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HORIZONTAL TURBULENCE AND DISPERSION IN LOW-WIND STABLE CONDITIONS

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Abstract: Describing turbulence and dispersion under low-wind conditions (roughly < 2 m s^{-1} at 10 m above ground level) is difficult as traditional assumptions fail or become no longer applicable and processes traditionally neglected or overlooked become important. When the winds are weak, the scalar average wind speed and the vector average wind speed need to be clearly distinguished and the variances of both lateral and longitudinal wind velocity fluctuations need to be considered in dispersion calculations. We examine commonly-used methods of estimating these variances from wind-speed and wind-direction statistics measured separately, for example, by a cup anemometer and a wind vane, and evaluate the implied relationship between the scalar and vector wind speeds, using field measurements collected under low-wind stable conditions. Several inconsistencies inherent in the existing methods are highlighted, and improved relations for the two wind variances are derived. It is observed that the commonly-used assumption that the two wind variances are equal is not necessarily valid. Most existing expressions for the lateral wind variance work well, but that is not the case with regards to the longitudinal wind variance. Although, as far as diffusion is concerned, the correct estimation of the latter is only important when the winds are low, it is important to get both wind variances right in turbulence characterisation. The analysis presented here is general and not just restricted to low-wind stable conditions that dominate the dataset used for validation. An analytical dispersion model is evaluated with different formulations of the lateral and longitudinal wind variances, and it is found that the new relations for the two variances lead to better concentration predictions.

Key words: *Weak winds, stable stratification, turbulence intensity, low wind diffusion.*

INTRODUCTION

The variances of longitudinal and lateral wind fluctuations (σ_u^2 and σ_v^2) provide important information about the nature of turbulent flow in the atmospheric boundary layer, and form essential inputs to models of atmospheric dispersion. The assumption in models, such as the Gaussian plume model, that the influence of σ_u^2 on dispersion is negligible compared to mean advection becomes questionable as the wind speed becomes low (approximately < 2 m s^{-1}). We examine the problem of parameterising σ_u and σ_v from the statistics of wind speed and wind direction obtained from routine measurements. These statistics are: the scalar average wind speed ($\overline{U}$), the scalar average wind direction ($\overline{\theta}$), the standard deviation of wind speed (σ_U) and the standard deviation of horizontal wind direction fluctuations (σ_θ), which are all determined directly from the instantaneous wind speed (U) and the instantaneous wind direction (θ) using 'single-pass' methods (Mori 1986, EPA 2000). An analytical dispersion model is evaluated with different formulations of σ_u and σ_v.

EXISTING METHODS

A simple formula to determine σ_v from wind data is (e.g., Hanna 1983):

$$\sigma_v = \overline{U}\tan\sigma_\theta . \tag{1}$$

Under low-wind stable conditions, because of the low-frequency meandering of the flow, σ_θ can be as large as 90° (Sagendorf and Dickson 1974). Under such conditions, Eq. (1) becomes inapplicable. For small σ_θ, Eq. (1) gives

$$\sigma_v \approx \overline{U}\sigma_\theta , \tag{2}$$

which is a very frequently used relationship. It is also common to assume that the turbulence is isotropic in the horizontal so that $\sigma_u \approx \sigma_v$. Although $\overline{U}$ is used in the above expressions as this is what is available from routine measurements, in reality researchers do not normally make it clear whether the use of $\overline{U}$ or that of the vector average wind speed ($\overline{u}$) is physically more realistic in these expressions. When σ_θ is small (e.g. under strong winds), it does not matter whether $\overline{u}$ or $\overline{U}$ is used because they are almost equal. However, under low wind conditions that is not necessarily true. It is important to point out that dispersion models also require average transport wind speed as a separate input, for which $\overline{u}$ should be used. This quantity can be obtained from $\overline{U}$ via Eq. (6) given below. Hereafter in this paper, we make a clear distinction between $\overline{U}$ and $\overline{u}$.

