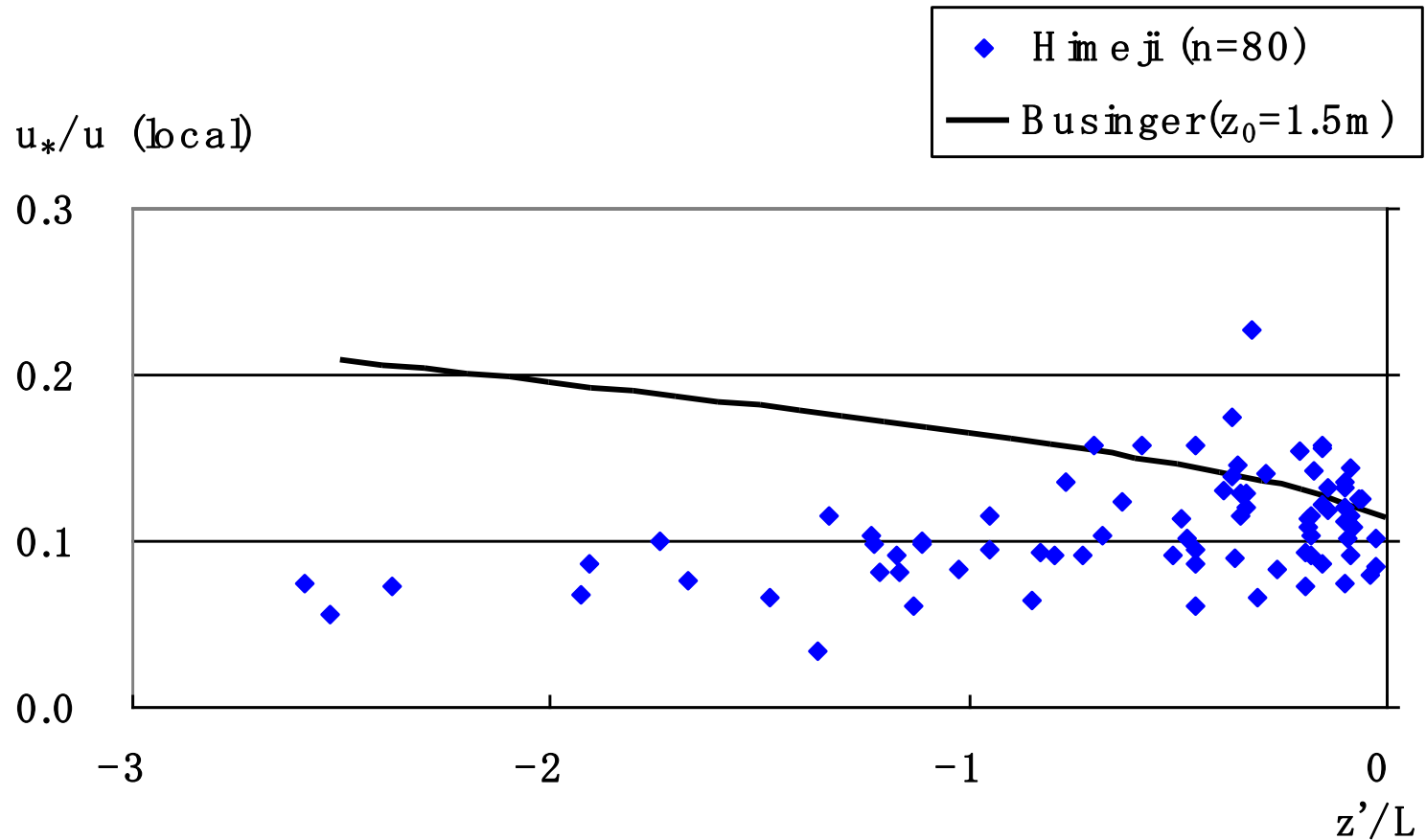


Figure 8. u_*/u observed at $z=54m=6.4h$ in Himeji versus calculated u_*/u using Businger's Ψ_m function

