IRSN INSTITUT DE RADIOPROTECTION ET DE SÛRETÉ NUCLÉAIRE

Model reduction via principal component truncation for the optimal design of atmospheric monitoring networks

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1. Context
2. Methodology
   - Construction of an optimal network
   - Truncation of the cost function
   - Validation of the reduction method
3. Applications
   - Taking into account foreign sources
   - Taking into account population density
   - Sequential deployment
4. Conclusion

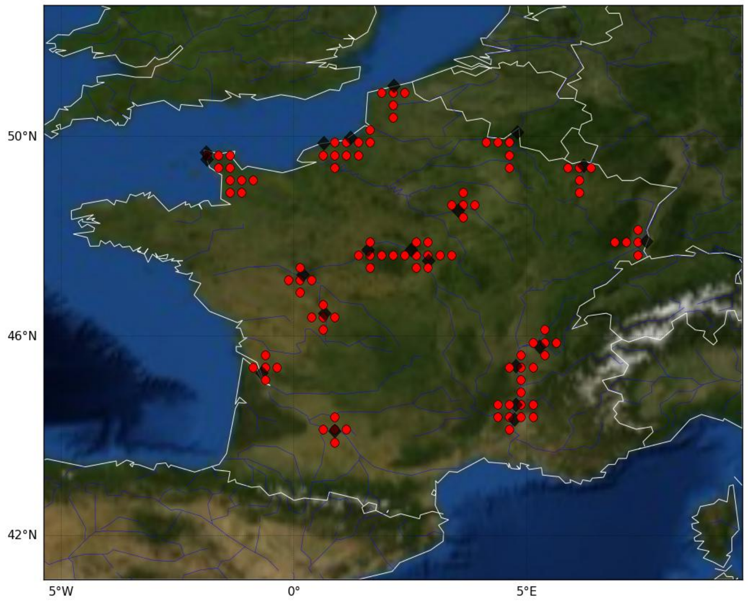
IRSN is planning the deployment of the renovated French nuclear monitoring network

Objectives of the network

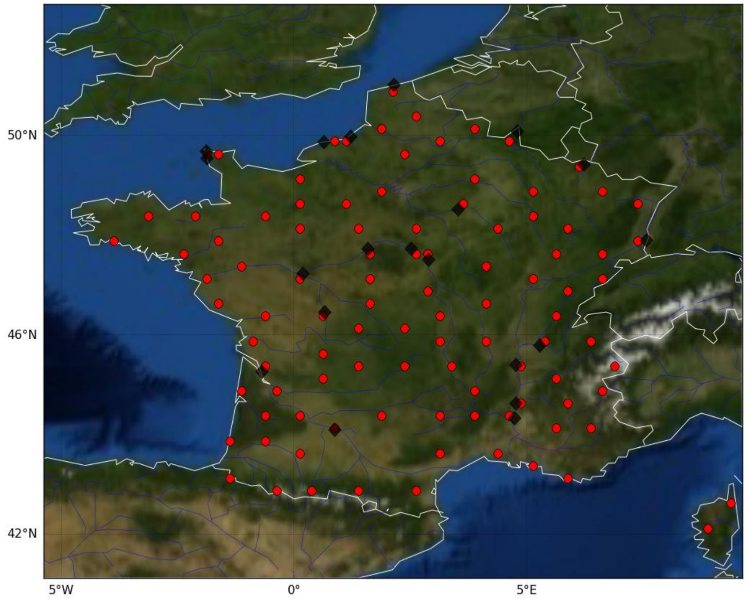
- To detect quickly an accidental release
- To monitor 20 nuclear French power plants
- To reconstruct accurately the accidental plume by using measurements from the network

development of an optimisation method

- location of the stations
- number of stations



Concentrated network



Uniform network

Construct a database of accidents

For each nuclear power plant, for each day of a reference year (2004), for iodine 131 and for a core melt-down accident: **8360 simulations** of 168 hours.

