EVALUATION OF THE MASS EXCHANGE BETWEEN A CAVITY AND THE EXTERNAL FLOW

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INTRODUCTION

The so called operational models of pollutant dispersion in urban areas describe the mass exchange between the recirculating regions within the urban canopy (street canyons) and the atmosphere aloft by means of few parameters, in order to provide simple relations. These are models with one-degree of freedom: the canyon is described as a box with uniform pollutant concentration within it and with a discontinuity surface at the top, where the mass exchange takes place. The velocity and the concentration of the external flow are assumed uniform too. The recirculating flow within each street is assumed to be driven by the external wind. These models require an estimation of the mass exchange velocity $u_d$ between the canyon and the external flow, depending on the turbulence intensity of the external flow. In this study we provide an experimental investigation of the dependence of $u_d$ on three different dynamical conditions of the external flow. A mass exchange model with two degree of freedom is proposed and compared with the experimental results.

PROBLEM SETTING

The velocity mass exchange $u_d$ is related to the turbulent fluctuations at the shear layer taking place at the interface between the cavity and the external flow. The turbulent transport between these two regions is characterized on one side by a momentum ‘diffusion’ induced by the local shear generated turbulence and on the other side by the turbulent kinetic energy fluxes coming from the external flow toward the cavity (Salizzoni, 2006). The dynamics of local generated instabilities depends on the mean velocity difference $\Delta U = U_2 - U_1$ within the shear layer ($U_2$ and $U_1$ are the velocities at the shear layer boundaries) and induces the vertical ‘flapping’ of the shear layer, producing the transport of coherent structures within the cavity: it is worth noting that these two mechanisms are not independent of each other. Moving from these arguments, $u_d$ has to scale with the mean velocity difference across the shear mixing layer at the interface; so, we can write:

$$u_d = \frac{1}{\Delta U} \alpha$$

where $\alpha$ is a parameter. As long as the fluxes of turbulent kinetic energy (from the outside to the inside of the canyon) are relevant in the dynamics of the flow within the shear layer and within the cavity, we have to consider that $\alpha$ depends on $\ell_{ext}$, the integral length scale, and on $i_{ext}$, the turbulence intensity of the external flow:

$$\alpha = f(i_{ext}, \frac{\ell_{ext}}{H}) \quad (1)$$

where $H$ is the canyon height and $i_{ext} = u_* / U_{ext}$ is defined by the ratio of the friction velocity and the mean velocity of the external flow.

Among the operational models, the model OSPM (Berkowicz et al., 1997) assumes that
\[ \alpha = 1/i_{\text{ext}} \]

where \( i_{\text{ext}} \) is taken as a fixed value, equal to 0.1, a representative value of the turbulence intensity in an urban atmospheric flow. Differently, in the model SIRANE, Soulhac (2000) assumes that

\[ \alpha = \pi \sqrt{\frac{W}{\ell_{\text{ext}}}} \]

where \( W \) is the canyon width. In fact, as Salizzoni (2006) showed, in case of a square cavity, the integral length scale within the cavity is not sensitive to the variation of the turbulence length scale in the external flow. The problem is then limited to the definition of the dependence of \( \alpha \) on the turbulence intensity of the external flow only, i.e. \( \alpha = f(i_{\text{ext}}) \).

The existing operational models assume that the mean velocity within the cavity is equal to zero and that the mean velocity in the external flow \( U_{\text{ext}} \) is uniform; in this case, \( \Delta U = U_{\text{ext}} \). Nevertheless, in the case we are interested in, the external flow is not uniform: the external flow is a boundary layer, whose height \( \delta \) is much greater than the canyon size (\( \delta \gg 10H \)). This means that the external velocity \( U_1 \) is not even approximately equal to \( U_\infty = U(\delta) \), the free stream velocity at the top of the boundary layer. Furthermore, the mean horizontal internal velocity \( U_2 \) varies with the stream-wise coordinate. Both factors make rather difficult the definition of an equivalent shear layer. In order to evaluate the vertical extension of the shear layer, we analyzed the Reynolds stress vertical profiles and identifying the boundary between the region where it varies relatively rapidly and the region outside the cavity where it is nearly constant (inertial region). The difference \( \Delta U \) was evaluated on the profile at the cavity centre assuming that, at that position, the mean flow within the cavity could be considered parallel to the mean flow in the external region.