Some researchers have attempted to derive better expressions for σ_v and σ_u that are also applicable under low wind conditions. Assuming that $\sigma_u \approx \sigma_v$, van den Hurk and de Bruin (1995) derived

$$\sigma_v^2 \approx \sigma_u^2 \approx \left[\sigma_U^2 - \overline{U}^2\{\exp(-\sigma_\theta^2) - 1\}\right]/2 . \tag{3}$$

An apparent physical inconsistency in Eq. (3) is that σ_v is non-zero even when $\sigma_\theta = 0$, but as will be seen later this turns out to be less significant that the problems caused by assuming $\sigma_u \approx \sigma_v$.

Cirillo and Poli (1992) assumed that U and θ are statistically independent, and by considering that θ is normally distributed, with standard deviation σ_θ, and that U is constant so that $P(U) = \delta(U - \overline{U})$, where δ is the Dirac delta function, they obtained the following expressions:

$$\sigma_v^2 = \overline{u}^2 \sinh(\sigma_\theta^2) , \quad (4)$$

$$\sigma_u^2 = \overline{u}^2 [\cosh(\sigma_\theta^2) - 1] . \quad (5)$$

For small σ_θ, $\sigma_v^2 \approx \overline{u}^2 \sigma_\theta^2$ and $\sigma_u^2 \approx \overline{u}^2 \sigma_\theta^4 / 2$. A limitation in the above formulation is that $\sigma_u = 0$ when $\sigma_\theta = 0$, which is not necessarily the case. The above formulation results in the following relationship between $\overline{u}$ and $\overline{U}$:

$$\overline{u} = \overline{U} \exp(-\sigma_\theta^2 / 2) , \quad (6)$$

which is also reported by Mori (1986), van den Hurk and de Bruin (1995) and others and gives the desired property $\overline{u} = \overline{U}$ when $\sigma_\theta = 0$. Eq. (6) can be substituted in Eqs. (4) and (5) to yield

$$\sigma_v^2 = \overline{U}^2 \exp(-\sigma_\theta^2) \sinh(\sigma_\theta^2) , \quad (7)$$

$$\sigma_u^2 = \overline{U}^2 \exp(-\sigma_\theta^2) [\cosh(\sigma_\theta^2) - 1] . \quad (8)$$

The formulae (4) and (5) have also been used by Sharan and Yadav (1998) and others for modelling diffusion under low-wind stable conditions, although it appears that these authors used the available $\overline{U}$ data instead of the required $\overline{u}$ data in these formulae (in other words, formulae (7) and (8) should have been used).

We assess the relationships (2), (3), (7) and (8) using field data obtained under stable stratification.

MEASUREMENTS USED

We use the meteorological and dispersion data from a field experiment conducted under low-wind stable conditions at the Idaho National Engineering Laboratory (INEL), now the Idaho National Laboratory, in south-eastern Idaho (USA) in 1974 (Sagendorf and Dickson, 1974). This remains one of the most commonly used datasets for evaluating dispersion models for applications involving low wind speeds and strong stability (e.g., Anfossi *et al.* 2006). The site was located in a broad, relatively flat plain, and the area was semi desert with dry climatic conditions. Wind measurements were taken by lightweight cup anemometers and bivanes located on a tower at six heights: 2, 4, 8, 16, 32 and 61 m above ground level (AGL). SF_6 was released at 1.5 m AGL (effective height 3 m AGL) and the resulting concentrations were measured by ground-level samplers located on three circular arcs laid out at radii of 100, 200 and 400 m from the source. The samplers were placed at intervals of 6° on each arc for a total of 180 sampling positions. Some elevated samples were also taken.

