16th International Conference on
Harmonisation within Atmospheric Dispersion Modelling for Regulatory Purposes
8-11 September 2014, Varna, Bulgaria

ANALYZING LOCALIZATION FEATURES OF A WEIGHTED LEAST-SQUARES TECHNIQUE IN A POINT SOURCE RECONSTRUCTION

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Abstract: Identification of unknown contaminant releases is crucial in the events of gas leakage, industrial accidents, safety and security concerns. For a point based release, the nature of a source is determined by fixed number of parameters, mainly its location and source strength. The identification of these parameters is addressed by using limited concentration measurements sampled by a network of receptors. This is formulated as over-determined linear inverse problem. To address this, an optimally weighted least-squares inversion technique is presented here. The inversion technique provides a minimum weighted norm solution and does not require initial guesses for the release parameters. This study proposes the optimality conditions under which a point source is identifiable. The inversion technique and optimality criteria are evaluated using field measurements of a single release trial# 7 conducted during Fusion Field Trials in September 2007. The point source is reconstructed within 3 m from the true release. The source strength is retrieved within a factor of 1.25 to true release mass. The optimality criteria show that the accuracy of reconstruction depends on the nature of model resolution, predictability of measurements and visibility of the true source from the receptors.

Keywords: *FFT07, inverse problem, localization, least-squares, source identification***.**

INTRODUCTION

The inverse problem of identifying a point release is a parametric estimation problem associated with the estimation of its location coordinate and strength. Several inversion techniques based on variational minimization and random search algorithms are evolved to estimate the unknown release parameters. The measured concentrations can provide information about the origin of release but they are often not sufficient to determine them uniquely. This requires additional information in the form of a priori. However, loss of information occurs during the measurement process and the inverse procedure by smoothness constraints. In this, it is important to analyze the merits and shortcoming of these inversion solutions. Furthermore, the quantification of the loss of information occurring during the reconstruction process would also allow for an optimal design of monitoring network and comparison of different reconstruction procedures.

The aim of this study is to propose a weighted least-squares technique to reconstruct a point release using limited concentration measurements from trial#7, Fusion Field Trial 2007 experiment (Storwald, 2007). The formalisms are discussed to analyze the reliability of the estimated solutions. The source reconstruction features are analyzed in view of its visibility from the monitoring network, nature of the reconstructed release and resolution of the reconstructed source.

INVERSION TECHNIQUE & LOCALIZATION FEATURES

The inversion technique addresses here the reconstruction of a point release in a discretized space. This is based on a linear relationship between an unknown source vector $\mathbf{s}\in\Re^N$ (N is the dimension of model state vector) and measured concentration field $\boldsymbol{\mu}\in\Re^m$ given as (Pudykiewicz, 1998),

$$\boldsymbol{\mu}=\mathbf{A}\mathbf{s} \qquad (1)$$

where $\mathbf{A}\in\Re^{m\times N}$ is the sensitivity matrix. In order to avoid the singularity artifact due to point nature of sensitivity functions (Issartel et al., 2007), regularization is provided in terms of a weight function which describes the available information from monitoring network. Accordingly, the equation (1) is modified as (Issartel et al., 2007),

$$\boldsymbol{\mu}=\mathbf{A}_{\mathbf{w}}\mathbf{W}\mathbf{s} \qquad (2)$$

where $\mathbf{A_w} = \lfloor a_{ij} / w_{jj} \rfloor$ is the modified sensitivity matrix and $\mathbf{W} \in \Re^{N \times N}$ is the weight matrix having only non-zero diagonal elements ($w_{ij} = 0,\ if\ i \neq j$) and satisfy following properties: (i) $w_{jj} > 0$ (ii) $\sum_{j=1}^{N} w_{jj} = m$ and (iii) $diag\left(\mathbf{A_w^T}\, \mathbf{H_w^{-1}} \mathbf{A_w}\right) \equiv 1$. The weight functions are computed iteratively using algorithm proposed by Issartel (2005). The last constraint corresponds to an appropriate optimality criterion (also called as "*Renormalizing condition*") in which $\mathbf{H_w} = \mathbf{A_w} \mathbf{W} \mathbf{A_w^T}$ is the Gram matrix of the weighted sensitivity functions and superscript "T" denotes the transposition. Using the properties of matrix theory and singular value decomposition, the emission estimate can be obtained as

$$\mathbf{s} = \mathbf{A_w^T} \mathbf{H_w^{-1}} \boldsymbol{\mu} \tag{3}$$

The equation (3) provides an estimate for the distributed emissions and is seen as a generalized inverse solution to the under-determined class of linear inverse problems.

