NUMERICAL ERROR QUANTIFICATION OF RANS MODELLING IN AN IDEALIZED CENTRAL EUROPEAN CITY CENTRE

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Abstract: To further increase the confidence in the results of numerical simulations of flow and dispersion phenomena in urban environments thorough verification and validation of the numerical models is required. For validation of the numerical models complete experimental data of high quality with known uncertainties are required. The measured mean velocities and turbulence quantities of the *Michel-Stadt* case with flat roofs, which are publicly available in the CEDVAL-LES database, are used here to show how well a Reynolds Averaged Navier Stokes modelling approach can perform in a complicated urban geometry. Solution verification with the help of error estimation was carried out together with a validation which included the numerical uncertainty in the evaluation. This was done by the calculation of validation uncertainty according to the Standard of the American Society of Mechanical Engineers on Verification and Validation in Computational Fluid Dynamics and Heat Transfer (ASME V&V 20-2009), which helps the modeller to differentiate between numerical and modelling error. The most difficult regions in the flow field for the conceptual model are shown to be the streamwise street-canyons.

Key words: *urban flow, RANS modelling, validation uncertainty, solution verification*

INTRODUCTION

In the framework of the Action COST 732 Quality assurance of microscale meteorological models (http://www.mi.uni-hamburg.de/Home.484.0.html) two validation data sets were used, the Mock Urban Setting Test (MUST) which consists of regular rows of containers and the Joint Urban 2003 Test, which models a part of Oklahoma City. It was found that a test case with a complexity in between the two is needed to investigate the deficiencies of the conceptual models in more detail. In the Action COST ES1006 "Evaluation, improvement and guidance for the use of local-scale emergency prediction and response tools for airborne hazards in built environments" (http://www.elizas.eu/) therefore the first test case is an idealized Central European city centre, Michel-Stadt, which represents this medium complexity, see Figure 1.

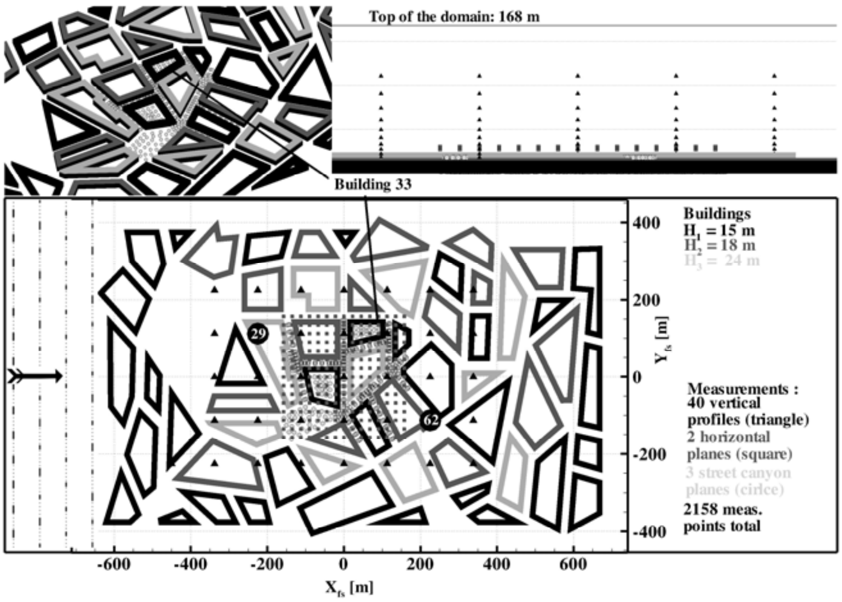

Figure 1. Building layout, computational domain height and measurement positions for the test case Michel-Stadt

The present study shows results of a solution verification and validation exercise for the mean velocity components carried out on this test case with four different mesh types according to the Standard for Verification and Validation in Computational Fluid Dynamics and Heat Transfer (ASME V&V 20-2009).