A total of eleven separate experiments were carried out: ten involving stable conditions and one with near-neutral stability. Each experiment lasted for one hour, except for one that lasted for 50 min. Sagendorf and Dickson (1974) only report hourly averages of scalar wind speed, scalar wind direction and the standard deviation of wind direction (σ_θ). This is not sufficient information for our analysis, and, therefore, we obtained raw measurements of horizontal wind speed and direction originally sourced from Dr. Sagendorf (with one missing stable experimental hour (Test#5)). The sampling frequency in six experiments was once every 3 s and that in the remaining four was once every 2 s. We calculated hourly averaged values of $\overline{U}$, $\overline{u}$, σ_U, σ_θ, σ_v and σ_u. Data from all six heights were used, so the total number of hourly averaged data points was 60.

EVALUATION

In Figure a, the expected trend of σ_θ decreasing with $\overline{u}$ can be seen, with σ_θ as high as 93° and as low as 2° being observed. In Figure b, although the magnitudes of the observed σ_v and σ_u are similar, there is little correlation ($r^2 = 0.14$) between the two, suggesting that the assumption of $\sigma_u = \sigma_v$ mentioned earlier is not satisfactory. This assumption becomes important in low wind conditions where σ_u cannot be neglected compared to the mean advection and needs to be explicitly included in dispersion calculations.

Some authors (e.g. Leung and Liu 1996) have derived empirical expressions for σ_θ in terms of the so called persistence $P_r (= \overline{u} / \overline{U} \leq 1)$. These can be contrasted with the following expression obtained from Eq. (6):

$$\sigma_\theta = \sqrt{2 \ln(1/P_r)} . \quad (9)$$

The relationship between σ_θ and P_r given by Eq. (9) is compared with the data in Figure c. Except for the three outliers, Eq. (9) agrees very well with the measured behaviour, which means that the assumption implicit in relations (6) and (9) that wind direction is normally distributed and is independent of wind speed during the averaging period of interest (i.e. 1 hour) is mostly satisfactory.

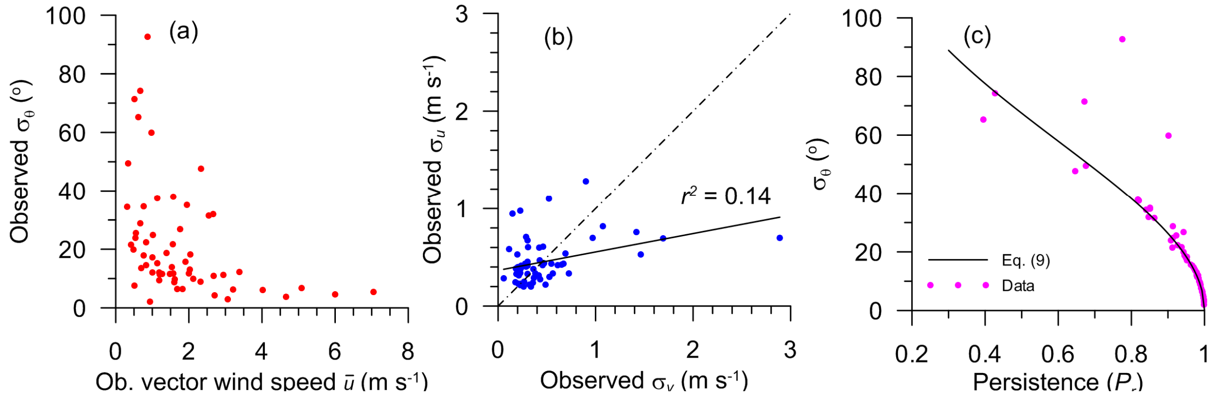

Figure 1. (a) Variation of the observed σ_θ with the observed vector wind speed $\bar{u}$, (b) comparison of the observed σ_u with the observed σ_v , and (c) variation of σ_θ with persistence (P_r) according to Eq. (9) and as obtained from the data.