- Mesoscale atmospheric model MM5 [Anthes and Warner, 1978], forced by the NCEP analysis is used to compute meteorological fields at 0.25 x 0.25 resolution
- Eulerian model POLAIR3D [Quelo et al., 2007] is used to simulate iodine 131 concentrations

Chemistry-transport equation:

$$\frac{\partial c_i}{\partial t} + \mathrm{div}(\mathbf{u} c_i) = \mathrm{div}\left(\rho \mathbf{K} \nabla \left(\frac{c_i}{\rho}\right)\right) - \Lambda_i^{\mathrm{s}} c_i - \Lambda_i^{\mathrm{d}} c_i + S_i$$

Ground boundary conditions:

$$-K_z \nabla c_i \cdot \mathbf{n} = -v_i^{\mathrm{dep}} c_i \,.$$

Deposition velocity:

$$v^{\mathrm{dep}} = 0.5\,\mathrm{cm\,s^{-1}}\,.$$

Wet scavenging (Belot):

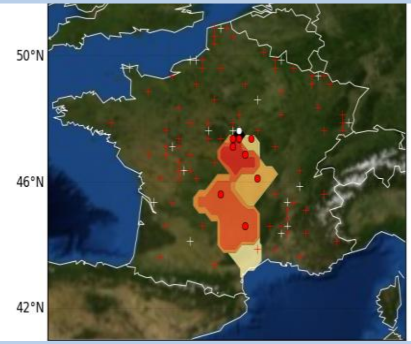
$$\Lambda_i^{\mathrm{s}} = a p_0^b\,, \qquad a = 8 \times 10^{-5}\,, \qquad b = 0.8\,.$$

Construction of an optimal network

1- Initial random network + accidental database

2- Reconstruction of concentrations from the network observations

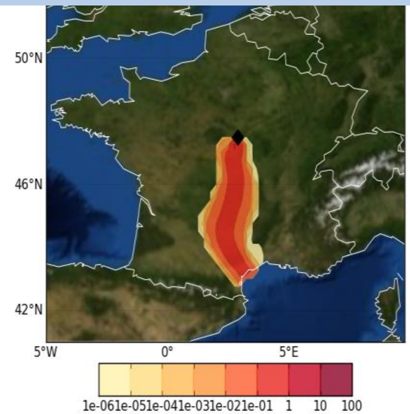
Choice of spatial interpolation method: ***closest point*** *approximation (but kriging is possible)*

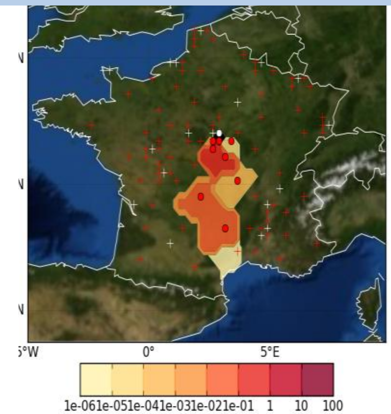


3- Comparison between interpolated and simulated fields

Evaluating the performance of given network: *cost function*

$$J_\alpha = \left(\frac{1}{N} \sum_{i=1}^{q} \sum_{k=1}^{n} \left| \overline{c_k^i} - c_k^i \right|^\alpha \right)^{\frac{1}{\alpha}}$$