WIND-TUNNEL AND NUMERICAL EXPERIMENT

Wind-tunnel

The chosen case study is an idealized Central-European city centre, Michel-Stadt. It was chosen as it is a complex geometry with detailed measurement results available. Two component LDV (Laser Doppler Velocimetry) measurements were carried out in the Environmental Wind Tunnel Laboratory of the University of Hamburg. They are part of the CEDVAL-LES database (http://www.mi.uni-hamburg.de/Data-Sets.6339.0.html) which contains different datasets for validation purposes. This case, Michel-Stadt, is the most complex case of the dataset. There are two versions of it, one with flat roofs and another with slanted roofs. In this work the flat-roof case is used. A detailed description of the measurements can be found in (Hertwig, D. et al. , 2012), here only the most relevant details are summarized. 2158 measurement points are available for the flow field, they can be seen in Figure 1. They consist of 40 vertical profiles (10-18 points depending on location for each), 2 horizontal planes (height 27m and 30m, 225 measurement points for each) and 3 so-called street canyon planes (height 2m, 9m and 18m, 383 measurement points for each), which are located inside the urban canopy.

The two available velocity components are the streamwise and lateral velocity component, and time series are available for each of them. The dataset also contains the statistically evaluated mean, rms (root mean square) and correlation values for comparison with steady state computations. Approach flow data are provided from 3 component velocity measurements. The approach flow is modelled as an atmospheric boundary layer in the wind tunnel with the help of spires and roughness elements.

Numerical model

The computational domain was defined to comply with the COST 732 Best Practice Guideline (Franke, J et al., 2011) (Fig. 1), which resulted in a 1575m · 900m · 168m domain, with a distance of the buildings of $11H_3$ from the inflow, $9.4H_3$ from the outflow and at least $6H_3$ from the top boundaries, where H_3 = 24m is the highest building's height. The computations were done in full scale, while the experiment was done at a scale of 1:225.

As inflow boundary condition a power law profile (exponent 0.27, with reference velocity $U_{ref} = 6.11 ms^{-1}$ at z_{ref} = 100m) fitted to the measured velocity values was given. This corresponds to a surface roughness length z_0 = 1.53m. Britter, R.E. and S. R. Hanna (2003) define this as a very rough or skimming approach flow. The turbulent kinetic energy and its dissipation profiles were calculated from the measured approach flow values by their definition and equilibrium assumption, respectively. At the top of the domain the inflow values corresponding to that height were fixed. The lateral boundaries were treated as smooth solid walls, as the computational domain's extension is the same as the wind tunnel width. The floor, roughness elements and buildings were also defined as smooth walls. Standard wall functions were used. As the roughness elements are included in the domain there is no need to use rough wall functions for the approach flow and also the problem of maintaining a horizontally homogeneous ABL (Atmospheric Boundary Layer) profile, which is reported by Blocken, B. et al. (2007) to be problematic for this kind of modelling, is avoided. The Reynolds Averaged Navier Stokes Equations were solved with standard k-ε turbulence model and SIMPLE (Semi-Implicit Method for Pressure Linked Equations) method was used for pressure-velocity coupling with the simpleFoam solver of OpenFOAM®. The convective terms were discretized with the second order limited linearUpwind scheme of OpenFOAM®, for more numerical details see Rakai, A. et al. (2013a).

Four mesh types are compared, their visual appearance is illustrated always for Building 33 (shown in Figure 1) in Figure 2. All the meshes were generated automatically which is a must for using this model for operational purposes and general building configurations. A detailed description on the generation is given in Rakai, A. et al. (2013a), here only the types are listed and the number of cells are given in Table 1. The four mesh types are an unstructured full tetrahedral Delauney mesh generated with ANSYS® Icem, an unstructured full polyhedral mesh created by ANSYS® Fluent from the tetrahedral mesh, a Cartesian hexahedral mesh created with snappyHexMesh of OpenFOAM® and a body fitted hybrid mesh with mostly hexahedral elements meshed with snappyHexMesh of OpenFOAM®.

Table 1. Investigated cell numbers (million cells)

Mesh type	coarse	medium	fine
Polyhedral	1.73	3.21	6.17
Tetrahedral	6.65	13.17	26.79
hexahedral Cartesian	8.04	14.23	27.52
hexahedral body fitted	8.04	14.23	27.52

