ADAPTATION OF THE REYNOLDS STRESS TURBULENCE MODEL FOR ATMOSPHERIC SIMULATIONS
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Outline

1. Introduction and motivations
2. Reynolds Stress model
3. Atmospheric RSM constants
4. Validation of the model
5. Conclusions and perspectives
1 – Introduction and motivations
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Research issues in atmospheric CFD modeling

• Atmospheric CFD simulations (particularly RANS k-ε) are often used
  • For local scale wind engineering
  • For dispersion in complex urban area with obstacles
  • For risk and safety assessment in industrial areas
1 – Introduction and motivations

Research issues in atmospheric CFD modeling

- **Different issues have to be properly solved**
  - Boundary conditions over the surface layer?
  - Anisotropy of the turbulence?
  - “Standard” or “atmospheric/Duynkerke” constants?
  - Value of the turbulent Schmidt number?

  ➔ Large uncertainty and user-dependent variability when comparing with field measurements

- **Objectives of this work**
  - Introduce anisotropy of turbulence using Reynolds Stress Model
  - Develop a 1D model for the entire Atmospheric Boundary Layer

  ➔ Provide parameterizations and boundary conditions for 3D CFD calculations
2 – Reynolds Stress model
2 – Reynolds Stress model
RSM equations

- Reynolds Stress Model equations
  - Reynolds Stress equation
    \[
    \frac{\partial \overline{u_i' u_j'}}{\partial t} + u_k \frac{\partial \overline{u_i' u_j'}}{\partial x_k} = \frac{\partial}{\partial x_k} \left( K_m \frac{\partial \overline{u_i' u_j'}}{\partial x_k} \right) + P_{ij} + \phi_{ij} - \frac{2}{3} \delta_{ij} \varepsilon
    \]
    with \( P_{ij} = u_i' u_j' \frac{\partial \overline{u_j}}{\partial x_k} + u_j' u_j' \frac{\partial \overline{u_i}}{\partial x_k} \)
  - Turbulent dissipation rate equation
    \[
    \frac{\partial \varepsilon}{\partial t} + u_i \frac{\partial \varepsilon}{\partial x_i} = \frac{\partial}{\partial x_k} \left( K_m \frac{\partial \varepsilon}{\partial x_k} \right) + C_{\varepsilon 1} \varepsilon P_{ii} + C_{\varepsilon 2} \varepsilon^2
    \]
  - Turbulent viscosity
    \[
    K_m = C_k \frac{\varepsilon^2}{\varepsilon}
    \]
2 – Reynolds Stress model

RSM constants

- The preceding equations depends on 5 constants

\[ C_\mu, \sigma_k, \sigma_\varepsilon, C_{\varepsilon 1}, C_{\varepsilon 2} \]

- Pressure-strain term \( \phi_{ij} \)
  - We choose the model of Gibson and Launder (1978)
  - This model introduces 5 other constants:

\[ C_1, C_2, C_1', C_2', C_L \]

- The “standard” values of these constants are:

<table>
<thead>
<tr>
<th>( C_\mu )</th>
<th>( \sigma_k )</th>
<th>( \sigma_\varepsilon )</th>
<th>( C_{\varepsilon 1} )</th>
<th>( C_{\varepsilon 2} )</th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( C_1' )</th>
<th>( C_2' )</th>
<th>( C_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09</td>
<td>1.0</td>
<td>1.3</td>
<td>1.44</td>
<td>1.92</td>
<td>1.8</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
<td>0.39</td>
</tr>
</tbody>
</table>
2 – Reynolds Stress model

RSM constants

• “Standard” constants are not adapted for the atmosphere

• Consider the Surface Boundary Layer
  • For example, the $C_\mu$ constant controls the level of turbulent kinetic energy $k$:
    \[
    B = \frac{k}{u_*^2} = \frac{1}{\sqrt{C_\mu}}
    \]
  • Wind tunnel measurements give $B = 3.33$ (i.e. $C_\mu = 0.09$)
  • Atmospheric measurements (Panofsky and Dutton, 1984) give $B = 5.48$ (i.e. $C_\mu = 0.033$)

• It is necessary to define “atmospheric” constants:
  • Duynkerke (1988) proposed a set of constants for the $k-\varepsilon$ model
  • In this work, we propose a new set of constants for the Reynolds Stress Model
3 – Atmospheric RSM constants
3 – Atmospheric RSM constants
Determination approach

• **In the Atmospheric Surface Layer**
  • We assume a 1D Atmospheric Surface Layer, horizontally uniform
  • We identify the 1D RSM equations with the following analytical profiles

\[
\bar{u}(z) = \frac{u_*}{\kappa} \ln \left( \frac{z}{z_0} \right) \\
\varepsilon(z) = \frac{u_*^3}{\kappa z} \\
\bar{u}^{r2} = \alpha_x^2 u_*^2 \\
\bar{v}^{r2} = \alpha_y^2 u_*^2 \\
\bar{w}^{r2} = \alpha_z^2 u_*^2 \\
\bar{u}'w' = -u_*^2
\]

with \( \alpha_x = 2.46 \), \( \alpha_y = 1.9 \) (Panofsky and Dutton, 1984) and \( \alpha_z = 1.17 \)
3 – Atmospheric RSM constants

