

**16th International Conference on  
Harmonisation within Atmospheric Dispersion Modelling for Regulatory Purposes  
8-11 September 2014, Varna, Bulgaria**

---

**ANALYSIS OF DISPERSION AND HIGHER ORDER STATISTICAL MOMENTS ABOVE  
RURAL AND URBAN AREAS**

*Evgeni Syrakov<sup>1</sup> and Kostadin Ganev<sup>2</sup>*

<sup>1</sup> University of Sofia, Faculty of Physics, Sofia, Bulgaria

<sup>2</sup> National Institute of Geophysics, Geodesy and Geography, Bulgarian Academy of Sciences, Sofia,  
Bulgaria

**Abstract:** The basic PBL resistance laws (RL-method) are modified and applied to areas with large urban roughness, accounting for a wide range of turbulent urban regimes. A number of urban and rural diffusion characteristics for these regimes are studied and juxtaposed.

**Key words:** *urban/rural turbulent regimes, modified resistance laws method, none Gaussian plume-MM model, statistical moments, dispersion, skewness, kurtosis.*

## INTRODUCTION

RL-method for rural conditions makes it possible a wide range of turbulent regimes to be studied. Respective modification of the RL-method for urban conditions, based on a simple parameterization scheme is given in the present work. On this basis a set of different diffusion characteristics – dispersions, higher order moments and their combinations are determined and analyzed.

## RURAL CONDITIONS

The generalized version of the RL-methods makes it possible the dependence of transfer-interaction characteristics  $C_g = u_* / G_0$  – geostrophic drag coefficient,  $\alpha$  – the angle of wind full rotation in PBL,  $\mu$  – internal stability parameter in PBL, etc. on the dimensionless external parameters:

$$R_0, S, R_{0i}, M, \phi, \psi, \mu_N, \quad (1)$$

to be obtained (Syrakov E., 2011, Syrakov E. et al., 2012), where  $R_0 = G_0 / fz_0$  – geostrophic drag coefficient,  $S = \beta \delta \theta / fG_0$  – external stability parameter,  $\delta \theta = \theta_H - \theta_0$  – increment of  $\theta$  in PBL,  $R_{0i} = G_0 / fhi$  – the inversion Rossby parameter,  $H$  (for inversion case =  $hi$ ) – inversion height,  $M, \phi$  – baroclinic parameters (module and angle),  $\psi$  – the angle of terrain slope,  $\mu_N = N / f$  – free flow stability parameter. The parameters  $C_g, \alpha$  and  $\mu$  are used as an input to steady-state stratified, baroclinic PBL over sloping terrain with vertical exchange coefficient for momentum (Syrakov E. and K. Ganev, 2003, Syrakov E., 2011):

$$k_z = [\kappa u_* z / \varphi_u(\zeta)] (1 - z/H)^m \quad (2)$$

and analogous formula for  $k_{z\theta}$ , where  $\varphi_u(\zeta)$  in (2) is replaced by  $\varphi_\theta(\zeta)$ . The values of the exponent is 2 for unstable and 1 for stable and neutral stratification. In (2)  $\zeta = z/L$ ,  $L$  is the Monin-Obukhov length. The explicit form of the universal functions  $\varphi_u(\zeta)$  and  $\varphi_\theta(\zeta)$ , which include also the asymptote  $\zeta^{-1/3}$  for free convection, is given in Syrakov E. (2011), Syrakov E. et al. (2012),  $H$  is PBL height. Formula (2) is a slight modification of the one suggested in Troen I. and L. Mahrt (1986).

The dynamic characteristics, obtained by the PBL model are used as an input to the none- Gaussian pume MM-model (Syrakov E. et al., 2012):

$$c(x, y, z, t) = \frac{c_0(x, z, t)}{\sqrt{2\pi} \sigma_y} \exp\left(-\frac{(y-Y)^2}{2\sigma_y^2}\right), \quad (3)$$

where the wind rotation effect is accounted by the mean displacement  $Y$ . The parameters  $Y$  and the dispersion  $\sigma_y$  are calculated by the definition formulae:

$$Y(x, z, t) = c_1/c_0, \quad \sigma_y^2 = c_2/c_0 - Y^2, \quad (4)$$

where zero moment – the cross-wind integrated concentration  $c_0(x, z, t)$  and the first and second moments  $c_1(x, z, t)$  and  $c_2(x, z, t)$  are calculated numerically on the basis of a system of equations describing the statistical moments – moment's method (MM).