If the nature of the release is known as point wise, the source can be defined as $s_k^{true} = q\,\delta\left(\mathbf{x} - \mathbf{x}_k\right)$, where q is the source strength, $\mathbf{x}_k = \left(x_k, y_k\right)$ is the source location and $\delta(.)$ is the Dirac delta function. In view of the assumption, the equation (2) can be reduced as

$$\boldsymbol{\mu} = q\, w(\mathbf{x}_k)\, \mathbf{a_w}(\mathbf{x}_k) \tag{4}$$

By substituting equation (4) into equation (3), the estimate $\mathbf{s}$ is transformed as

$$\mathbf{s} = q\, w\left(\mathbf{x}_k\right) \mathbf{A_w^T} \mathbf{H_w^{-1}} \mathbf{a_w}(\mathbf{x}_k) \tag{5}$$

Using the Cauchy-Schwartz inequality and equation (5), the vector $\mathbf{s}$ attains its maximum value at the point source location $\left(\mathbf{x}_k\right)$ due to the third property of the weight function. Thus, the source location can be retrieved by searching out the location/grid coinciding with the maximum of estimate $\mathbf{s}$. Further, source strength q is derived as (Sharan et al., 2012),

$$q = s(\mathbf{x}_k) / w(\mathbf{x}_k)\ . \tag{6}$$

Besides a model estimate, it is important to yield measures for resolution, noise in the reconstruction and information on the efficiency of the monitoring design. These are described by the model resolution matrix, data resolution matrix and the variations of the weight (visibility) functions (Menendez et al., 1996). The resolution matrix comprises information about model, measurement, solution point and the a priori information. Therefore, re-writing equation (2) in terms of their true value and equation (5) in terms of the their estimated values as,

$$\mathbf{s}^{retrieved} = \mathbf{A_w^T} \mathbf{H_w^{-1}} \mathbf{A}\, \mathbf{s}^{true} \tag{7}$$

$$\boldsymbol{\mu}^{retrieved} = \mathbf{A} \mathbf{A_w^T} \mathbf{H_w^{-1}} \boldsymbol{\mu}^{true} = \mathbf{A_w} \mathbf{W} \mathbf{A_w^T} \mathbf{H_w^{-1}} \boldsymbol{\mu}^{true} = \mathbf{H_w} \mathbf{H_w^{-1}} \boldsymbol{\mu}^{true} = \boldsymbol{\mu}^{true} \tag{8}$$

In view of equations (7) and (8), the model resolution matrix ($\mathbf{R_M}$) and data resolution matrix ($\mathbf{R_D}$) are given as

$$\mathbf{R_M} = \mathbf{A_w^T}\, \mathbf{H_w^{-1}} \mathbf{A}\,; \qquad\qquad \mathbf{R_D} = \mathbf{I}^{m \times m} \tag{9}$$

This is observed that the solution to equation (1) is based on an inverse operator described in equation (3). The quality of this inverse operator is described by the resolution matrices ($\mathbf{R_M}$ and $\mathbf{R_D}$). An operator is said to be good inverse only when these resolution matrices becomes identity matrices. It is already shown that the data resolution matrix is identity ($\mathbf{R_D} = \mathbf{I}^{m \times m}$) and thus, measurements can be predicted exactly. The model resolution matrix contains precise information about the quality of the estimated solution on each grid point for the inverse procedure used. Computationally, it is quite expensive to compute the full rank $\mathbf{R_M}$. However, the column (point spread function ($\mathbf{A_w^T} \mathbf{H_w^{-1}} \mathbf{a}\left(\mathbf{x}_k\right)$)) and row (resolution kernel ($\mathbf{a_w^T}\left(\mathbf{x_k}\right) \mathbf{H_w^{-1}} \mathbf{A}$)) corresponding to the solution point can be analyzed to determine the quality of the solution. The resolution kernel describes how well the unknown release can be independently predicted or resolved. The point spread function illustrates the nature of the reconstructed source and the good solution corresponds to the spiky feature centered about the corresponding point. The maximum of point spread function can be given as

$$q_k^{retrieved} = \frac{\mathbf{a}_\mathbf{w}^{\mathrm{T}}(\mathbf{x}_k)\mathbf{H}_\mathbf{w}^{-1}\mathbf{a}(\mathbf{x}_k)}{w(\mathbf{x}_k)} q_k^{true} = q_k^{true} \qquad \text{since } \mathbf{a}_\mathbf{w}^{\mathrm{T}}(\mathbf{x}_k)\mathbf{H}_\mathbf{w}^{-1}\mathbf{a}_\mathbf{w}(\mathbf{x}_k) = 1 \tag{10}$$

In view of equation (8) and (10), it can be noticed the proposed inversion technique is able to optimally discriminate the unknown point release.