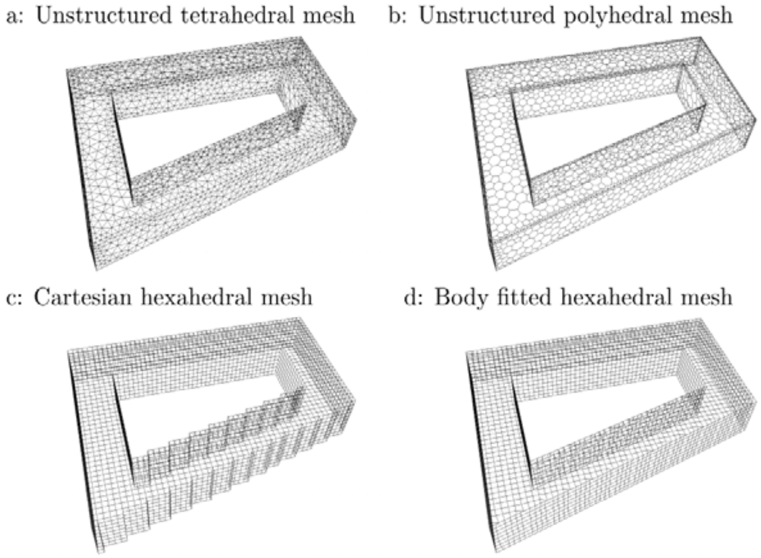

Figure 2. Coarsest surface meshes on Building 33

VERIFICATION AND VALIDATION

Evaluation is carried out according to the Standard for Verification and Validation in Computational Fluid Dynamics and Heat Transfer (ASME V&V 20-2009). Its definition is used: "In V&V, the ultimate goal of engineering and scientific interest is validation, which is defined as the process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model. However, validation must be preceded by code verification and solution verification. Code verification establishes that the code accurately solves the mathematical model incorporated in the code, i.e. that the code is free of mistakes for the simulations of interest. Solution verification estimates the numerical accuracy of a particular calculation." A more detailed discussion on verification and validation in Computational Wind Engineering (CWE) can be found in Franke, J. (2010).

Code and solution verification

Initial code verification with the Method of Manufactured Solutions for the simpleFoam solver of OpenFOAM® has been carried out by Fisch, R. et al. (2012). The current solution verification was implemented with a method based on Richardson extrapolation. The ASME V&V 20-2009 Standard suggests the use of Grid Convergence Index (GCI), however here numerical uncertainty was estimated with the Global Averaging Method (Phillips, T. S. and C. J. Roy, 2011) which was found to be the most efficient method. It is also based on Richardson extrapolation but uses a global value for the order of convergence for the error estimation. Phillips, T. S. and C. J. Roy (2011) compared test cases with analytical solutions and efficiency was defined as estimating the exact error with the smallest difference. Although the method is not the most conservative (a conservative estimator always overestimates the exact error) from those that they investigated, for the purpose of the present study efficiency was decided to be more important. For more details on the methods the reader is referred to Phillips, T. S and C. J. Roy (2011) or Rakai, A. et al. (2013b). Here only the results of the numerical uncertainty estimation for each measurement point are used referred to as U_{num}.

Validation

When comparing results at more than 2000 measurement points the use of metrics is unavoidable. The decision on which metrics to use on the other hand has a significant effect on the evaluation of the results. To demonstrate this, three different metrics are used. A simple one based on the L2 norm of the differences between experimental (E_i) and simulation (S_i) results, Equation (1), the well known hit rate (HR), taking into consideration an absolute (W) and relative allowed (25%) deficiency for the results, Equation (2), and a suggested new metric, the validation rate (VR), which also incorporates the numerical uncertainty, Equation (3).

$$L2 = \frac{\sqrt{\sum_{i=1}^{n}(E_i - S_i)^2}}{\sqrt{\sum_{i=1}^{n} E_i^2}} \tag{1}$$

$$HR = \frac{1}{N}\sum_{i=1}^{N}\delta_i \ where \ \delta_i = \begin{cases} 1 & \text{for} \quad \left|\frac{S_i - E_i}{E_i}\right| \leq 0.25 \quad \text{or} \quad |S_i - E_i| \leq W \\ 0 & \text{for} \quad \text{else} \end{cases} \tag{2}$$

With W=0.0165 for U_{mean} /U_{ref}, W=0.0288 for V_{mean} /U_{ref}, W=0.00434 for U_{rms} /U_{ref} and W=0.00874 for V_{rms} /$U_{ref.}$.

