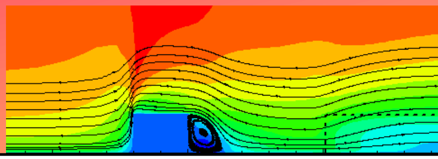
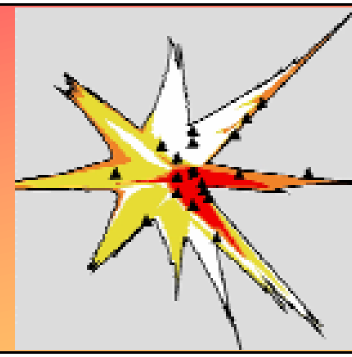
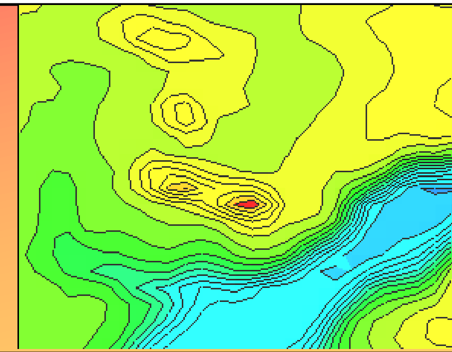






A FAST RELIABLE ALGORITHM FOR POINT SOURCE LOCALIZATION: APPLICATION TO A NEW KITFOX DATA SET

Grégory TURBELIN

gregory.turbelin@ufrst.univ-evry.fr

S. K. Singh, H. Kouichi, N. Bostic, A. A. Feiz, P. Ngae, J.P. Issartel

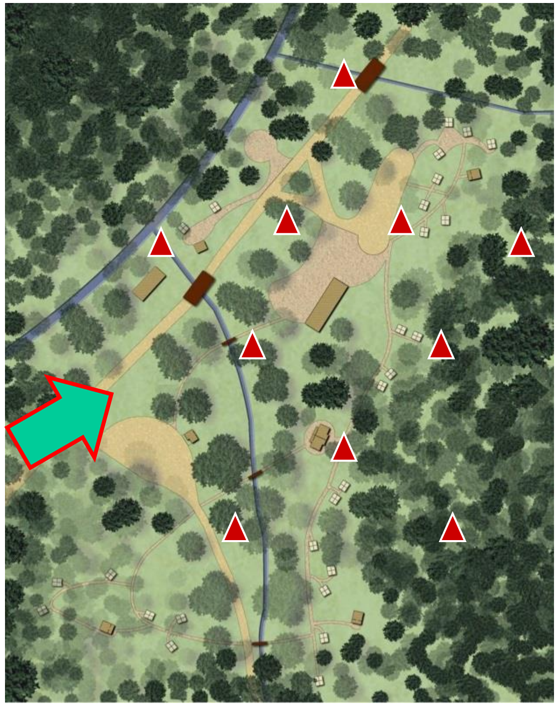
Laboratoire de Mécanique et d'Energétique d'Evry
Université d'Evry, France

http://lmee.univ-evry.fr/

❑Context

➔ Accidental or intentional atmospheric contaminant release (local scale)

- Source estimation methods aim to estimate
  - source(s) location(s)
  - source type, strength, and number
  - release start time and duration
- Given :
  - site description (terrain, vegetation, building)
  - available meteorological information
  - *m* concentrations measured by a network