Determination approach

- It provides a system of equations for the constants:

\[
\bar{u}^{r^2} \text{ equation } - C_1 \frac{\alpha_x^2}{B} + C'_1 C' L B^2 \kappa \alpha_z^2 - \left[ \frac{2}{3} C_2 \left( 2 - C'_2 \kappa B^2 C_L \right) - 2 \right] \frac{\kappa z}{u_*} \frac{\partial u}{\partial z} + \frac{2}{3} (C_1 - 1) = 0
\]

\[
\bar{v}^{r^2} \text{ equation } - C_1 \frac{\alpha_y^2}{B} + C'_1 C' L B^2 \kappa \alpha_z^2 - \frac{2}{3} C_2 \left( -1 - C'_2 \kappa B^2 C_L \right) \frac{\kappa z}{u_*} \frac{\partial u}{\partial z} + \frac{2}{3} (C_1 - 1) = 0
\]

\[
\bar{w}^{r^2} \text{ equation } - C_1 \frac{\alpha_z^2}{B} - 2C'_1 C'_ L B^2 \kappa \alpha_z^2 - \frac{2}{3} C_2 \left( -1 - 2C'_2 \kappa B^2 C_L \right) \frac{\kappa z}{u_*} \frac{\partial u}{\partial z} + \frac{2}{3} (C_1 - 1) = 0
\]

\[
\bar{u}'w' \text{ equation } \frac{1}{B} \left( C_1 + \frac{3}{2} C'_1 \right) + \left[ C_2 \left( 1 - \frac{3}{2} C'_2 \kappa B^2 C_L \right) - 1 \right] \frac{\kappa z}{u_*} \frac{\partial u}{\partial z} = 0
\]

- Which provides, after resolution, a set of “atmospheric” constants:

<table>
<thead>
<tr>
<th>$C_\mu$</th>
<th>$\sigma_k$</th>
<th>$\sigma_e$</th>
<th>$C_{\varepsilon 1}$</th>
<th>$C_{\varepsilon 2}$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C'_1$</th>
<th>$C'_2$</th>
<th>$C_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.033</td>
<td>1.0</td>
<td>2.38</td>
<td>1.46</td>
<td>1.83</td>
<td>1.8</td>
<td>0.6</td>
<td>0.94</td>
<td>0.03</td>
<td>0.19</td>
</tr>
</tbody>
</table>
4 – Validation of the model
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Methodology of the 1D numerical model

- We have developed a numerical 1D model for the ABL
  - Flow is horizontally homogenous:
    \[ \bar{w} = 0 \quad \text{and} \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0 \]
  - Pressure gradient and Coriolis force in geostrophic balance
  - 1D equation model of mean wind speed, including Coriolis effect:
    \[ \frac{\partial \bar{u}}{\partial t} = -\frac{\partial \bar{u}' \bar{w}'}{\partial z} + f(\bar{v} - V_g) \quad \text{and} \quad \frac{\partial \bar{v}}{\partial t} = -\frac{\partial \bar{v}' \bar{w}'}{\partial z} - f(\bar{u} - U_g) \]
  - Reynolds Stress Model as turbulence closure model
  - Equations are solved numerically until a steady state
- We apply the model on the overall ABL and compare it with empirical results
4 – Validation of the model
Results in neutral conditions

- **Velocity profile:**
  - For example for $U_g = 5\text{m.s}^{-1}$, $z_0 = 0.01\text{m}$, $\varphi = 45^\circ$
4 – Validation of the model
Results in neutral conditions

- Turbulence profiles (normalized by the ground value):
  - For example for $U_g = 5 \text{ m.s}^{-1}$, $z_0 = 0.01 \text{ m}$, $\varphi = 45^\circ$
4 – Validation of the model

Results in neutral conditions

- Ekman’s theory predicts the twisting of the flow in the Ekman layer:

\[
\begin{align*}
    u &= U_0 \left[ 1 - \exp(-az) \cos(az) \right] \\
    v &= \text{sgn}(f) \ U_0 \exp(-az) \sin(az)
\end{align*}
\]

with \( a = \frac{|f|}{2K_M} \)

- Simulation of the Ekman Layer with the 1D model:
4 – Validation of the model

Results in neutral conditions

- The Rossby similarity theory gives:
  \[ \frac{U_g}{u_*} = \frac{1}{\kappa} \left[ \ln \left( \frac{u_*}{fz_0} \right) - B \right]^2 - A^2 \]
  
  \( A \) and \( B \) are empirical constants

- Sensibility of the 1D model to control parameters \( U_g, f \) and \( z_0 \)
4 – Validation of the model

Results in neutral conditions

- Atmospheric Boundary Layer height:
  - The Rossby-Montgomerry equation

\[ h = \frac{c u_*}{f} \text{ with } c \approx 0.26 \]
4 – Validation of the model
Results in stratified conditions

- **Velocity profile:**
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4 – Validation of the model
Results in stratified conditions

• Relation between $U_g$ and $u_*$:

$$\frac{U_g}{u_*} = \frac{1}{\kappa} \sqrt{\ln \left( \frac{u_*}{fz_0} \right) - B} - A^2$$

$A$ and $B$ are dependent on stability, measured empirically

Unstable conditions

Stable conditions
4 – Validation of the model

Results in stratified conditions

- The Rossby similarity theory gives:

\[ \sin(\alpha) = -\frac{A_u}{\kappa U_g} \text{sign}(f) \]

- \( A \) is dependent on stability, measured empirically
5 – Conclusions and perspectives
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• **Conclusions**
  • Development of a *new set of atmospheric constants* for the Reynolds Stress turbulence model
  • *1D simulations* of the Atmospheric Boundary Layer with this Reynolds Stress Model, *including Coriolis effects*
  • **Validation against empirical results**

• **Perspectives**
  • Validation in more complex configurations
  • Evaluation of the effect of anisotropy on dispersion modeling
  • Unified approach between “atmospheric” and “standard” RSM constants (see Poster H15-166)
Thank you for your attention 😊

Questions ?