### MODIFICATION FOR URBAN CONDITIONS

Applying the modified RL-method to urban boundary layer (UBL) with height  $H$  allows parameterization of basic characteristics for a wide set of turbulent regimes in similarity format of internal/external parameters. Above urban canopy layer of the urban roughness sub layer (URL) is situated with depth  $(z_* - h)$ , or more precisely depth  $[z_* - (d + z_{ou})]$  when using the displacement height  $d$ , where  $z_* = (2 \div 5)h$  (for example the lower limit  $z_* = 2h$  is used for typical European cities (Fischer B. et al., 1999),  $h$  is the average roof height. In this layer basic input parameters are generated - effective friction velocity  $u_*^{eff} = u_*(z = z_*)$  and analogically  $q^{eff} = q(z = z_*)$  (the typical value of the effective cinematic heat flux),  $z_{ou}$ ,  $z_{oT}$  (respectively the effective aerodynamic and temperature urban roughness lengths), which are important for the UBL as a whole. The depth of URL is significantly smaller than  $H$  - at  $z_* = 2h$   $[z_* - (h + z_{ou})] = 0.8h \ll H$  (the evaluations  $d = 0.7h$ ,  $z_{ou} = 0.1h$  are used (Grimmond and Oke, 1999)). This makes is possible, as a first approximation, this thin layer to be interpreted as elevated equivalent surface layer above level  $z = d + z_{ou}$  (where the velocity becomes zero), forcing the whole UBL trough typical effective parameters  $u_*(z_*)$ ,  $q(z_*)$ . This is analogical to the role, which the surface layer plays in PBL at rural conditions.

**Table 1.** Input PBL external parameters for turbulent regimes

PBL type	barothropic			inversions			baroclinic Unstable/neutral				terrain slope		
	1	2	3	4	5	6	7/11	8/12	9/13	10/14	15	16	17
$R_0$	$10^7$	$10^7$	$10^7$	$10^7$	$10^7$	$10^7$	$10^7$	$10^7$	$10^7$	$10^7$	$10^7$	$10^7$	$10^7$
rural													
$R_0$	$5.10^4$	$5.10^4$	$5.10^4$	-	$5.10^4$	$5.10^4$	-/	-/	-/	-/	$5.10^4$	$5.10^4$	$5.10^4$
urban													
$S$	-500	0	500	-500	0	500	-500 /0	-500 /0	-500 /0	-500 /0	-500	0	500
$R_{ou}$	-	-	-	400	400	400	-	-	-	-	-	-	-
$M$	0	0	0	0	0	0	10	10	10	10	0	0	0
$\phi [^\circ]$	0	0	0	0	0	0	0	180	220	270	0	0	0
$\psi$	0	0	0	0	0	0	0	0	0	0	0.1	0.1	0.1
[rad]													

In the framework of this analogy a simplified urban modified scheme for the RL-method can be developed analogically as for the case of rural conditions, moreover that in first approximation Rossby number similarity is fulfilled also over level  $z_*$  in UBL (Fischer B. et al., 1999). The urban modification of the RL-method consists of introducing the displacement height  $d$  and the respective above commented effective parameters, which adequate describe the specific urban conditions.

The urban modified RL-method makes it possible determining of the basic unknown urban modified flux parameters  $C_g = u_*(z_*)/G_0$  and  $\mu = \kappa u_*(z_*)/fL(z_*)$  ( $L(z_*)$  is the value of the local MO-length at

$z = z_*$ ) and angle  $\alpha$  of full wind rotation in UBL in dependence of the modified external parameters:  $R_o = G_0/fz_{ou}$ ,  $S = \beta\delta\theta/fG_0$  ( $\delta\theta = \theta_H - \theta(z_{oT} + d)$  is temperature increment in UBL) and the other from (1), which remain practically unchanged. Obviously the urban influence is manifested mostly trough parameters  $R_o$  and  $S$ , accounting effects connected with urban roughness length and temperature, respectively at levels  $z = z_{ou} + d$  and  $z = z_{oT} + d$ . It should be noted that  $\delta\theta_{urban} \neq \delta\theta_{rural}$  (in integral plan the thermal stability in UBL is diminished and respectively the instability is increased, due to the city heat island). In order to make turbulent regimes comparable the external parameters  $S_{urban}$  and  $S_{rural}$  are chosen equal. Attention is paid mostly to the Rossby parameter, which influents significantly not only the roughness effects, but also all the external conditions, which determine the turbulent regime. At urban conditions the following parameters are applied:  $h = 15m$ ,  $z_{ou} = 1.5m$ ,  $G_0 = 7.5m/s$ , i.e.  $R_o - urban = 5 \cdot 10^4$ , while at rural:  $G_0 = 7.5m/s$ ,  $z_0 = 0.01m$ , i.e.  $R_o - rural = 7.5 \cdot 10^6 \approx 10^7$ . The other external parameters values are given in Table 1.

