A NON-DIMENSIONAL URBAN DISPERSION MODEL AND SPURIOUS SELF-CORRELATION ESTIMATE

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Abstract: A Gaussian plume urban dispersion model is presented, where the horizontal and vertical diffusion coefficients are determined according to the theories of G.I. Taylor (1921) and Hunt, J.C.R. and A.H. Weber (1979) respectively. The model is validated with dispersion measurements from field experiments conducted in Oklahoma City, Salt Lake City, London, and St. Louis, which are presented in non-dimensional form. A statistical analysis of robustness is conducted on the non-dimensional data in order to determine the spurious self-correlation of the relationship between the non-dimensional variables.

Key words: *Urban dispersion, Gaussian plume model, experiments, atmospheric stability.*

INTRODUCTION

Risk assessment and health impact of toxic materials released in the atmosphere are based to a large extent on the hazard criteria of the contaminants, and on the mean concentration field at ground level. Several simple analytical models are available to predict mean dispersion in urban areas (e.g., Hanna *et al.*, 2003; Venkatram *et al.*, 2005; Hanna *et al.*, 2007). However, the extent of stratification in urban areas, and its effects on dispersion are not well understood. The mechanical generation of turbulence and the release of thermal energy accumulated during the day contribute to weakening the stability of the flow. We present a Gaussian plume dispersion model, where the horizontal diffusion coefficient is determined by the theory of G.I. Taylor (1921), and the vertical diffusion coefficient by the theory of Hunt, J.C.R. and A.H. Weber (1979). The model is compared to daytime and nighttime data collected in four urban experiments, namely at Oklahoma City (codename JU2003), Salt Lake City (codename URBAN 2000), Saint Louis, and London (codename DAPPLE). The model and data are non-dimensionalized using the atmospheric boundary layer turbulence length and time scales. The data collapse well, suggesting that the scaling variables are appropriate. However, since the non-dimensional variables contain a common variable, the issue of spurious self-correlation needs to be addressed. We performed two statistical tests which indicate a negligible amount of spurious correlation between the non-dimensional variables. The model agrees well with both the nighttime and daytime experiments, and predicts the existence of different trends between near and far field. The results indicate that stratification in urban areas is weak, but its effects on dispersion are not negligible.

URBAN DISPERSION MODEL

We use a reflected Gaussian plume model:

$$c = \frac{Q}{\pi U \sigma_y \sigma_z} \exp\left(-\frac{y^2}{2\sigma_y^2} - \frac{z^2}{2\sigma_z^2} \right) \quad (1)$$

where Q is the mass emission rate, U is the mean wind speed at building height, and σ_y and σ_z are the spread coefficients. The horizontal coefficient σ_y is determined according to the theory of G.I. Taylor (1921) assuming homogeneous turbulence in the horizontal direction and exponential velocity decorrelation with time scale T_y; the vertical coefficient σ_z is derived according to the theory of Hunt, J.C.R. and A.H. Weber (1979) assuming ground level releases in neutral atmosphere. More details and the analytical expressions of σ_y and σ_z are reported in Franzese, P. and P. Huq (2010).

We focus here on the non-dimensional form of the model, which provides insight on the variables driving dispersion in urban areas. The model is tested with four urban experiments conducted in Oklahoma City (Dugway Proving Ground, 2005), Saint Louis (McElroy, J.L. and F. Pooler, 1968), Salt Lake City (Allwine *et al*., 2002), and London (DAPPLE, 2002). Following Davidson *et al*. (1995), and Davidson *et al*. (1996), the concentration is scaled as CUL_yL_z/Q, where L_y and L_z are the horizontal and vertical atmospheric boundary layer turbulence length scales, respectively. We used L_y = 2000 m and L_z = 800 m for the daytime experiments, and L_y = 1000 m and L_z = 200 m for the nighttime experiments. A time scale is defined as $T = (T_yT_z)^{1/2}$, with the turbulent time scales T_y and T_z calculated as $T_y = L_y / \sigma_v$ and $T_z = L_z / \sigma_w$, where σ_v and σ_w are the horizontal and vertical turbulent velocity standard deviations. Then, the distance from the source x is non-dimensionalized as $x/(UT)$. Figure 1 shows the non-dimensional data along with the Gaussian model prediction.