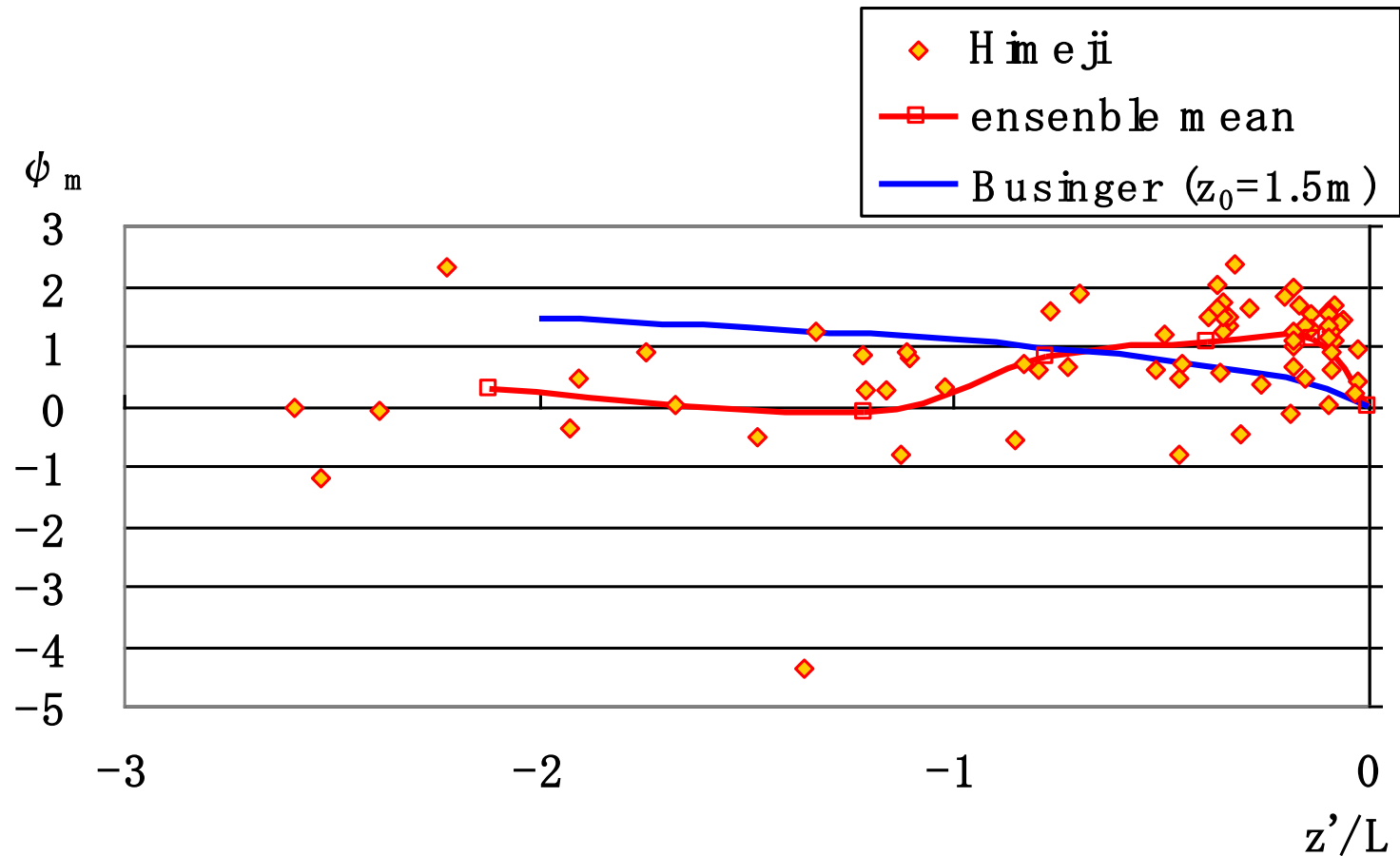


Figure 9. Ψ_m calculated from the data of Himeji and their ensemble mean versus Businger's Ψ_m function.