FUSION FIELD TRIALS 2007

The FFT-07 was a short range dispersion experiment conducted at DPG (Dugway Proving Ground), Utah in September 2007, over flat terrain (Storwald, 2007). This study utilizes real measurements from Trial#7 corresponding to single point source releasing propylene (C_3H_6) over a period of 10 min with a release rate 5.53 g s^{-1}. The concentration measurements were recorded at the height of 2 m about ground level by fast-response digital photo-ionization (DGPID) detectors at the frequency response of 50 Hz.

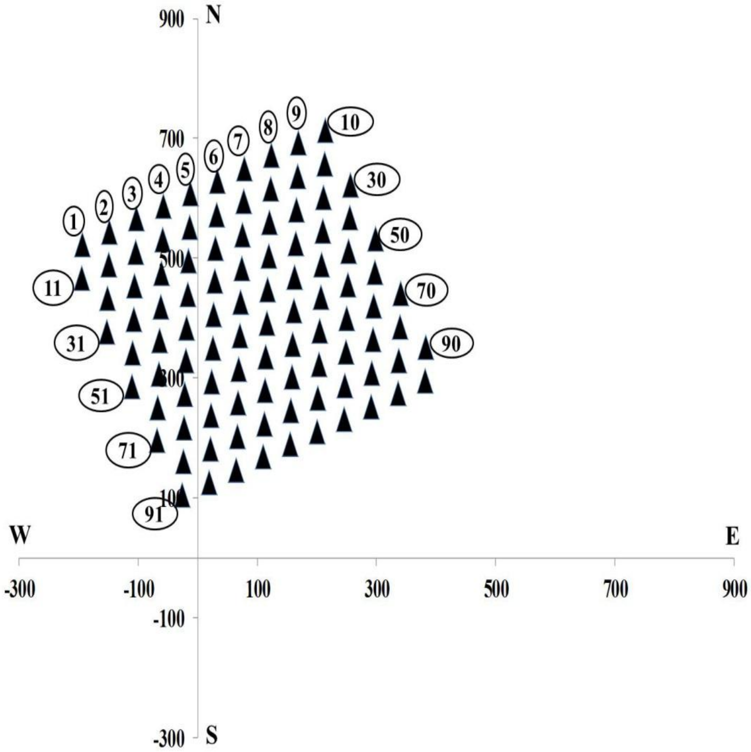

Figure 1: Layout of the FFT07 receptor's grid in a computational domain of 1200 m × 1200 m. The black triangles denote receptors. The receptor's index numbers are denoted in circles.

A total of 100 DGPID's are arranged in a rectangular staggered grid/array of area 475 m×450 m in 10 rows and 10 columns to measure the concentrations (Figure 1). The odd numbered rows (1, 3, 5, 7 and 9) of PID start at a location 25 m from the western grid edge of the grid and are spaced every 50 m. The even numbered rows (2, 4, 6, 8 and 10) begin exactly on the western grid edge and are also spaced every 50 m. Spacing between rows for all PIDs is 50 m. The staggered spacing scheme of PID's creates north-south columns of 25 m spacing. The index number of the DGPID's is mentioned within the circles in figure (1). The height of the source as well as samplers is 2 m above the ground level. The meteorological measurements (wind components and temperature) are taken from a 32 m ultrasonic tower with five levels (2, 4, 8, 16 and 32) located at a distance of 750 m north-west of the grid centre. In the data set, the locations of the source and the receptors are given in the latitude and longitude coordinates which are converted into UTM (Universal Transverse Mercator) coordinate system. The high frequency observed concentrations at all the receptors are averaged over the sampling duration for inversion computations.

RESULTS & DISCUSSION

The inversion technique is implemented on a discretized domain of 399×399 grids with a resolution of 3 m. The sensitivity coefficients are computed using an adjoint of an analytical dispersion model (Sharan et al., 1996). The dispersion parameters are obtained by using Gryning et al. (1987) formulations.

With the real measurements, the source location is retrieved within 3 m of the true release and the source strength is over-predicted by 25% of the true release rate. The function $w(\mathbf{x})$ is maximum at the receptors and decreases while moving in the upwind direction of the monitoring network. The source location lies in the well illuminated region of the monitoring network and retrieved as a distinct maximum region of $s(\mathbf{x})$. The upwind extension of source estimate is obtained in the poorly illuminated region due to lack of monitoring network.