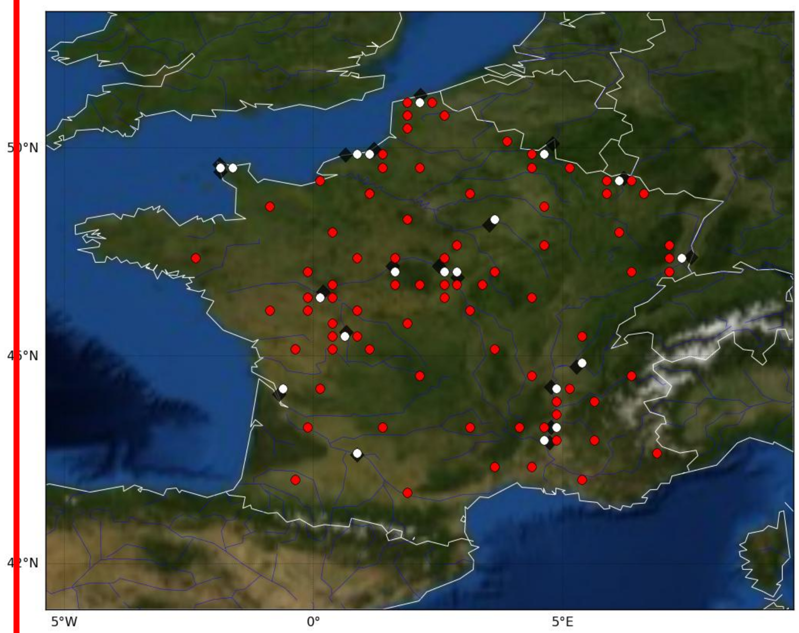


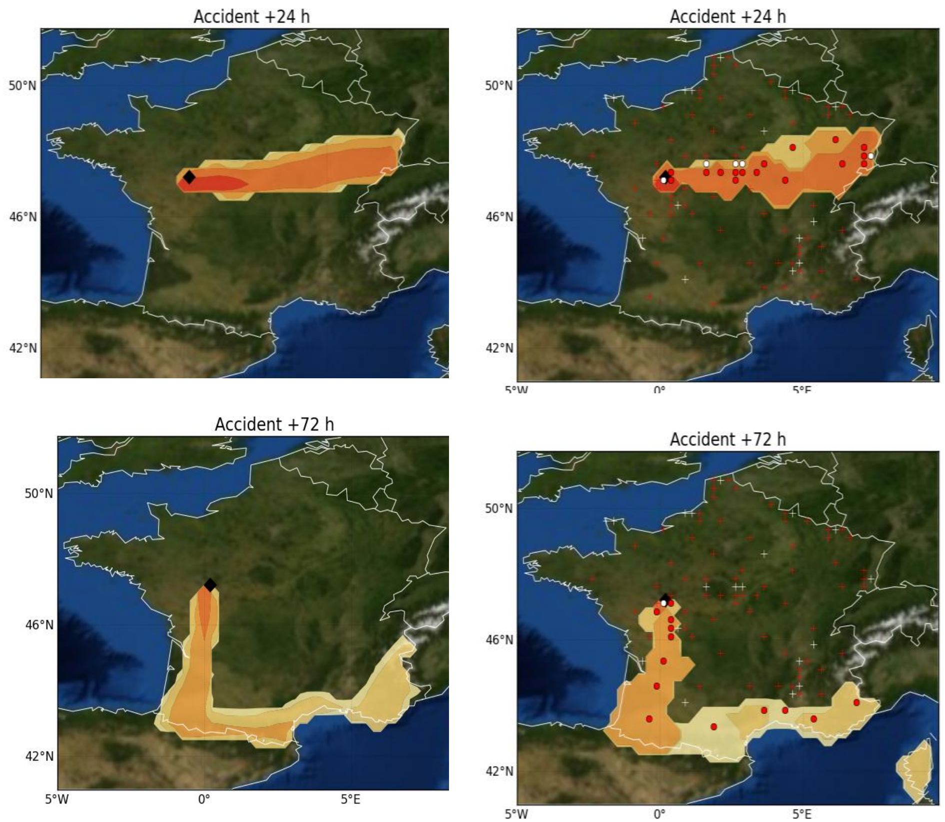
4- New network configuration

Minimising the cost function: ***simulated annealing*** *algorithm*

Construction of an optimal network

Optimal network : reference case, α = 1





simulation

Reconstruction plume

CPU time : 3 weeks

⇨ ***Truncation of the cost function***

Too long CPU time : controlled truncation of the cost function

Principal component analysis consistency with the cost function

1- Matricial form of the cost function

Interpolated concentrations $\overline{\mathbf{c}}^i = \mathbf{A}_\xi \mathbf{c}^i$

The residual error is $\overline{\mathbf{e}}^i = \left(\mathbf{A}_\xi - \mathbf{I}\right)\mathbf{c}^i .$

Real concentrations $c_k^i = [H]_{k,i}$ with H the Jacobian matrix which represents the set of all accidents.