### SIMULATION RESULTS

The respective diffusion characteristics for urban conditions are calculated according to the same scheme (1-4) as for rural conditions: implementation of RL-method with input external parameters, given in Table 1  $\rightarrow$  determining  $C_g$ ,  $\alpha$ ,  $\mu$  parameters  $\rightarrow$  PBL dynamics characteristics  $\rightarrow$  none Gaussian plume MM. It will be noticed that  $u$  in (2) is replaced by  $u(z_*)$ ,  $L$  by  $L(z_*)$  and  $z$  by  $z' = z - d$ .

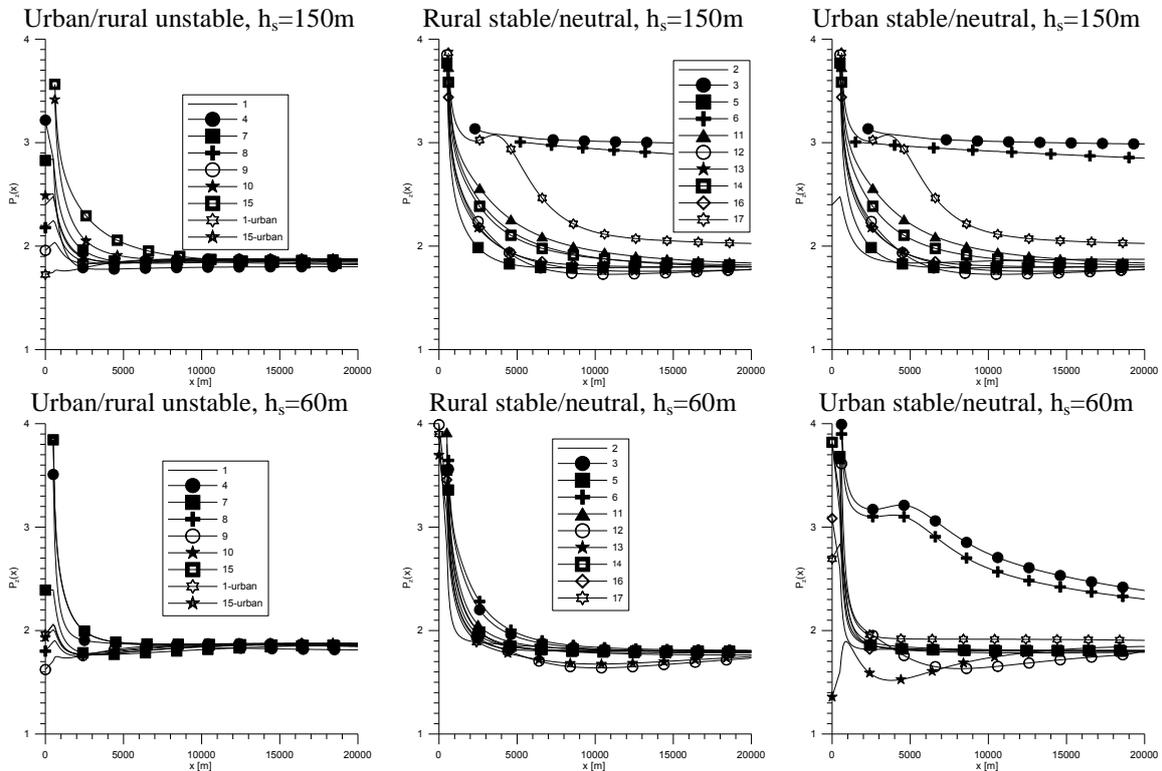


Figure 1. Comparison of parameter  $P_z(x) = Ku(x)/(Sk^2(x)+1)$  for urban and rural conditions at different turbulent regimes.

Some Plume-MM characteristics: dispersions Lagrangian vertical coordinate  $Z(x)$ , lateral and vertical dispersions  $\sigma_y(x)$ ,  $\sigma_z(x)$  and skewness-kurtosis combination  $P_z(x) = Ku(x)/(Sk^2(x)+1)$  - their surface values for rural cases ( $z = z_0$ ) and a little above roof level for urban cases, obtained for elevated sources,

are shown in Figs. 1-3. The 17 turbulent regimes (barotropic, inversions, baroclinic, terrain slope), for which the simulations are made, are determined by the external parameters, shown in Table 1.