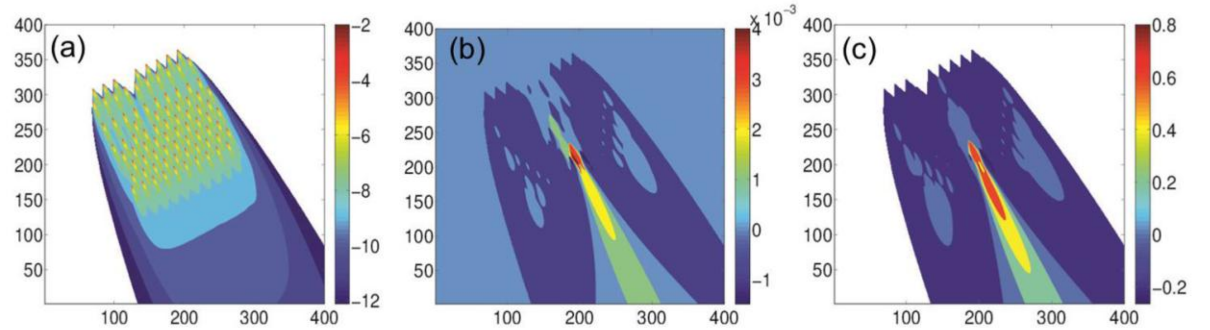

Figure 2: Isopleths of $w(\mathbf{x})$ (m^{-3}) (panel a), $s(\mathbf{x})$ (g m^{-3}) (panel b) and source strength resolution (panel c) in the Euclidian domain.

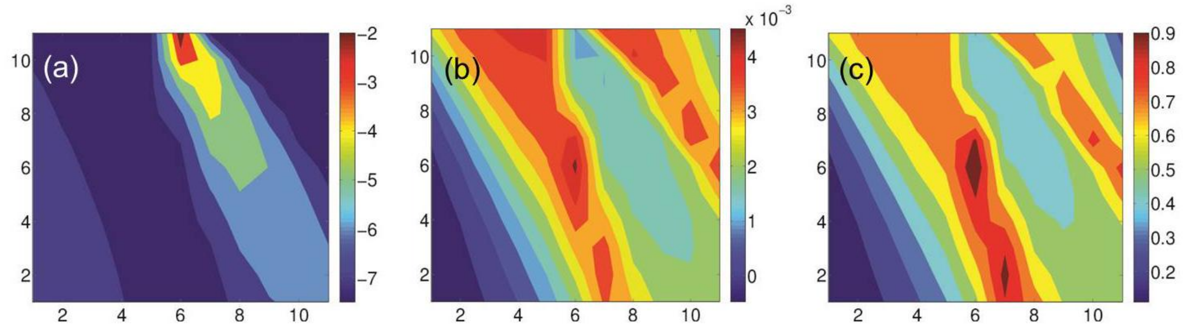

Figure 3: Isopleths of $w(\mathbf{x})$ (m^{-3}) (panel a), $s(\mathbf{x})$ (g m^{-3}) (panel b) and source strength resolution (panel c) in the five point window around retrieved source location. The center of the domain denotes the source location.

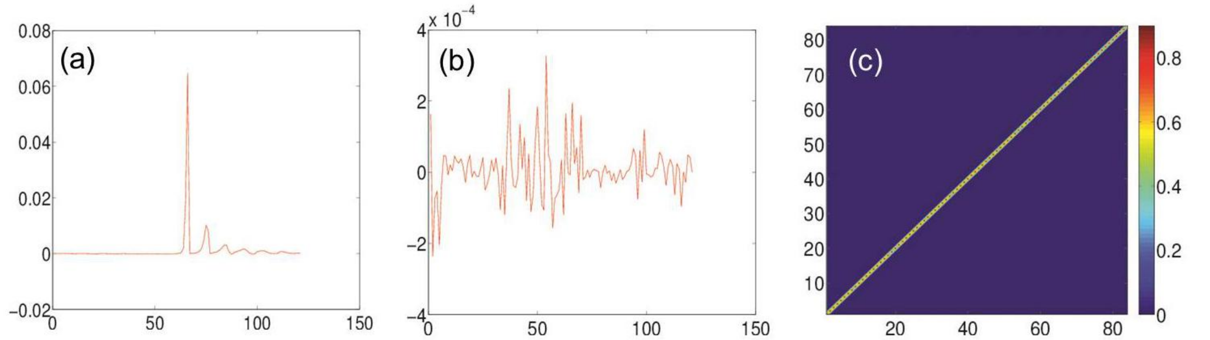

Figure 4: Point spread function (panel a) and resolution kernel (panel b) for the model resolution matrix ($\mathbf{R_M}$) in its neighborhood of five-point window. In panels (a & b), the horizontal axis denotes number of total grid point and vertical axis denotes the amplitude of the function. Panel c denotes the data resolution matrix ($\mathbf{R_D}$). In panel c, the horizontal and vertical axis denotes number of measurements.

