A Lagrangian stochastic model for estimating the high order statistics of a fluctuating plume in the neutral boundary layer

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May 7, 2013
Target

- Impact assessment of risks related to the dispersion of flammable gases and toxic substances.
- Simulation of the combined effects of the turbulent mixing and molecular diffusivity.
- Estimate of the concentration fluctuations and prediction of the higher statistics and the concentration PDFs.


New experiment data set in wind tunnel:

- Measures of high order concentration statistics in a fluctuating plume in a neutral boundary layer.
- Poster session T8.

Numerical simulations:

- Comparison between experiments and computed solutions.
- Evaluation of the accuracy of the model.
Lagrangian Stochastic model

Equations describing the evolution of the position $X_i$ and velocity $U_i$ of a set of independent fluid particles.

$$dX_i = (\langle u_i \rangle + U'_i) \, dt$$

deterministic term

$$dU'_i = a_i (X, U', t) \, dt + b_{ij} (X, U', t) \, d\xi_j$$

stochastic diffusive term

- $U'_i$: Lagrangian velocity fluctuation related to the Eulerian mean velocity $\langle u_i \rangle$.
- $a_i$ is estimated according to the well-mixed conditions\(^1\).
- $b_{ij}$ is defined from the Kolmogorov’s hypotheses of self-similarity and local isotropy in the inertial subrange\(^2\).
- $d\xi_j$ incremental Wiener process with zero mean and variance $dt$.

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Micromixing model

Molecular diffusivity is simulated by an Interaction by Exchange with the Conditional Mean (IECM) model.

\[
\frac{dC}{dt} = - \frac{C - \langle C | u_i \rangle}{\tau_m}
\]

- \( C \) is the concentration associated to a fluid particle and \( \langle C | u_i \rangle \) is the mean scalar concentration conditioned on the local position and velocity.

- The micromixing time \( \tau_m \) represents the temporal scale of the molecular diffusion:
  - parametrization of \( \tau_m \) follows the formulation of Cassiani et al. (2005a)\(^3\);
  - \( \tau_m \) is assumed to be proportional to the time scale of the relative dispersion process, \( \tau_m = \mu_t \tau_r \);
  - \( \tau_m = f(\sigma_u, \varepsilon, \sigma_0, t) \)

SLAM: computational algorithm

1. **Pre-processing** ($X$ and $U$):
   - simulation of the trajectories of an ensemble of particles released at the source location;
   - estimate of the conditional mean concentration $\langle C|u_i \rangle$ and the micromixing time $\tau_m$.

2. **Simulation of the concentration fluctuations** ($X$, $U$, $C$):
   - instantaneous release of a uniform particle distribution in the domain;
   - initialization of the particle properties ($X$, $U$, $C$);
   - main time loop:
     - loop on all the particles:
       - update particle velocity and position;
       - apply boundary conditions;
       - update particle concentration;
     - update cell-centred statistics;
   - update time-averaged statistics.
Experimental set-up

Velocity field → Hot Wire Anemometry measures.

1) $\frac{\langle u \rangle}{u^*}$ vs $\frac{z}{\delta}$

2) $\frac{\sigma_u}{u^*}$, $\frac{\sigma_v}{u^*}$, $\frac{\sigma_w}{u^*}$ vs $\frac{z}{\delta}$

3) $\frac{\varepsilon \delta}{u^3}$ vs $\frac{z}{\delta}$

- Boundary layer depth $\delta = 0.8$ m.
- Friction velocity: $u^* = 0.185$ m/s.
- Source height $\frac{h_s}{\delta} = 0.19$.
- Two source diameters: $\frac{\phi_1}{\delta} = 3.75e-3$, $\frac{\phi_2}{\delta} = 7.5e-3$.

Concentration field → measures of ethane (passive scalar) concentration by means of Flame Ionization Detector.
Numerical discretization parameters

1) \( x/\delta = 0.625 \)

2) \( x/\delta = 5 \)

Figure: \( M_2^* \) vs \( y/\delta \)

- **a:** \( \Delta t = 1e-3, \Delta x = 0.02, \Delta y = \Delta z = 5e-3; \)
- **b:** \( \Delta t = 5e-4, \Delta x = 0.02, \Delta y = \Delta z = 5e-3; \)
- **c:** \( \Delta t = 1e-3, \Delta x = 0.01, \Delta y = \Delta z = 3e-3. \)
Computational set-up

<table>
<thead>
<tr>
<th>C₀</th>
<th>σ₀</th>
<th>Cᵣ</th>
<th>µₜ</th>
<th>velocity classes</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>( \sqrt{2/3}d_s )</td>
<td>0.3</td>
<td>0.6</td>
<td>3 × 3 × 3</td>
</tr>
</tbody>
</table>

Table: Free parameter values adopted in the simulations

Non-dimensional concentration centred moments:

\[
Mᵢ^* = \left[ \frac{1}{N_c} \sum_{p=1}^{N_c} (C_p - C_c)^i \right]^{1/i} \frac{u_\infty \delta^2}{Q} \quad i = 1, 2, 3, 4
\]

- \( u_\infty \): the velocity at the boundary layer height;
- \( N_c \): number of particles in a discrete volume;
- \( C_c \): mean concentration in a discrete volume;
- \( C_p \): concentration associated to a particle.
Results: $M_i^*$ vs $y/\delta$ evaluated at the source height and $x/\delta = 0.625$
Results: $M_i^*$ vs $y/\delta$ evaluated at the source height and $x/\delta = 3.75$
Results: $M_i^* \text{ vs } y/\delta$ evaluated at the source height and $x/\delta = 5.0$

1) $M_3^*$ vs $y/\delta$

2) $M_4^*$ vs $y/\delta$

$$D_{rel} = \sqrt{\frac{\int_{-\infty}^{\infty} [(M_i^*)_{S1} - (M_i^*)_{S2}]^2 dy}{\int_{-\infty}^{\infty} [(M_i^*)_{S2}]^2 dy}}$$

<table>
<thead>
<tr>
<th>$x/\delta$</th>
<th>$D_{rel}$</th>
<th>$M_3^*$</th>
<th>$D_{rel}$</th>
<th>$M_4^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.75</td>
<td>0.17</td>
<td>0.36</td>
<td>0.12</td>
<td>0.29</td>
</tr>
<tr>
<td>5.0</td>
<td>0.12</td>
<td>0.29</td>
<td></td>
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</tr>
</tbody>
</table>

Table: Relative difference of the third and fourth moments.
The ability of the Lagrangian Stochastic Micromixing model SLAM to estimate concentration fluctuations was investigated.

The dispersion of a fluctuating plume produced by a continuous release from a point source in a neutral boundary layer was simulated and a comparison with a new experimental data set was performed.

Good agreement of the first four moments of the concentration close to the source.

Good agreement of the mean concentration and variance in the far-field.

Some discrepancies in the third and fourth moments of the concentration in the far-field.
Thank you for your attention!
Any questions?