The validation rate is defined as

$$VR = \frac{1}{N}\sum_{i=1}^{N}\delta_i \quad where \quad \delta_i = \begin{cases} 1 & D_i \le U_{val} \\ 0 & \text{for} \quad \text{else} \end{cases} \tag{3}$$

The validation uncertainty (U_{val}), first suggested by Stern, F. et al. (2001) is defined in Equation (4) and the validation comparison error (D_i) in Equation (5).

$$U_{val} = \sqrt{U_{num}^2 + U_{input}^2 + U_E^2} \tag{4}$$

$$D_i = S_i - E_i \tag{5}$$

For the validation uncertainty the strong model concept is used, see CFDUncertaintyWorkshop (2008), so U_{input}=0 and U_E is the experimental uncertainty equal to W. For model validation excercises high VR means that simulations with higher resolution may be needed.

RESULTS

Results are shown in Table 2 for the metrics on different mesh types for the two mean velocity components. The resulting numerical error estimate is always given for the finest mesh.

Table 2. Results of the metrics for each mesh type

	U_{mean}			V_{mean}		
Mesh type	**L2**	**HR**	**VR**	**L2**	**HR**	**VR**
polyhedral	0.21	69%	81%	0.47	66%	88%
tetrahedral	0.15	72%	81%	0.39	68%	90%
hexahedral Cartesian	0.18	66%	75%	0.57	60%	88%
hexahedral body fitted	0.17	71%	81%	0.45	66%	87%

It is observed that the best metrics are found for the tetrahedral meshes in all cases, namely lowest L2 values, which can be regarded as a relative error norm, and highest HR. This can be explained by the better resolution with the tetrahedral mesh. Although the number of cells is similar in case of the tetrahedral and hexahedral meshes, the local resolution in the areas of high gradients is better for the tetrahedral mesh. It must be noted however that the tetrahedral mesh was the least stable during the computations, in many cases severe underrelaxation was necessary to even reach first order convergence.

The validation rate is also highest for the tetrahedral meshes. That metric has however a very different interpretation. A high VR value shows that in most of the measurement points no conclusion can be drawn about the conceptual model, e.g. turbulence model, as the numerical and experimental uncertainty combined to validation uncertainty is higher than the actual difference between simulation and measurement result.

In Figure 3 the 2m street-canyon planes are shown and non-validated points are highlighted with black dots for the three different mesh types. It can be seen that independent of the mesh type the streamwise street canyons contain several non-validated points, so they are identified as a critical area for the conceptual model.

CONCLUSION

An idealized Central European city centre, Michel-Stadt, was investigated according to the Standard of the American Society of Mechanical Engineers on Verification and Validation in Computational Fluid Dynamics and Heat Transfer. Validation uncertainty was defined for the more than 2000 measurement points for the mean horizontal velocity components and four different mesh types were compared with the help of validation metrics. The best is obtained for the locally best refined tetrahedral meshes, which are at the same time the numerically least stable. A new metric, the validation rate, was introduced. It shows the ratio of the measurement points where conceptual model development is still possible with a given numerical resolution. The streamwise street canyons were identified as critical areas for the model even with the coarse numerical resolution.

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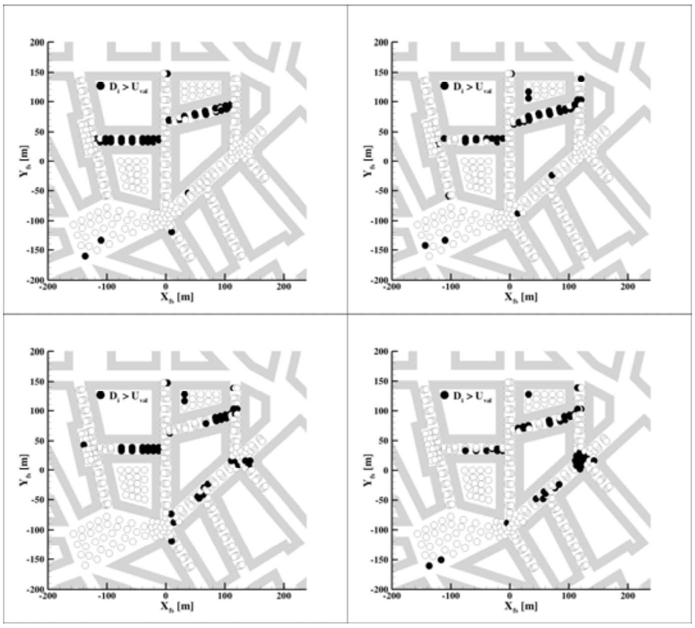

Figure 3. *Non validated points in the 2m street canyon*
(tetra – up-left, poly – up-right, body fitted hexahedral – down -left, Cartesian hexahedral – down-right)

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