⇨ expression of the cost function

$$\mathscr{I}(\xi) = \sum_{i=1}^{q}\sum_{k=1}^{n} |\overline{c}_k^i - c_k^i|^2 = \sum_{i=1}^{q}\left(\overline{\mathbf{c}}^i - \mathbf{c}^i\right)^{\mathrm{T}}\left(\overline{\mathbf{c}}^i - \mathbf{c}^i\right)$$

$$= \mathrm{Tr}\left(\sum_{\mathrm{i}=1}^{\mathrm{q}}\left(\overline{\mathbf{c}}^{\mathrm{i}} - \mathbf{c}^{\mathrm{i}}\right)\left(\overline{\mathbf{c}}^{\mathrm{i}} - \mathbf{c}^{\mathrm{i}}\right)^{\mathrm{T}}\right) = \mathrm{Tr}\left(\left(\mathbf{A}_\xi - \mathbf{I}\right)\mathbf{H}\mathbf{H}^{\mathrm{T}}\left(\mathbf{A}_\xi - \mathbf{I}\right)^{\mathrm{T}}\right) .$$

2- To evaluate HH^T : Principal component analysis

The singular value decomposition of jacobian matrix H $\mathbf{H} = \sum_{i=1}^{q} \lambda_i \mathbf{v}_i \mathbf{u}_i^{\mathrm{T}}$

Then one has: $\mathbf{H}\mathbf{H}^{\mathrm{T}} = \sum_{i=1}^{q} \lambda_i^2 \mathbf{v}_i \mathbf{v}_i^{\mathrm{T}}$.

The cost function becomes

$$\mathscr{J}(\xi) = \sum_{i=1}^{q} \lambda_i^2 \mathrm{Tr}\left(\left(\mathbf{A}_\xi - \mathbf{I}\right)\mathbf{v}_i\mathbf{v}_i^{\mathrm{T}}\left(\mathbf{A}_\xi - \mathbf{I}\right)^{\mathrm{T}}\right) = \sum_{i=1}^{q}\sum_{k=1}^{n} \lambda_i^2 |\overline{v}_k^i - v_k^i|^2 .$$

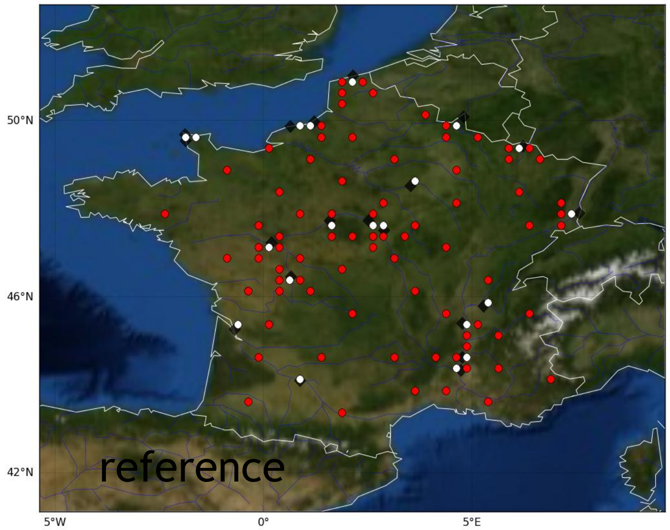
$$\propto J_2^{\;2} \quad \text{with} \quad J_\alpha = \left(\frac{1}{N}\sum_{i=1}^{q}\sum_{k=1}^{n}\left|\overline{c_k^i} - c_k^i\right|^\alpha\right)^{\frac{1}{\alpha}}$$

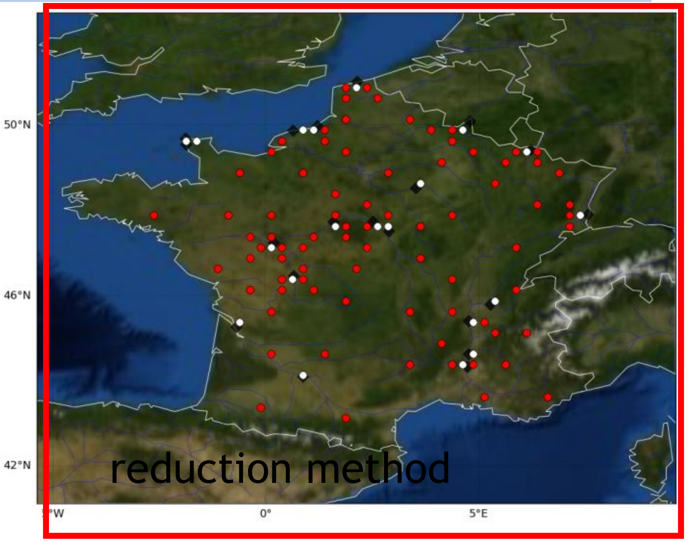
3- Truncation

Depending on the inertia $\sum_{i=1}^{r}\lambda_i^2$ choose a truncation order $r \leq q$