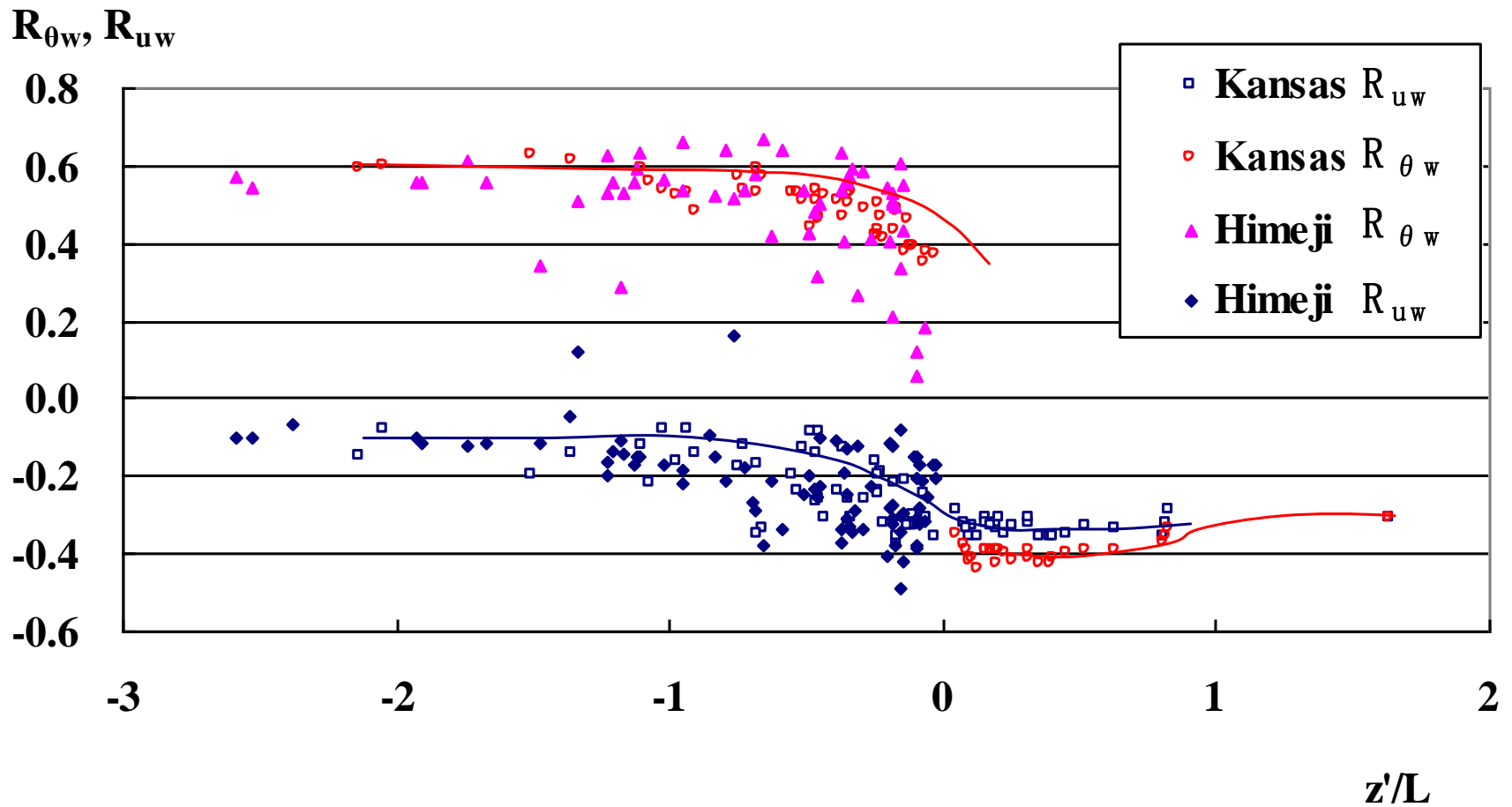


Figure 10. Correlation coefficient R_{u_w} and R_{θ_w} observed in Himeji and Kansas, the solid line is the ensemble mean of Kansas.

CONCLUSION

- (1) The friction velocity at approximately 1.5h in height, u_{*r} is better than a local value as the scaling velocity for Monin-Obukhov length, L .
- (2) Both σ_w/u_* and power spectra of vertical wind velocity obey Monin-Obukhov similarity theory in urban areas as well as rural areas.
- (3) Calculated heat flux using Businger's similarity function overestimated observations for heat flux in the range larger than 250 Wm⁻², however most of the calculations were within 1/2 to 2 times observations. Therefore Monin-Obukhov similarity theory is generally valid for calculation of heat flux in the urban inertial sublayer. However more data are necessary to estimate exact ϕ_h in urban areas.

- (4) The mixture of mechanical turbulence and convective turbulence probably reduces momentum transport in urban areas and rural areas as well. It reduces the correlation between u and w for $z'/L < -1$.
- (5) For $z'/L < -1$, the ensemble mean of Ψ_m approaches 0 which is the same value under neutral condition. The results correspond to the conclusion (4). It seems that the difference in Ψ_m between Himeji and Kansas for $-2.6 \leq z'/L < -1$ is caused by the strong mechanical turbulence in urban areas at low measurement height of $z/h = 6.4$.

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