The theory is in good agreement with the data. Here we are interested in assessing the correlation between the variables. Specifically, since the non-dimensional groups CUL_yL_z/Q and $x/(UT)$ both include the variable U, the apparent correlation may be partially due to the scaling variables used to non-dimensionalize C and x. In the following section, we assess the magnitude of the spurious contributions introduced in the correlation by the common variable U.

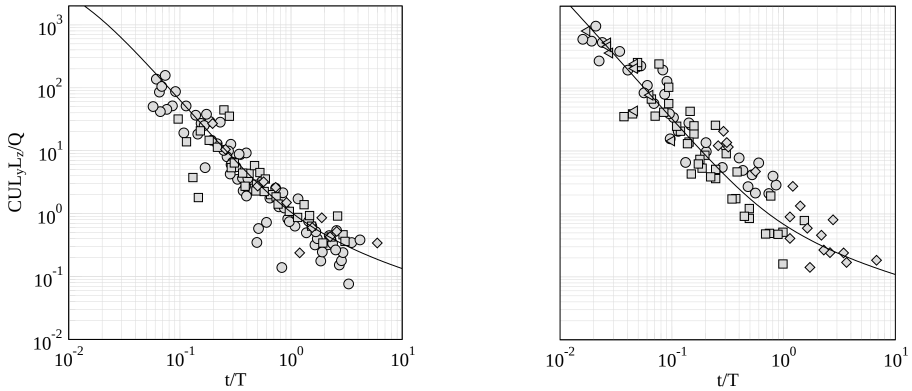

Figure 1. Observed non-dimensional concentration data from four experiments as functions of the non-dimensional distance from the source, along with the theoretical prediction, Eq. (1). Left panel, nighttime data; right panel, daytime data. Circles, Salt Lake City; squares, Oklahoma City; diamonds, St. Louis; triangles, London; solid line, theory.

SPURIOUS SELF-CORRELATION ESTIMATION

An assessment of the spurious component of the correlation can be conducted by performing statistical tests on the variables $Y = \log(CL_yL_z/Q)$, $X = \log(x/T)$, and $Z = \log(U)$. The logarithms of the variables are used in order to linearize the problem.

Table 1 reports the correlation coefficient r for the relations Y vs Z, Y vs X, and Z vs X, for the daytime and nighttime data. The results show very clearly that the variable Z is very weakly correlated with both X and Y. The correlation coefficient of Y vs X, which are the physical variables we are interested in, is about $r = -0.9$ for both the nighttime and the daytime data. By contrast, the correlation coefficient of all the relations involving Z is never larger than 0.5 for the nighttime cases, and is negligible for the daytime cases.

Table 1. Correlation coefficient r for the relations Z vs Y, Z vs X, and Y vs X, for the daytime and nighttime data.

	Nighttime	Daytime
Z vs Y	-0.3215	-0.0954
Z vs X	0.5047	0.0445
Y vs X	-0.9163	-0.9500

The second test is the evaluation of the coefficients of the multiple regression $Y = a + bX + cZ$. Table 2 reports the results of the test, including the regression coefficients b and c, their standard errors, the t-test statistics of the null hypothesis $b = 0$ and $c = 0$, respectively (which indicate unrelated variables), the probability p of obtaining a larger t assuming $b = 0$ and $c = 0$, and the 95% confidence intervals for the daytime and nighttime cases.

The absolute value of the regression coefficient b is much larger than c, with a relatively small standard error. The absolute values of t are also very high, and the confidence interval quite contained, indicating rejection of the null hypothesis $b = 0$ in all cases. By contrast, c is always quite small in absolute terms, and in addition is positive for the nighttime cases and negative for the daytime cases. The t-tests provide small t, and thus do not give unambiguous results, suggesting that c could be even closer to zero for larger datasets.

The combined results of the two tests strongly indicate an almost complete absence of spurious correlation introduced by the common variable U.

Table 2. Statistical quantities characterizing the coefficients of the multiple regression $Y = a + bX + cZ$, including the values of b and c, their standard errors, t-test results, probability p of obtaining a larger t under the null hypothesis, and the 95% confidence intervals.

		Regr. Coeff.	Stand. Error	t	$P > \|t\|$	[95% Conf. Interval]	
Nighttime	b	-1.460383	0.0562867	-25.95	0.000	-1.571846	-1.34892
	c	0.577883	0.1191898	4.85	0.000	0.341855	0.81391
Daytime	b	-1.601748	0.0523202	-30.61	0.000	-1.705563	-1.49793
	c	-0.440304	0.2559577	-1.72	0.089	-0.948179	0.06757

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