The panel c (figure 2) describes the resolution of source strength which is quite similar in distribution with $s(\mathbf{x})$ (panel b, figure 2) and attains maximum value of resolution (=1) at the point source location. This assures that the retrieved source strength exists uniquely. For a clear visualization, the figure (2) is re-plotted in figure (3) for a five-point window around the retrieved point source location. It is observed that the maxima of $s(\mathbf{x})$ is unique and distinctly observed (panel b, figure 3). Two peaks are observed for the resolution of the Dirac-delta source strength. However, the maximum value is attained at the center.

To analyze the model resolution matrix, the point spread function and the resolution kernel corresponding to the retrieved source location is shown in panels a and b, (figure 4) respectively. It is observed that the a Dirac-delta function is obtained at the retrieved source location which shows that the model is able to capture the nature of the release and model elements are predicted close to the observations. However, variations are observed in the resolution kernel around the point source location. This shows that the source strength is moderately resolved. The resolution kernel amplitude is maximum at the retrieved source and side lobes decreases with the increasing distance from the solution point. It is observed that the regions around the point source have good resolution and illumination of monitoring network. The data resolution matrix is observed as close to the identity matrix having peaks of magnitude unity only at the diagonal (panel c, figure 4).

CONCLUSION

The study presents a weighted least-squares inversion technique for the identification of a point source using limited concentration measurements. The obtained inverse solution belongs to the category of generalized inverse solution of the under-determined inverse problems. However, the point source parameters are obtained for an over-determined case assuming measurements are higher than number of unknowns. In addition, the study discusses the optimal localization features which are fulfilled by the proposed inversion technique. The weight functions are proposed based on the geometry of the monitoring network. The study is evaluated using the measurements from FFT07 (Trial# 7) experiment.

The point source is retrieved within 3 m of the true source location and the strength is retrieved as slightly over-predicted in comparison to the true release strength. The source is observed to be distinctly located and lies in a highly illuminated region of the monitoring network. The spread function of the retrieved source is observed as Dirac-delta type function and source strength is moderately resolved by the model. This assures that the retrieved source is close and unique to the true source. Also, the measurements are closely predicted by the model and model errors are minimum as possible. The proposed inversion solution minimizes the spread of the resolution kernels for a given weighted solution.

REFERENCES

Gryning, S.-E., Holtslag, A.A.M., Irwin, J.S. and Sivertsen, B., 1987: Applied dispersion modelling based on meteorological scaling parameters. *Atmospheric Environment*, **21**, 79–89.

Issartel, J.-P., 2005: Emergence of a tracer source from air concentration measurements, a new strategy for linear assimilation. *Atmos. Chem. Phys*., **5**, 249–273.

Issartel, J.-P., M. Sharan and M. Modani, 2007: An inversion technique to retrieve the source of a tracer with an application to synthetic satellite measurements. *Proc. Roy. Soc. A*, **463**, 2863-2886.

Menendez, R. G. P., Andino, S. L. G. and Lutkenhoner, B., 1996: Figures of Merit to compare distributed linear inverse solutions. *Brain Topography*, **9**, 117-124.

Pudykiewicz, J., 1998. Application of Adjoint Tracer Transport Equations for Evaluating Source Parameters. *Atmospheric Environment*, **32**, 3039-3050.

Sharan, M., M. P. Singh, A. K. Yadav, P. Aggarwal and S. Nigam, 1996: A mathematical model for the dispersion of pollutants in low wind conditions. *Atmospheric Envirnment*, **30**, 1209-1220.

Sharan, M., J.-P. Issartel, S. K. Singh and P. Kumar, 2009: An inversion technique for the retrieval of single-point emissions from atmospheric concentration measurements. *Proc. Roy. Soc. A*, **465**, 2069-2088.

Sharan, M., J.-P. Issartel and S. K. Singh, 2012: A point-source reconstruction from concentration measurements in low-wind stable conditions. *Q.J.R. Meteorol. Soc*., **138**, 1884–1894.

Storwald, D.P., 2007: Detailed test plan for the fusing sensor information from observing networks (Fusion) Field Trial (FFT-07). Meteorology Division, West Desert Test Center, U.S. Army Dugway Proving Ground WDTC Document No. WDTC-TP-07-078.