The study of the relations between skewness and kurtosis is of interest. Kendal and Stuart (1977) found from statistical considerations, that for vertical skewness and kurtosis such a limiting relation exists and is of quadratic nature:  $Ku \sim Sk^2 + 1$ , i.e. the relation  $P_z(x) = Ku(x)/(Sk^2(x)+1) \sim \text{const.}$  tends to be universal. Experimental results show similar approximately quadratic behaviour (Durst et al., 1987, Lewis et al., 1997) in both shear- and buoyancy- produced turbulence. The results, shown in Figure 1 for relation  $P_z(x)$  confirm and significantly generalise these conclusions for a wider range of turbulent regimes, including for urban conditions. Some deviations can be observed for regimes 3 and 6. As a whole  $P_z(x)$  tends towards typical values 1.7-1.8, while these values are first (closer to the source) reach for unstable conditions.

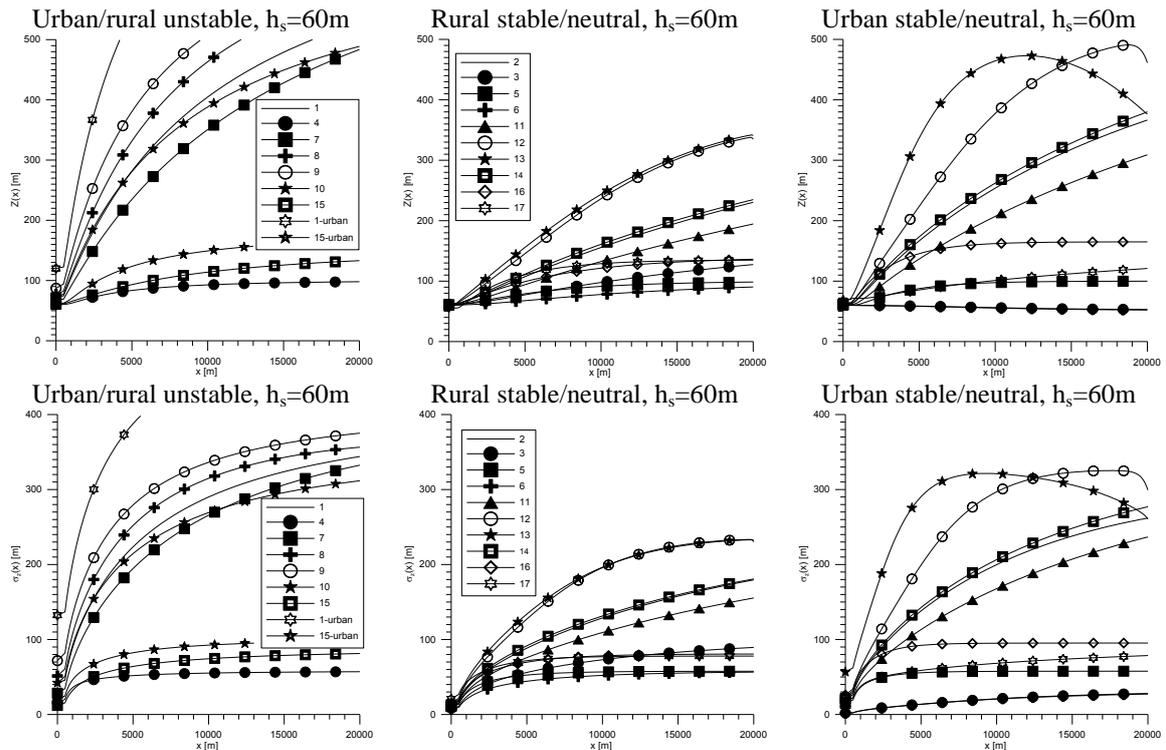


Figure 2. Comparison of  $\sigma_z(x)$  and  $Z(x)$  for urban and rural conditions at different turbulent regimes.

From Fig.2 it can be seen that for urban/rural unstable cases  $\sigma_z(x)$  and  $Z(x)$  have similar behaviour and curve distribution in dependence to turbulent regimes. The comparison rural/urban here is made for regimes 1 and 15. For the first regime  $\sigma_z(x)$  and  $Z(x)$  are significantly larger for the urban case. This tendency is valid also for the second case, but is much weakly manifested.

For the stable/neutral conditions the comparison rural/urban is given for the whole set of considered turbulent regimes. Largest  $\sigma_z(x)$  and  $Z(x)$  values can be observed for baroclinic regimes 11-14, while for urban conditions the respective values are bigger than for rural conditions. The smallest  $\sigma_z(x)$  and  $Z(x)$  values are observed for regimes 3, 5, 6, in particular for urban conditions.