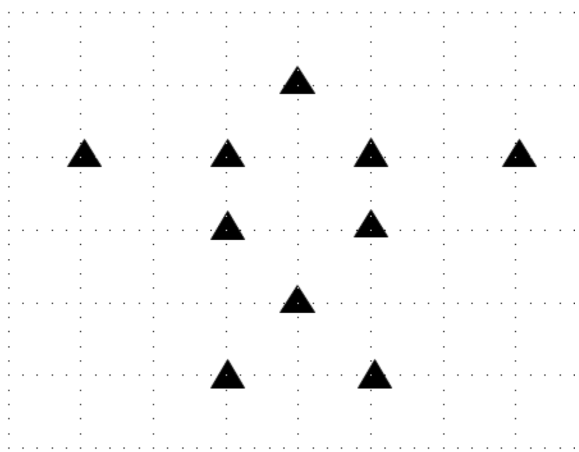


Several source estimation algorithms are currently being developed

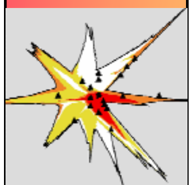
❑The discrete inverse problem

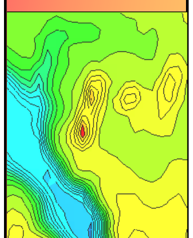
- ➔ Source described, on a grid of *N* points, by a source vector **s**
- ➔ It generates a field of concentrations only known through *m* observations $\mu_i = C(\mathbf{x_i})$ at locations $\mathbf{x_i}$ $(i = 1 \ldots m)$

The problem consists in determining the *N* unknown components of the source vector from the *m* measurements



$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_m \end{pmatrix} \overset{?}{\Longrightarrow} \mathbf{S} = \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ \vdots \\ s_N \end{pmatrix}$$





❑The renormalization technique

➔ Deterministic technique:

- use of a minimum of a priori information
- use of adjoint transport equations (receptors oriented modeling technique)
- computation of a renormalizing function

➔ It returns a source estimate which is linear with respect to the observations

- Issartel et al. (2005, 2007): utility of the renormalization to minimize inversion artifacts
- Sharan et al. (2009): reconstruction of a single ground-level point source
- Singh et al. (2013): identification of multiple-point sources releasing similar tracer
- Turbelin et al. (2014): generalization for discrete inverse problems

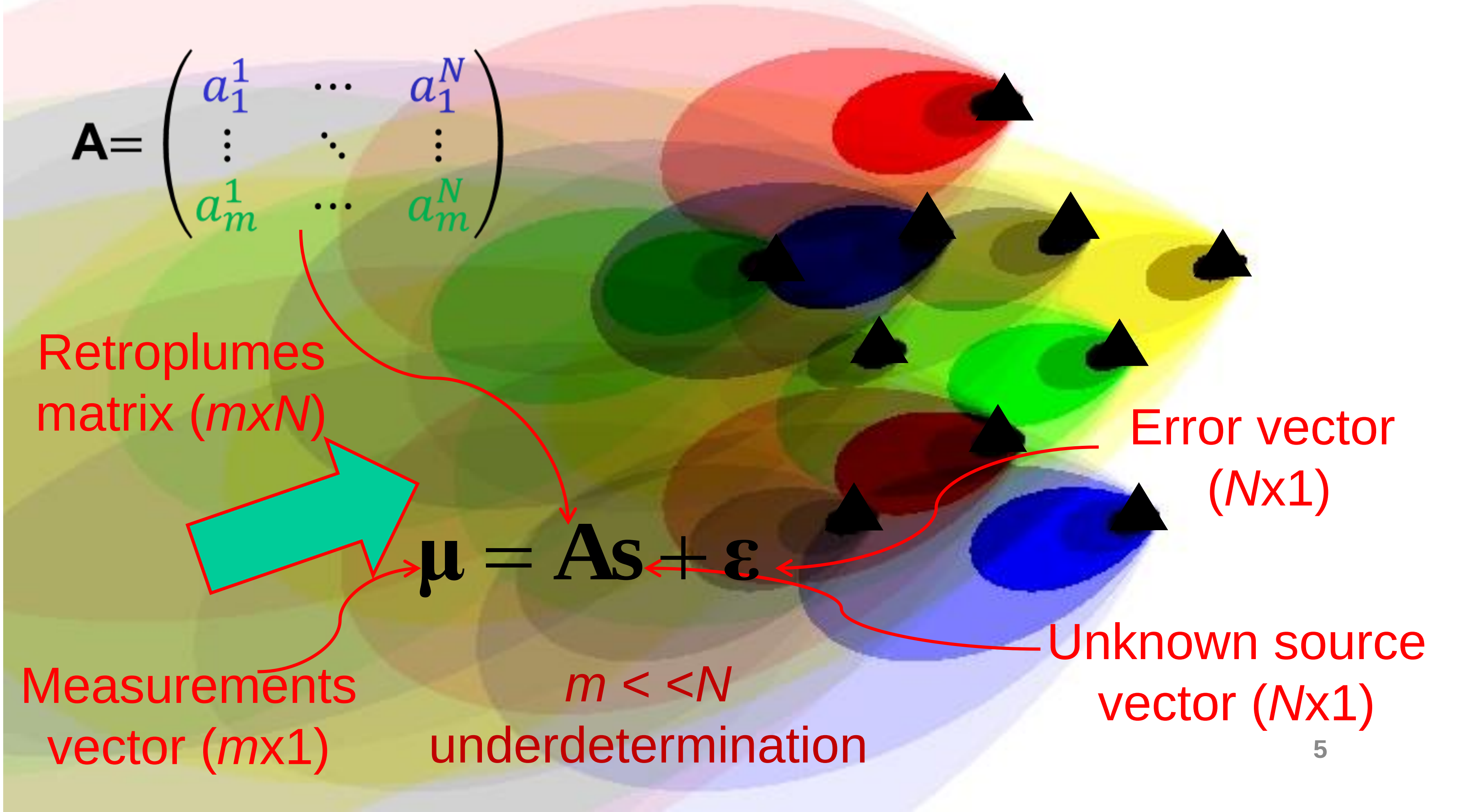
For a matter of simplicity, this presentation only deals with continuous releases, for time varying releases see Issartel et al. (2007)