**EXPERIMENTAL FACILITIES**

In order to define a typical velocity exchange between the cavity and the external flow, several approaches can be adopted. Some authors (Caton et al., 2003; Dezso-Weidinger et al., 2003) evaluated the wash-out time of the cavity by measuring the spatially averaged concentration within the cavity as it empties by means of a Particle Tracking Velocimetry technique. Some others (Barlow and Belcher, 2002; Barlow et al., 2004) evaluated the sublimation-time of naphthalene from the canyon walls.

In our case, the wash-out time of the cavity was evaluated using a Flame Ionisation Detector (FID), by measuring the temporal evolution of ethane concentration at different position within the cavity as it empties. The tracer was injected in a two-dimensional square canyon \((H/W=1)\) by means of a linear ground level source, placed at the centre of the canyon (Figure 1); the source strength is referred here to as \( M_q \). For each point of measure, the experiment was repeated 50 times to allow an ‘ensemble’ average of the signals. The experiments were performed for different dynamical conditions of the external flow: three different oncoming wind profiles have been reproduced, with different mean velocity and turbulent intensities characteristics, i.e. different ratios \( u_\ast /U_\infty \) (see Table 1). The velocity measurements were performed by means of a hot-wire anemometer in the external flow and by means of a Particle Image Velocimetry system inside the canyon (Salizzoni, 2006).

**AN ANALYTICAL MODEL WITH TWO DEGREE OF FREEDOM**

In Figure 1 are shown some of the wash-out curves that we measured at different positions within the canyon. It is evident that the curves measured at different positions differ
significantly one from the other and that all curves have a horizontal tangent for \( t \to 0 \). This behaviour cannot be modelled by means of a box model with one degree of freedom only, which would lead to an exponential curve with a negative tangent for \( t \to 0 \).

![Diagram of wash-out curves](image)

\( \text{Fig. 1: Normalized wash-out curves measured at different positions within the cavity as a function of time, } C_0 = C(0). \)

In order to describe the pollutant transfer between the canyon and the external flow, we have therefore adopted a model with two degrees of freedom. As it is represented in Figure 2, the flow in the cavity consists of two regions and the mass transport is described in terms of a sequence of transfers between three regions, each with a different mean concentration. One region represents the external flow, referred to as box 0; the two other boxes give a rough description of the pollutant distribution inside the canyon: the box 2 represents the core of the flow inside the cavity, while the box 1 represents the recirculating part of the flow. Both concentrations \( C_1 \) and \( C_2 \) are assumed to be uniform within the boxes.

![Two degree of freedom model](image)

\( \text{Fig. 2. Two degree of freedom model for a square cavity.} \)

Assuming the scheme represented in Figure 2, we can write a mass balance for the two boxes within the cavity:

\[
\begin{align*}
V_1 \frac{dC_1}{dt} &= S_{10} u_d (C_{\text{ext}} - C_1) + S_{12} \tilde{u}_d (C_2 - C_1) + M_d \\
V_2 \frac{dC_2}{dt} &= S_{12} \tilde{u}_d (C_1 - C_2)
\end{align*}
\]

(2)

where \( V_1 \) and \( V_2 \) are the volumes of box 1 and box 2 respectively, \( u_d \) and \( S_{10} \) are the velocity and the surface exchange between the box 1 and the box 0 and \( \tilde{u}_d \) and \( S_{12} \) are the velocity and the surface between box 1 and box 2.
We set:

\[ V_1 = \beta V_0 \quad \text{and} \quad V_2 = (1 - \beta)V_1 \]

and we define:

\[ \frac{1}{T_1} = \frac{S_0 \mu_d}{\beta V_0} \quad \text{and} \quad \frac{1}{T_2} = \frac{S_0 \bar{u}_d}{(1 - \beta)V_1} \]

In order to evaluate the typical time scale for the mass transfer between the recirculating region and the external flow, we evaluated the temporal evolution of a passive scalar concentration in the cavity as it empties, after having stopped the injection, i.e. \( M_d = 0 \). We set the external concentration equal to zero, \( C_{\text{ext}} = 0 \), and we adopt a change of variables:

\[ C_1 \rightarrow C_1 C_{10} \quad C_2 \rightarrow C_2 C_{20} \]