The lateral spread of the plume  $\sigma_y(x)$  is shown in Fig. 3. The behaviour of  $\sigma_y(x)$  is demonstrated for a wide set of unstable rural conditions. The comparison rural/urban is made only for regimes 1 and 15. In

both of the regimes  $\sigma_y(x)$  is significantly larger for urban conditions. For stable/neutral conditions in general, compared with the rural case, the urban cases show a much more spread “fan” of the  $\sigma_y(x)$  curves. Largest values for  $\sigma_y(x)$  are reached for the baroclinic regimes 12, 14 for urban conditions, while the most limited decrease of  $\sigma_y(x)$  can be observed for regimes 3 and 6, both for rural and urban conditions.

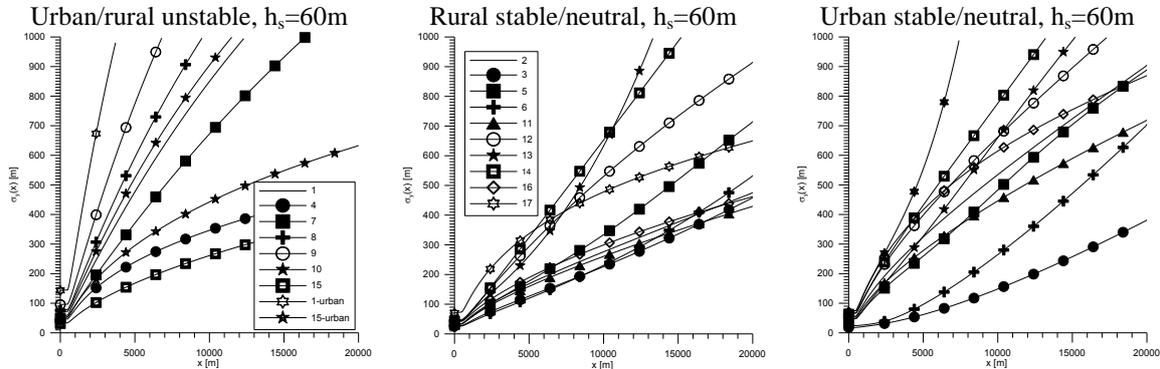


Figure 3. Comparison of  $\sigma_y(x)$  for urban and rural conditions at different turbulent regimes.

## CONCLUSIONS

The RL-method was modified for urban conditions using a simple parameterization scheme. This allows the basic dynamic and dispersion urban parameters to be obtained and compared with the respective rural parameters for a wide set of turbulent regimes.

The performed analysis demonstrates the significant influence of the turbulent regimes, as well as of the change of conditions rural-urban, on the studied diffusion characteristics. Their more precise determination will result in more accurate calculation of the concentration fields for rural/urban conditions and different meteorological conditions. The results can be used in many applied air pollution tasks, study of parameters sensitivity to external factors, parameterization of numerical urban models, etc.

## ACKNOWLEDGMENTS

The present work is supported by the Bulgarian National Science Fund (grant ДЦБП-02/1/29.12.2009).

## REFERENCES

- Durst, F., Jordanovich, J. and Kanevce, L.J., 1987: Probability density distribution in turbulent wall boundary layer flow. *Turbulent shear flow* **5**, Springer, 198-220
- Fischer B., S. Joffre, J. Kukkonen, M. Pringe, M. Rotach and M. Schatzmann, 1999: Meteorology applied to urban air pollution problems, Final report COST Action 715, Luxembourg, OPEC
- Grimmond C. and T. Oke, 1999: Aerodynamic properties of urban areas derived from analysis of surface form. *J Appl Meteor*, **38**, 1261-1292
- Kendal, sir M. and A. Stuart, 1977: The advanced theory of statistics, v.1, 472p.
- Lewis, D., Chatwin, P. and Mole, N., 1997: Investigation of the collapse of the skewness and kurtosis exhibited in atmospheric dispersion data. *Il Nuovo Cimento C*, **20** (3), 385-398
- Syrakov E. and K. Ganev, 2003: Accounting for effects of wind rotation in the PBL on the plume characteristics. *Int J Environment & Pollution*, **vol. 20**, No.1-6, 154-164.
- Syrakov E., 2011: Atmospheric boundary layer: Structure, Parameterization, Interactions, Heron Press, Sofia, p.394
- Syrakov E., K. Ganev, M. Tsankov and E., Cholakov, 2012: Estimation and parameterization of the influence of synoptic conditions on pollution characteristics in the PBL, *Int. J. Environment and Pollution*, Vol. 50, Nos. 1/2/3/4, 327-341
- Troen I. and L. Marth, 1986: A simple model of atmospheric boundary: sensitivity to surface evaporation, *Boundary Layer meteorology*, **37**, 129-148.