❑The linear model

- ➔ The concentrations measured at the captors locations are linear functions of the sources, the multiplicative factor being the retroplumes matrix **A**
  - **components obtained by solving adjoint equations**

$$\mathbf{A}=\begin{pmatrix} a_1^1 & \cdots & a_1^N \\ \vdots & \ddots & \vdots \\ a_m^1 & \cdots & a_m^N \end{pmatrix}$$

$$\boldsymbol{\mu} = \mathbf{A}\mathbf{s} + \boldsymbol{\varepsilon}$$



❑A minimum weighted norm solution

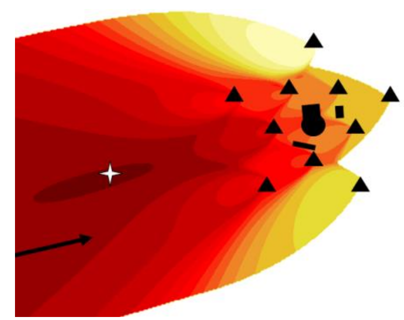
- ➔ Any solution to the problem can be written as $\hat{\mathbf{s}} = \mathbf{G}\boldsymbol{\mu}$
  - – **G** (N×m): some generalized inverse of **A**
- ➔ Discrete renormalized solution given by

$$\mathbf{s}_{//\mathbf{w}} = \mathbf{A}_{\mathbf{w}}^{\mathrm{T}}\mathbf{H}_{\mathbf{w}}^{-1}\boldsymbol{\mu} = \mathbf{W}^{-1}\mathbf{A}^{\mathrm{T}}(\mathbf{A}\mathbf{W}^{-1}\mathbf{A}^{\mathrm{T}})^{-1}\boldsymbol{\mu}$$

  - – unique minimum **W**-weighted norm solution of the problem, i.e.

$$\mathbf{s}_{//\mathbf{w}} \ \text{minimizes} \ \|\mathbf{s}\|_{\mathbf{w}} = \sqrt{\mathbf{s}^{\mathrm{T}}\,\mathbf{W}\mathbf{s}}$$