The σ_v values calculated by the often-used Eq. (2) compare well with the data in Figure a. However, the comparison is not as good when the same formula is used to represent σ_u (Figure b) ($r^2 = 0.24$ as opposed to 0.92) although the estimated magnitudes are comparable to the data. In Figure d, the performance of the σ_u formulation (3) by van den Hurk and de Bruin (1995), which requires additional information about σ_U , is significantly better than that of Eq. (2) shown in Figure b. But Figure c, together with Figure a, demonstrates that this formulation is not as good as the simple Eq. (2) when it comes to estimating σ_v . The reason for this is discussed later.

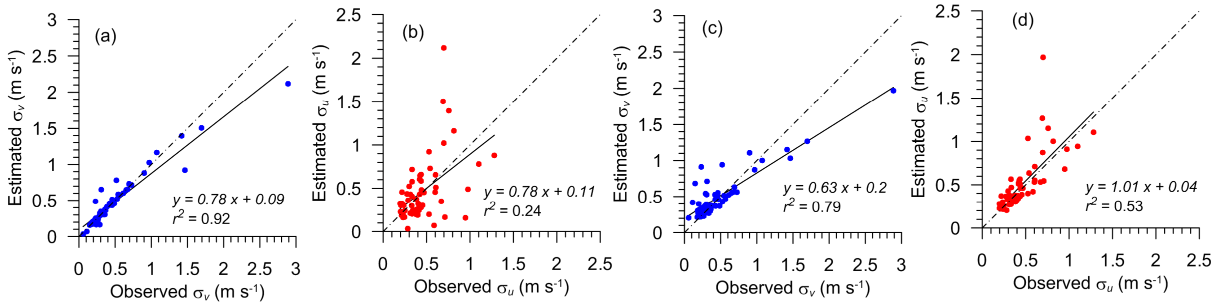

Figure 2. Comparison of the observed (a) σ_v and (b) σ_u , with that estimated from Eq. (2) assuming that $\sigma_u = \sigma_v$, (c) σ_v and (d) σ_u , with that estimated from Eq. (3) assuming that $\sigma_u = \sigma_v$. The solid lines are the linear best-fits.

Figure a shows that Eq. (7) by Cirillo and Poli (1992) for σ_v describes the data well, and its performance is very similar to that by Eq. (2) (Figure a). However, their formulation (8) for σ_u performs very poorly (Figure b). Weber (1998) also notes that this particular formulation does not satisfactorily describe the ratio σ_v / σ_u obtained from measurements. The results above indicate that Eq. (7), or even the simple Eq. (2), is able to describe the σ_v data satisfactorily, but as far as σ_u is concerned, Eq. (3) by van den Hurk and de Bruin (1995) is the best of the three.

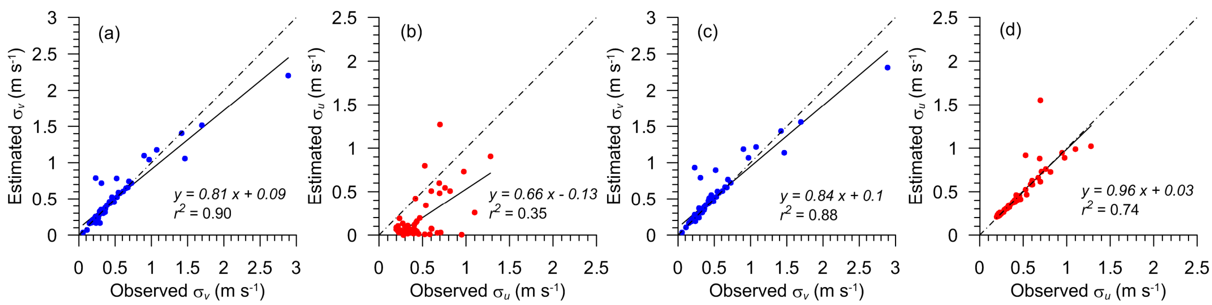

Figure 3. Comparison of the observed (a) σ_v with that estimated from Eq. (7), (b) σ_u with that estimated from Eq. (8), (c) σ_v with that estimated from Eq. (10), and (d) σ_u with that estimated from Eq. (11). The solid lines are the linear best-fits.

