Point Source Reconstruction : Analyzing Localization Features of a Weighted Least-Squares Technique

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Motivation

• Importance in emergency actions and national security.
• Necessity of finding optimal solution to inverse problems.
• Localization features of the inverse solution.
• Resolution of the source estimates.
Objective

- To propose an inversion technique.
- To discuss optimal localization properties of the inverse solution.
- Evaluation using Fusion Field Trials 2007 dataset.

The presentation is focused for inversion of a continuous point release.

- Point source reconstruction: Parametric estimation
- Unknowns: location & strength
Approach

• Source–receptor relationship

A discrete version of the source is retrieved in finite dimension.

For non-parametric source, $m << N$

\[ \mu = A \sigma \]

- Concentration measurements $dim \ m$
- Sensitivity matrix $dim \ m \times N$
- Source $dim \ N$
Weight matrix

\[ \mu = A_w W_s \]

- **Properties**
  - Purely diagonal and \( w_{jj} > 0, \quad \text{for } j = 1, \ldots, N \)
  - \( \sum_{j=1}^{N} w_{jj} = m \)
  - Criterion for best choosing \( w \) is
    \[ \text{diag} \left( A_w^T H_w^{-1} A_w \right) \equiv 1 \]

\( W \) is computed by an iterative algorithm (Issartel, 2005).
**continue...**

- **Source estimate**

\[ \hat{s} = A_w^T H_w^{-1} \mu \]

where, \[ H_w = A_w W A_w^T \]

- **In case of a point source of strength** \( q \) **located at** \( x_0 \)

\[ s(x) = q \delta(x - x_0) \]

\[ \mu = q w(x_0) a_w(x_0) \]

The source estimate is,

\[ s(x) = q w(x_0) a_w(x_0)^T H_w^{-1} a_w(x) \]

Maximum of the \( s \) coincides with the point source location.

Now, **intensity** can be computed as, \[ \tilde{q} = s(x_0) / w(x_0) \]
Desired localization features

1. Measurements should be well retrieved
   \emph{data resolution matrix, ideally, identity $m \times m$}

2. Source should be well retrieved in spite of:
   - information sparsity (limited number of meas.)
   - $N \times N$ model resolution matrix, ideally, identity.
   - information accuracy (errors in the measurements)
   - Variance of the source, ideally, zero.
Measurements should be well retrieved.

- Data resolution matrix

$$\mu_{\text{retrieved}} = A_{\text{retrieved}} s$$

$$= A A_w^T H_w^{-1} \mu_{\text{true}}$$

$$= A_w W A_w^T H_w^{-1} \mu_{\text{true}}$$

$$= H_w H_w^{-1} \mu_{\text{true}}$$

$$= \mu_{\text{true}}$$
Source should be well estimated from exact measurements

• Exact source retrieval is not feasible

\[ s^\text{retrieved} = A_w^T H_w^{-1} \mu = A_w^T H_w^{-1} A \mu \]

\[ R_M = A_w^T H_w^{-1} A = A_w^T H_w^{-1} A_w W \]

\[ \text{diag}(A_w^T H_w^{-1} A_w) = 1 \]

\[ \text{diag}(R_M) = \text{diag}(W) \]
Continue: point source at $x_k$

How the current solution is influenced by all possible sources?

How well is the quality of a source reconstruction?

Resolution kernel

Point spread function

$R_M = \begin{bmatrix} w_1 & \ldots & w_k & \ldots & w_N \end{bmatrix}_{N \times N}$
Source retrieval and measurement errors

- covariance matrix of estimated source versus measurement errors cov matrix
  \[
  \text{cov}[s^{\text{retrieved}}] = A_w^T H_w^{-1} \text{cov}[\mu] H_w^{-1} A_w
  \]

- **Assumption**: measurement errors are mainly due to model and : 
  Least-squares cost function is written as:  
  \[
  J = (\mu - A_w Ws)^T H_w^{-1} (\mu - A_w Ws)
  \]

- Issartel et al. (2012) have shown that $H_w$ provides an optimal discrimination to the measurements.

  If  \[
  \text{cov}[\mu] = H_w
  \]

  Then  \[
  \text{var}[s^{\text{retrieved}}] = \text{diag}(A_w^T H_w^{-1} A_w) = 1
  \]
Fusion Field Trial 2007

• 10 min Propylene release in a flat terrain (Storwald, 2007)
• Source height = 2m
• 100 DGPID fast response conc
• Sampling height = 2m
• Trial 7
• North-West wind direction
Computations

- Discretized domain, 1200 m × 1200 m.
- 400 grid points in each direction with uniform grid size 3m.
- 89 concentration measurements.
- Gaussian dispersion model (Sharan et al. 1996).
- Iterative computation of matrix $W$.
- True source (200, 200)
- True source strength = 5.53 g/s
Results

- A priori information apparent to monitoring network
- Well & poorly monitored regions are distinguished.

Distribution of weights
Retrieval

- Source is retrieved at a distance of 3m from true source.
- Strength is over-predicted by 25%.

Distribution of source information
continue...

(a) Synthetic data

(b) Real data

(b) is the best resolution, achieved with limited measurements.
Model Resolution Matrix

(a) Resolution kernel

(b) Point spread function
Conclusions

• Inversion technique retrieve the point source close to the true source.

• Technique fullfills the proposed localization criteria.

• Source is observed to be distinctly located and lies in a highly illuminated region of the monitoring network.

• Sharpness of the resolution is maximum at the retrieved location of the point source.
Thank You for Your Kind Attention

References