- ➔ Optimal diagonal weight matrix **W** (N×N), in case of a **single point source**
  - – the maximum value of the estimate corresponds to the location of the source
  - – the release intensity of the source is given by



$$\text{Intensity} = \frac{s_{//\mathbf{w}}(\mathbf{x}_0)}{w(\mathbf{x}_0)}$$

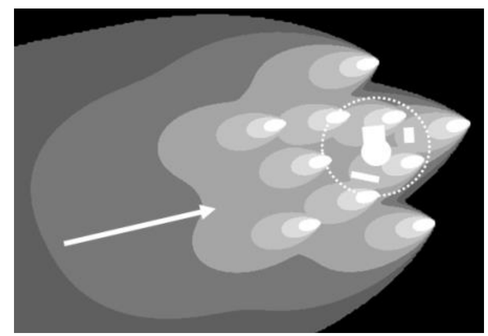
❑The renormalization condition

➔ Optimal reconstruction of position and intensity of all single sources, when (renormalization condition)

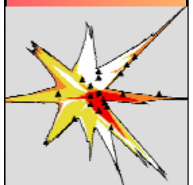
$$diag\left(\mathbf{A}_{\mathbf{w}}^{\mathrm{T}}\,\mathbf{H}_{\mathbf{w}}^{-1}\mathbf{A}_{\mathbf{w}}\right)\equiv 1 \quad \text{with} \quad w_{jj}>0 \quad \text{and} \quad \sum_{j=1}^{N} w_{jj}=m$$

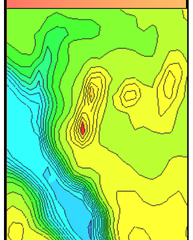
➔ The components of the optimal weight function are the discrete values of the visibility function

- characterizes the regions well or poorly monitored by the network
    - focus at the detectors locations
    - decreases with increasing downwind distance



It has been interpreted as the prior distribution of the emissions apparent to the monitoring system





❑Computation of the solution

➔ The inverse operator is the **W**-weighted generalized inverse of **A**

$$\mathbf{S}_{//\mathbf{W}} = \mathbf{W}^{-1}\mathbf{A}^{\mathrm{T}}(\mathbf{A}\mathbf{W}^{-1}\mathbf{A}^{\mathrm{T}})^{-1}\boldsymbol{\mu} = \mathbf{A}_{\mathbf{W}}^{+}\boldsymbol{\mu}$$

- computed by “classical” matrix operations
- or by making use of the pseudo inverse concept

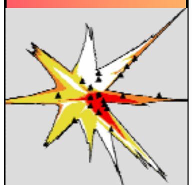
$$\boxed{\mathbf{A}_{\mathbf{W}}^{+} = \mathbf{W}^{-1/2}(\mathbf{A}\mathbf{W}^{-1/2})^{+}}$$

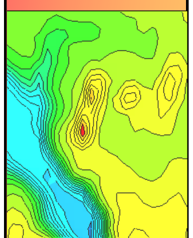
- “(.)$^{+}$”: Moore–Penrose inverse of a matrix

➔ Several efficient algorithms to obtain a pseudo-inverse

- the most reliable one is based on the Singular Value Decomposition method

But the optimal matrix **W** has first to be computed





❑Computation of the optimal weights

➔ "The components of **W** are the diagonal elements of the resolution matrix **R** when the diagonal elements of the symmetric matrix $\mathbf{R_w}$ are equal to one"

$$\mathbf{R} = \mathbf{A}_\mathbf{w}^\mathrm{T}\,\mathbf{H}_\mathbf{w}^{-1}\mathbf{A} \qquad\qquad \mathbf{R_w} = \mathbf{A}_\mathbf{w}^\mathrm{T}\,\mathbf{H}_\mathbf{w}^{-1}\mathbf{A_w}$$

$$w_{jj} = R_{jj} = w_{jj}^{-1}\mathbf{a_j}^\mathrm{T}\mathbf{H}_\mathbf{w}^{-1}\mathbf{a_j} \ \text{ when } \ R_{wjj} = w_{jj}^{-2}\mathbf{a_j}^\mathrm{T}\mathbf{H}_\mathbf{w}^{-1}\mathbf{a_j} = 1$$