IMPROVED EXPRESSIONS FOR THE VARIANCES

The reason why the σ_u formula (5) by Cirillo and Poli (1992) does not do well is its assumption that the probability density function of *U* can be represented by the Dirac delta function. This assumption leads to only the fluctuations in wind direction contributing to σ_u with the fluctuations in wind speed (σ_U) playing no part, whereas in reality the latter may actually be the dominant component of the two when it comes to the composition of σ_u. The σ_u formula (3) by van den Hurk and de Bruin (1995) explicitly accounts for σ_U , and, therefore, its performance is much better. However, van den Hurk and de Bruin's assumption that $\sigma_u = \sigma_v$ leads to inferior estimates of σ_v compared to those obtained by Eq. (4) of Cirillo and Poli. Hence, although one can use Eq. (4) or Eq. (2) for σ_v and Eq. (3) for σ_u, in the following we derive a more self-consistent formulation for σ_u and σ_v. It is not necessary to assume a particular form of *P*(*U*), such as the Dirac delta function used by Cirillo and Poli, because the integrals $\int_0^\infty U\,P(U)\,dU$ and $\int_0^\infty U^2\,P(U)\,dU$ involved in the derivation are simply equal to $\overline{U}$ and $\overline{U}^2 + \sigma_U^2$, respectively. The final expressions in terms of $\overline{U}$ are:

$$\sigma_v^2 = \overline{U}^2 \exp(-\sigma_\theta^2) \sinh(\sigma_\theta^2)[1 + (\sigma_U / \overline{U})^2] \; , \tag{10}$$

$$\sigma_u^2 = \overline{U}^2 \exp(-\sigma_\theta^2)[\cosh(\sigma_\theta^2)\{1 + (\sigma_U / \overline{U})^2\} - 1] \, . \tag{11}$$

So for small σ_θ , $\sigma_v^2 \approx \overline{U}^2 \sigma_\theta^2 [1 + (\sigma_U / \overline{U})^2]$ and $\sigma_u^2 \approx \sigma_U^2 (1 - \sigma_\theta^2)$. These expressions in terns of $\overline{u}$ are:

$$\sigma_v^2 = \overline{u}^2 \sinh(\sigma_\theta^2)[1 + (\sigma_U / \overline{u})^2 \exp(-\sigma_\theta^2)] \; , \tag{12}$$

$$\sigma_u^2 = \overline{u}^2 [\cosh(\sigma_\theta^2)\{1 + (\sigma_U / \overline{u})^2 \exp(-\sigma_\theta^2)\} - 1] \; , \tag{13}$$

which for small σ_θ reduce to, $\sigma_v^2 \approx \overline{u}^2 \sigma_\theta^2 [1 + (\sigma_U / \overline{u})^2]$ and $\sigma_u^2 \approx \sigma_U^2 (1 - \sigma_\theta^2)$. This indicates that the principal contribution to σ_v is from σ_θ while that to σ_u is from σ_U .

Figure c and Figure d compare the observed σ_v and σ_u with those determined using Eqs. (10) and (11), respectively. While the performance of Eq. (10) for σ_v is very similar to that by Eq. (7) and even to the simple Eq. (2), albeit with a slightly better slope of the best-fit line and a slightly worse correlation, the formula (11) gives the best estimates of σ_u out of all the formulae considered in this paper. A small number of substantial deviations of the estimated values from the data are mostly probably due to the implicit assumption in the above analysis, that wind direction is normally distributed and is statistically independent of wind speed, not holding valid.