We impose the initial conditions \( C_{10} = C_1(0) \) and \( C_{20} = C_2(0) \) and we set \( \gamma = C_{20} / C_{10} \). Given these conditions and substituting in equation (2), we obtain the following initial value system:

\[
\begin{align*}
\frac{dC_1}{dt} &= -\frac{C_1}{T_1} + \frac{(1 - \beta)}{\beta T_2} (\gamma C_2 - C_1) \\
\frac{dC_2}{dt} &= \frac{1}{T_2} \left( \frac{1}{\gamma} C_1 - C_2 \right) \\
C_1(0) &= 1 \\
C_2(0) &= 1
\end{align*}
\]

As long as the initial conditions of the process corresponds to a steady state, we have to assume that \( \gamma = 1 \); to evaluate \( \beta \) we can represent the core region (box 2) as a circle placed in the cavity centre with a radius \( \Re = RH \), where \( R \) is set equal to 0.31 (Salizzoni, 2006).

**DISCUSSION OF THE RESULTS**

The experimental wash-out curves have been fitted with the analytical solution of the initial value problem given by the equations (3) and the mass exchange velocities \( (u_d \text{ and } \bar{u}_d) \) have been estimated.

![Fig. 3: Comparison between analytical model (dashed line) and experimental wash-out curves (solid line) in two different positions within the cavity without (see text for details).](image-url)
The experimental curves (solid line) and the model results (dashed line) are shown in Figure 3 for one of the three cases considered: in Figure 3a and 3b are shown the experimental curves and the model curves, in Figure 3c and 3d the measured curves have been translated on the time axis of a time interval $\Delta t$. Actually the model predicts well the time evolution of the wash-out curve, except for this initial delay. This may be due to the fact that the system needs an initial time interval to reach the initial conditions that the model implicitly takes into account. For example, the condition $\gamma = 1$ is related to the assumption that there is no direct exchange between the outer region and the core of the cavity (box2), which may be not true: the core of recirculation region within the cavity may be directly perturbed by the ‘flapping’ of the shear layer and a mass transfer may take place between the core of the cavity (box 2) and the external flow box (0).

Table 1. Variation of the wash-out times and of the exchange velocities in function of the turbulence intensity $u_*/U_\infty$ of the external flow.

<table>
<thead>
<tr>
<th>Case</th>
<th>$u_*/U_\infty$</th>
<th>$\Delta U$ (m/s)</th>
<th>$T_1$ (s)</th>
<th>$T_2$ (s)</th>
<th>$u_d / U_\infty$</th>
<th>$\bar{u}<em>d / U</em>\infty$</th>
<th>$u_d / \Delta U$</th>
<th>$u_d / \Delta U$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.049</td>
<td>1.38</td>
<td>0.5</td>
<td>0.18</td>
<td>0.01</td>
<td>0.009</td>
<td>0.051</td>
<td>0.044</td>
<td>19.9</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.053</td>
<td>1.1</td>
<td>0.58</td>
<td>0.18</td>
<td>0.01</td>
<td>0.009</td>
<td>0.063</td>
<td>0.055</td>
<td>15.8</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.061</td>
<td>1.1</td>
<td>0.41</td>
<td>0.15</td>
<td>0.013</td>
<td>0.011</td>
<td>0.08</td>
<td>0.06</td>
<td>12.2</td>
</tr>
</tbody>
</table>

In Table 1 we show the dependence of $\alpha$ on $u_*/U_\infty$, which can be considered a representative normalized parameter of the external turbulence, as long as $u_*$ is the only relevant scale of the external flow field. As we can see, $\alpha$ depends on the external turbulence level: by increasing the external turbulence level the wash-out times are reduced.

CONCLUSIONS

By means of a two-degrees of freedom model we could simulate the time dependence of the wash-out curves of the cavity. Despite an initial time delay, the model agrees well with the experimental results and allows to compute the mass exchange velocities between cavity and external flow. The results enlighten the dependence of the velocity exchange on the dynamical conditions of the external flow.

REFERENCES