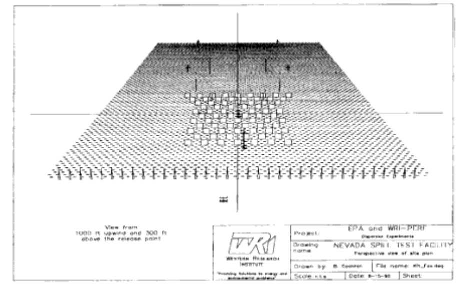
➔ This algorithm converges uniformly to the optimal weights matrix **W**

Algorithm 1. Computing the optimal weighted matrix **W**

Require: Let $\mathbf{A} \in \mathbb{R}^{N\times m}$.

```
N=columns[A]
m=rows[A]
W=m/N*I_N                          (initialization of W)
while d_min <=0.99                 (definition of the convergence criteria)
    H^-1=(AW^-1 A^T)^-1            (computation and inversion of the weighted Gram matrix)
        For j=1 to N
            a_j=A(j)               (writing the j columns of the matrices A as vectors)
            d_j= a_j^T H^-1 a_j*w_jj^-2   (computation of the diagonal elements of R_w,
        End for                     stored in a vector d)
    d_min = min(d)                 (convergence verification)
    W=W*(diag[d])^1/2              (definition of a new weight matrix)
end while
return W
```

❑The Kit Fox series



- ➔ Conducted at the Nevada Test Site (USA) in late August and early September 1995
  - – CO_2 was released as a tracer

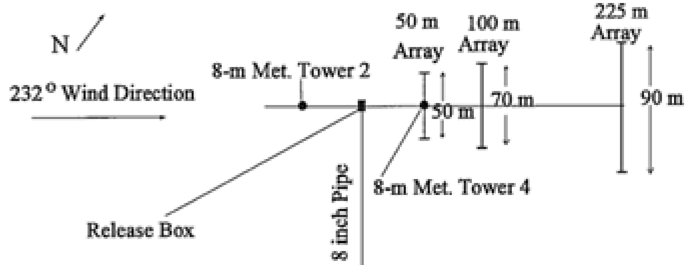
- ➔ 3 different surface roughness configurations:
  - – ERP + URA
  - – URA only
  - – Smooth Desert Surface

- ➔ The smooth desert releases, also referred to as the "DRI /WRI CO2-II" experiments, have been described in a separate report (Coulombe et al., 1999) never published and only mentioned by King et al. (2002)

These late experiments have never been used for evaluation purposes

❑The smooth desert kitfox experiments

➔ Sensors on 3 arrays oriented perpendicular to the centreline of the predicted transport course of the cloud



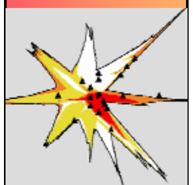
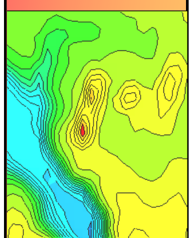
➔ 30 releases under neutral to extremely stable conditions

– 22 short duration releases (1.5 kg/s over 20s)

– 8 continuous releases (1-1.5 kg/s over 150-360s)

Table 1: Characteristics of the continuous release experiments

Test No.	Average wind speed 2m a.g.l. (ms^{-1})	Average wind direction 2m a.g.l. (degree)	Release rate (kgs^{-1})	Release duration mm:ss	Stability Class
9-4	3.5	234	1.527	2:31	D-E
9-7	2.5	229	1.497	3:31	F
9-9	1.9	235	1.438	5:31	F-G
10-5	2.0	232	1.037	5:58	G+
10-6	1.9	198	0.995	5:00	F-G
12-7	1.6	211	1.019	4:59	F-G
13-6	3.0	227	1.114	3:32	E
13-7	2.3	213	1.028	3:00	F