DISPERSION CALCULATIONS

Several analytical approaches have been suggested by various researchers (e.g., Cirillo and Poli, 1992; Sharan and Yadav, 1998; Thomson and Manning, 2001) for application under low wind conditions. The approach by Thomson and Manning, which is based on the Gaussian puff principle, is particularly attractive because it is consistent with both small time and large time behaviours of the puff spread; however, it has not previously been tested using any experimental data. We apply this model to simulate the SF_6 concentrations from the INEL field experiment described above, in order to examine how some of the different methods of calculating σ_v and σ_u affect dispersion. Figure presents the model results in terms of quantile-quantile (q-q) plots involving the variation of the sorted predicted concentrations with the sorted observed concentrations (both scaled by wind speed and emission rate).

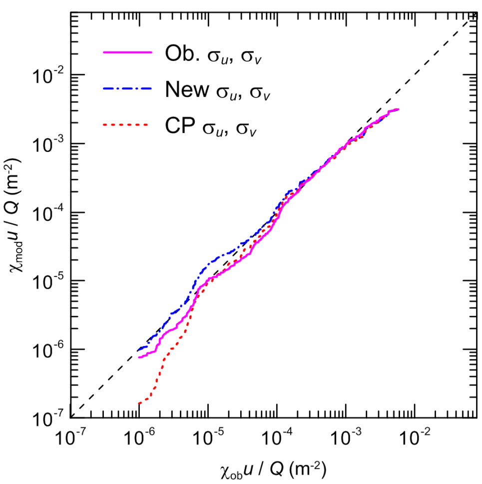

Figure 4. Quantile-quantile (q-q) plots of the observed concentrations vs. the concentrations predicted using various methods of calculating σ_u and σ_v.

It can be seen that the model performs reasonably well throughout the concentration range when it uses the observed σ_v and σ_u, and also the new parameterisations (10) and (11). The new parameterisations give slightly better performance at the lower-end of the concentration distribution than the σ_v and σ_u data, and this demonstrates that there is some uncertainty in the model with regards to its formulations and/or other input parameters. When the Cirillo and Poli (CP) σ_v and σ_u are used, the model considerably underestimates the lower-end concentration distribution, indicating that their formulation (5) for σ_u, which does not describe the data well, mostly affects the low concentrations.

CONCLUSIONS

Using measurements taken under low-wind stable conditions, we examined existing techniques of calculating the variances of longitudinal and lateral wind fluctuations (σ_u^2 and σ_v^2, respectively) from routine wind measurements. Some inconsistencies inherent in these techniques considered were highlighted, and it was observed that the commonly used assumption of $\sigma_v = \sigma_u$ is not necessarily valid. The paper makes it clear that the leading order term in determining σ_v is the standard deviation of horizontal wind direction fluctuations (σ_θ), whereas that in determining σ_u is the standard deviation of horizontal wind speed (σ_U). Most existing expressions for σ_v work well, but that is not the case with regards to σ_u. Although, as far as diffusion is concerned, the correct estimation of σ_u is only important when the winds are low (in other cases, the effects of σ_u are simply ignored compared to the mean advection), it is important to get both σ_v and σ_u right in turbulence characterisation, e.g. in calculating the turbulent kinetic energy. A more consistent set of formulae for σ_v and σ_u was derived, which provides better estimates, especially of the latter quantity. The vector average wind speed $\overline{u}$ which should be used as the average transport wind speed in dispersion models can be obtained from the scalar average wind speed $\overline{U}$ using Eq. (6). The paper also demonstrates that it is useful to report both measured σ_θ and σ_U, in addition to the measured $\overline{U}$ and wind direction ($\overline{\theta}$), for their potential application in the calculation of σ_u and σ_v. The analysis presented here is general and not just restricted to low-wind stable conditions. For example, it can also be applied to unstable conditions that involve large-scale meandering motions in the horizontal due to convection. These motions can potentially cause significant differences between $\overline{U}$ and $\overline{u}$ under low wind conditions.

The performance of different formulations of σ_v and σ_u was tested within an existing analytical dispersion model using the INEL SF_6 concentration data. It was found that the new formulations of σ_v and σ_u lead to a better simulation of the observed concentration distribution, particularly the low values.

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