An inverse modelling technique for emergency response application

Alison Rudd

Department of Meteorology
University of Reading

S. Belcher and A. Robins
Malicious or accidental release in an urban area
What area should the first responders cordon off or evacuate?
What are the source characteristics? - uncertainty
Where will the plume spread?
Malicious or accidental release in an urban area

What area should the first responders cordon off or evacuate?

What are the source characteristics? - uncertainty

Where will the plume spread?
The DYCE consortium

DYnamic deployment planning for monitoring of ChEmical leaks using an ad-hoc sensor network

- Chemical sensors
- Communications & networking
- Inverse modelling to estimate the source characteristics
- Wind tunnel & tracer trial validation studies

Funding
Inverse modelling

Inverse problem: extracting source characteristics from a set of concentration measurements

1. Make a first guess of the source characteristics \((Q, X_s, Y_s)\)

2. First guess \(\rightarrow\) forward model \(\rightarrow\) model-predicted concentrations

3. Model-predicted concentrations vs. measured concentrations \(\rightarrow\) Minimisation algorithm \(\rightarrow\) ‘best’ estimate of source characteristics.

4. ‘Best’ estimate \(\rightarrow\) forward model \(\rightarrow\) predicted plume.
Forward model

Forward model $\rightarrow$ model-predicted concentrations

Gaussian plume model - well known and understood

Inputs: source strength and position, wind speed and stability

We assume

- one continuous point source
- a ground level release, i.e. $Z_s = 0$
- concentration measurements at ground level

\[
C = \frac{Q}{\pi u \sigma_Y \sigma_Z} \exp\left(\frac{-\left(Y - Y_s\right)^2}{2\sigma_Y^2}\right)
\]
Optimisation

Minimise a cost function

\[ J = \frac{1}{2} \sum_{i=1}^{N} \left( \frac{C_i^o - C_i^m}{\sigma_i} \right)^2 \]

Concentration measurements \( C^o \)  
Model-predicted concentrations \( C^m \)

Measures the discrepancy between the measured and model-predicted concentrations

Minimise \( J \), which is the same as finding the values of the source characteristics for which the gradient of \( J \) is zero. This is your `best’ estimate of the source characteristics.

Least squares fit plus error weighting which leads to an uncertainty estimate of the source characteristics.
Need a rapid algorithm

Time is important in emergency situations

Estimate of uncertainty associated with the ‘best’ estimate from second derivative of the forward model w.r.t the source characteristics
Sources of error

- **Measurement error**
  the accuracy of the concentration measurement from the sensor *may be known*

- **Model error**
  how good is the model at representing reality? *can only estimate*

- **Sampling error**
  this is dependent on the averaging time of the data due to the natural variability of the concentrations *likely to dominate*

*Could prevent the inverse algorithm from making a good estimate of the source characteristics*
Wind tunnel data

Gaussian plume model tuned to the wind tunnel data

Difference due to model error and instrument error?

\[ C^* = \frac{CUH^2}{Q} \]
Sampling error

How to quantify the sampling error associated with taking a short time average to estimate the true mean in a turbulent flow

Standard deviation of the shorter time mean estimate of the true mean concentration

\[
\sigma_{\bar{C}_t} = \left( \frac{1}{n} \sum_{i=1}^{n} \left( \bar{C}_i^t - \bar{C}^T \right)^2 \right)^{\frac{1}{2}}
\]

- \( t \) is the shorter averaging time
- \( T \) is the total time length
- \( n \) is the n° of shorter averaging time samples

\( \bar{C}_i^t \) = mean concentration averaged over time \( t \)

\( \bar{C}^T \) = true mean concentration
Sampling error

\[ \frac{\sigma_{C'}}{C} = \text{the uncertainty in the short time mean estimate compared to the true mean concentration} \]

- 70% uncertainty on 10 sec average
- 60% uncertainty on 1 min average
- 20% uncertainty on 15 min average

Wind tunnel
\[ U_{ref} = 2.5 \text{ m/s} \]
\[ H = 1\text{m} \]

Equivalent full scale
\[ U_{ref} = 10 \text{ m/s} \]
\[ H = 500\text{m} \]
Inverse modelling - WT data

<table>
<thead>
<tr>
<th>Source parameter</th>
<th>True value</th>
<th>First guess</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>0.1</td>
<td>1</td>
<td>m³ s⁻¹</td>
</tr>
<tr>
<td>Xs</td>
<td>-47</td>
<td>-24</td>
<td>m</td>
</tr>
<tr>
<td>Ys</td>
<td>47</td>
<td>22</td>
<td>m</td>
</tr>
</tbody>
</table>

27 data points from wind tunnel data

<table>
<thead>
<tr>
<th>Source parameter</th>
<th>Estimate</th>
<th>Uncertainty</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>0.075</td>
<td>0.002</td>
<td>m³ s⁻¹</td>
</tr>
<tr>
<td>Xs</td>
<td>-30.37</td>
<td>1.54</td>
<td>m</td>
</tr>
<tr>
<td>Ys</td>
<td>43.70</td>
<td>0.20</td>
<td>m</td>
</tr>
</tbody>
</table>

The true values of (Q, Xs, Ys) do not lie within the uncertainty range of the estimates.
Inverse modelling - WT data

Sub set of 4 data points where the data values were accurately predicted by the Gaussian plume model.

<table>
<thead>
<tr>
<th>Source parameter</th>
<th>True value</th>
<th>First guess</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>0.1</td>
<td>1</td>
<td>m³ s⁻¹</td>
</tr>
<tr>
<td>Xs</td>
<td>-47</td>
<td>-24</td>
<td>m</td>
</tr>
<tr>
<td>Ys</td>
<td>47</td>
<td>22</td>
<td>m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source parameter</th>
<th>Estimate</th>
<th>Uncertainty</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>0.097</td>
<td>0.010</td>
<td>m³ s⁻¹</td>
</tr>
<tr>
<td>Xs</td>
<td>-46.57</td>
<td>7.84</td>
<td>m</td>
</tr>
<tr>
<td>Ys</td>
<td>46.51</td>
<td>1.37</td>
<td>m</td>
</tr>
</tbody>
</table>

The true values of (Q, X_s, Y_s) lie within the uncertainty range of the estimates.
Conclusions

• Characterising the errors is essential for inverse modelling
  – can quantify the measurement error
  – can estimate the model error for the wind tunnel data
  – however, it is sampling error that appears to be the most important, it could potentially hamper the inverse algorithm from finding the `best’ estimate.

• We have a method for estimating the uncertainty due to sampling error that can feed into the inverse algorithm – need to test it.

• Other studies we have done with synthetic data showed that measurements scattered about the plume in a square configuration lead to better estimates of the source characteristics because they contain direct information on the lateral spread of the plume.

Further work

• Test the inverse algorithm with a different forward model – the network model approach for urban dispersion.

• Use wind tunnel data collected using rectangular blocks to represent buildings in an urban area for validation.