❑Inputs for the renormalization method

- ➔ Components of **A** computed from an analytical Gaussian dispersion model (Sharan et al., 1996) used in a backward mode
  - – use of Briggs' model for dispersion parameters

- ➔ Concentrations from 24 captors of the 50m and 100m arrays averaged to obtain the measured concentrations vector μ (i.e. $m=24$)

- ➔ Technique implemented on a discretized domain of 300×300 points (i.e. $N=90000$) with $\Delta x = \Delta y = 1$m

On a machine Intel® Core™ i5-3427U CPU 1.80GHz, 8Go RAM, the CPU time involved in estimating the components of **A**, **W** and $\mathbf{s}_{//w}$ was approximately <30 seconds

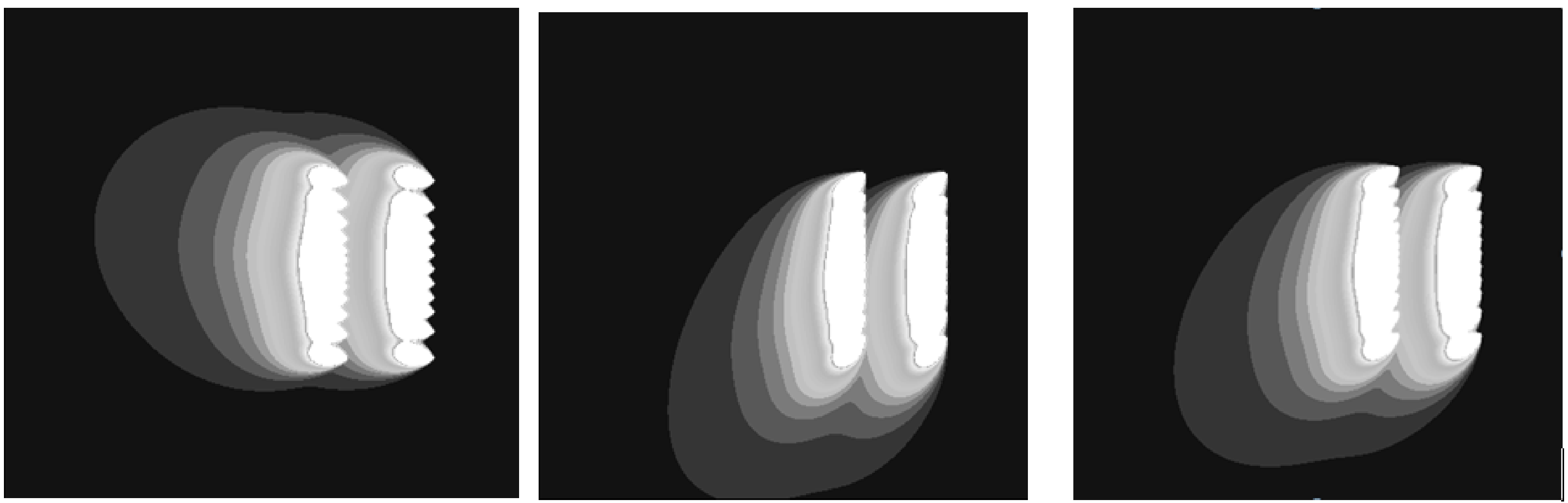
❑Results and discussion

➔ Regions

- well monitored by the network: white
- poorly monitored by the network: black

The source location (middle of the domain) lies in a well monitored region of the network

Visibility of the monitoring network for cases 9-9,12-7 and 13-7

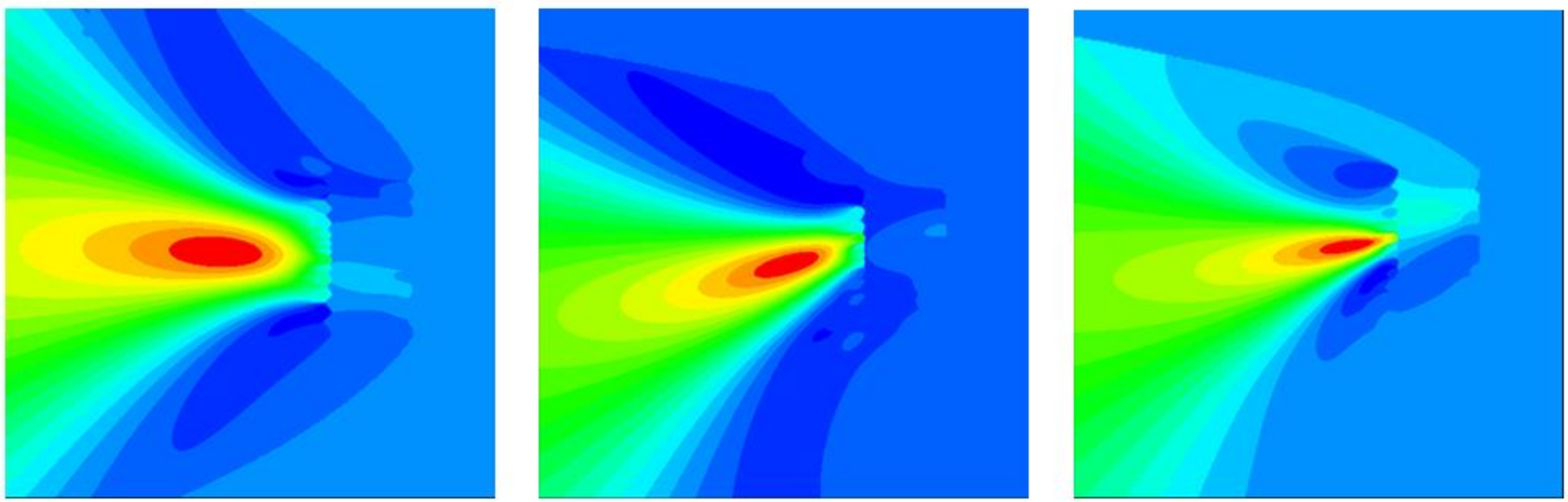


❑Results and discussion

➔ The maxima of $\mathbf{s}_{//w}$ is unique and sharp at position (x_s,y_s)

- lateral direction: $0 \leq \Delta y_s/x_m \leq 0.08$
- longitudinal direction: $0 \leq \Delta x_s/x_m \leq 0.2$
  - x_s is placed upstream of the true position, basically because **A** has been derived from a Gaussian model with constant mean wind speed, direction and empirical dispersion parameters

$\mathbf{s}_{//w}$ for cases 9-9,12-7 and 13-7



❑Conclusions

- The discrete source estimate given by the renormalization technique is

$$\mathbf{s}_{//w} = \mathbf{W}^{-1}\mathbf{A}^{T}\left(\mathbf{A}\mathbf{W}^{-1}\mathbf{A}^{T}\right)^{-1}\boldsymbol{\mu} \quad \text{with} \quad \mathbf{W} = \mathrm{diag}(w_1, w_2, \ldots, w_N)$$

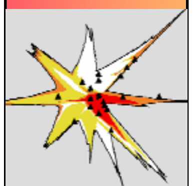
  - corresponds with the minimum **W**-norm solution of the underdetermined linear inverse problem

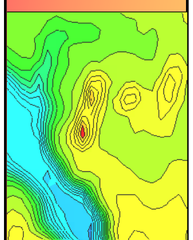
- It can be expressed by making use of the concept of generalized inverse

$$\mathbf{s}_{//w} = \mathbf{A}_{\mathbf{W}}^{+}\boldsymbol{\mu}$$

  - a computationally reliable way to compute the pseudo inverse is by using the Singular Value Decomposition (SVD)
  - but a specific algorithm must be used to compute the optimal weight matrix

- Applied to a new KITFOX data set, the source is observed to be distinctly located and converges onto reasonable estimates
  - new results needed with a more appropriate dispersion model





Thank you for your attention

gregory.turbelin@ufrst.univ-evry.fr

http://lmee.univ-evry.fr/