$$\mathscr{J}(\xi) \simeq \sum_{i=1}^{r}\sum_{k=1}^{n} \lambda_i^2 |\overline{v}_k^i - v_k^i|^2 .$$

- q = 8360 accidents, r = 400 modes retained, CPU time divided by 20!





Measures of differences between reduction and reference methods

	reference	reduction	randomize
J_1	7.74 mBq m^{-3}	8.03 mBq m^{-3}	12.23 mBq m^{-3}
$\Delta(\xi^1, \xi^2)$		63.3 km	75.7 km

$$\Delta\left(\xi^1, \xi^2\right) = \max_{1 \leq i_1 \leq m_1} \min_{1 \leq i_2 \leq m_2} D\left(\mathbf{s}^1_{i_1}, \mathbf{s}^2_{i_2}\right).$$

⇨ **Efficient network design method** : it allows to carry out a large number of case studies

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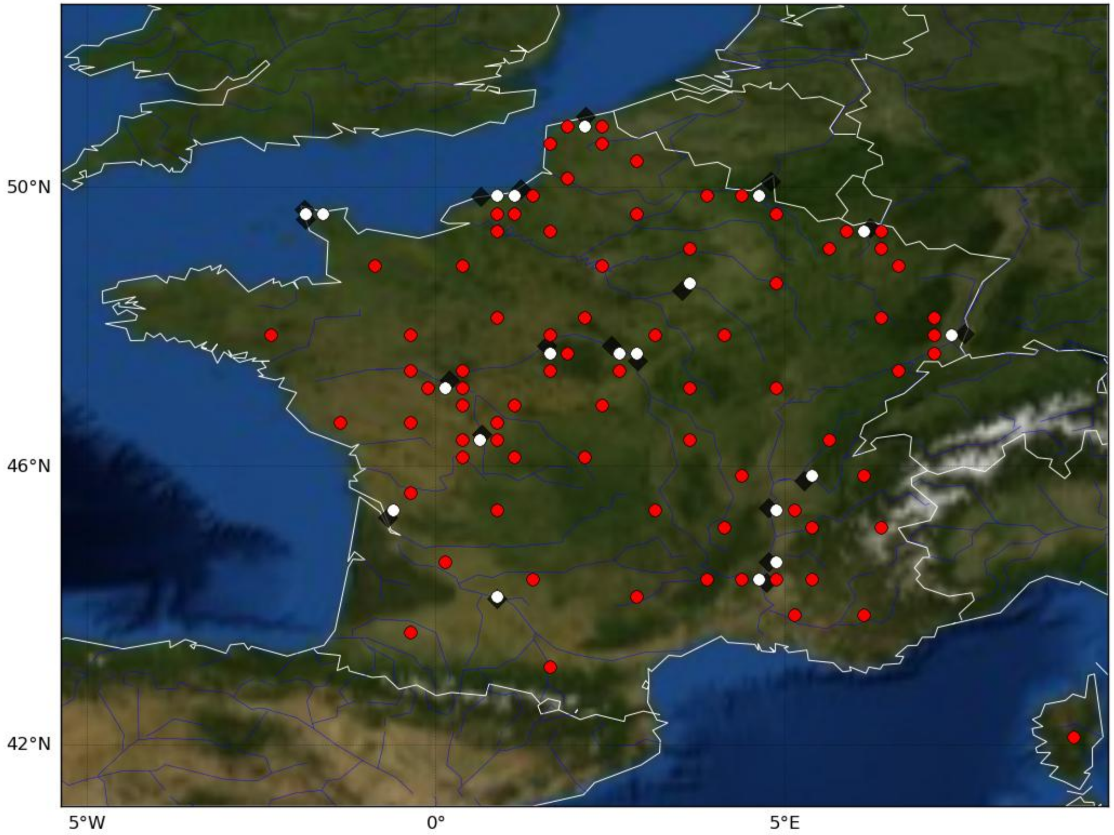
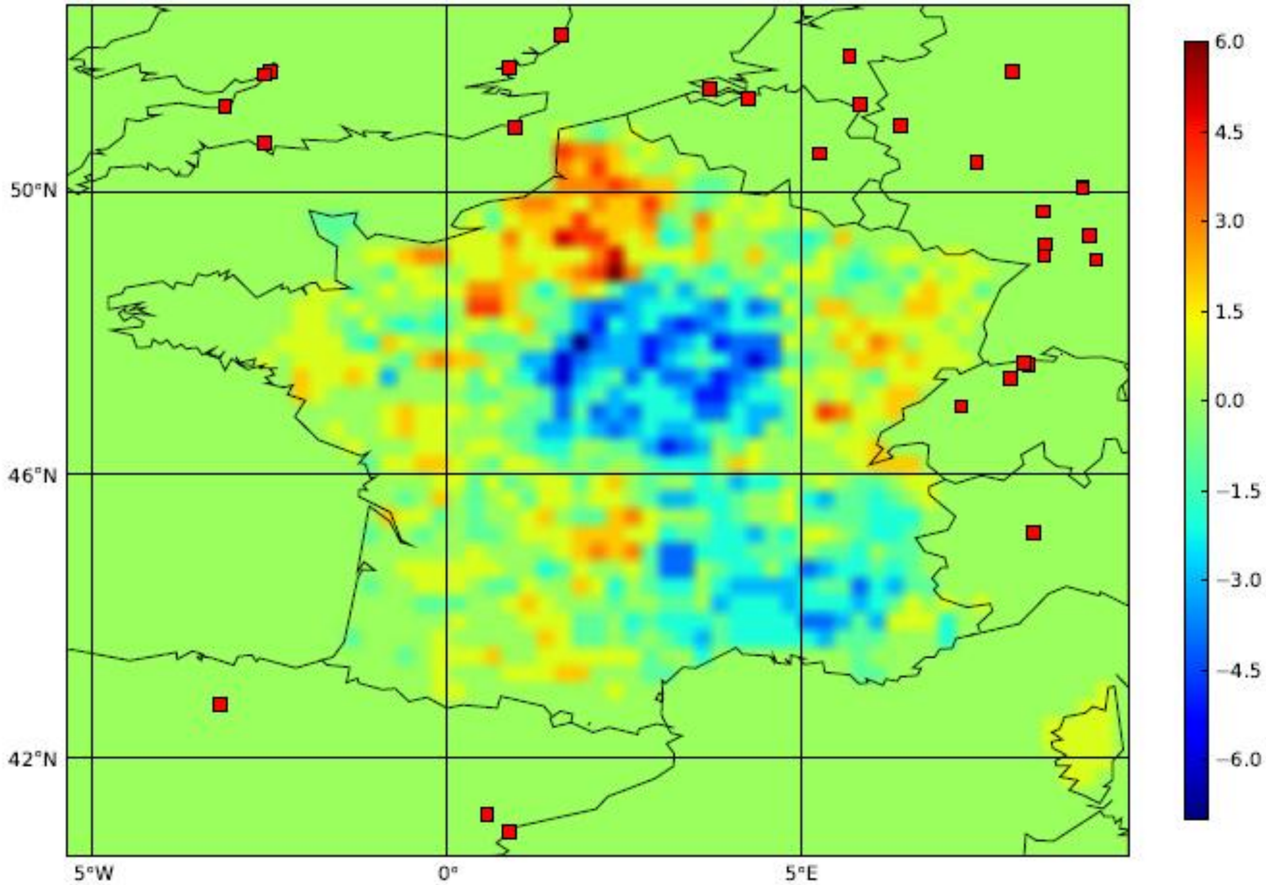
- 34 foreign power plants considered, 700 km from one arbitrary central location (2 25'E, 48 37'N)
- New accidental database : q = 22572 accidents ⇨ reduction method : r = 1080

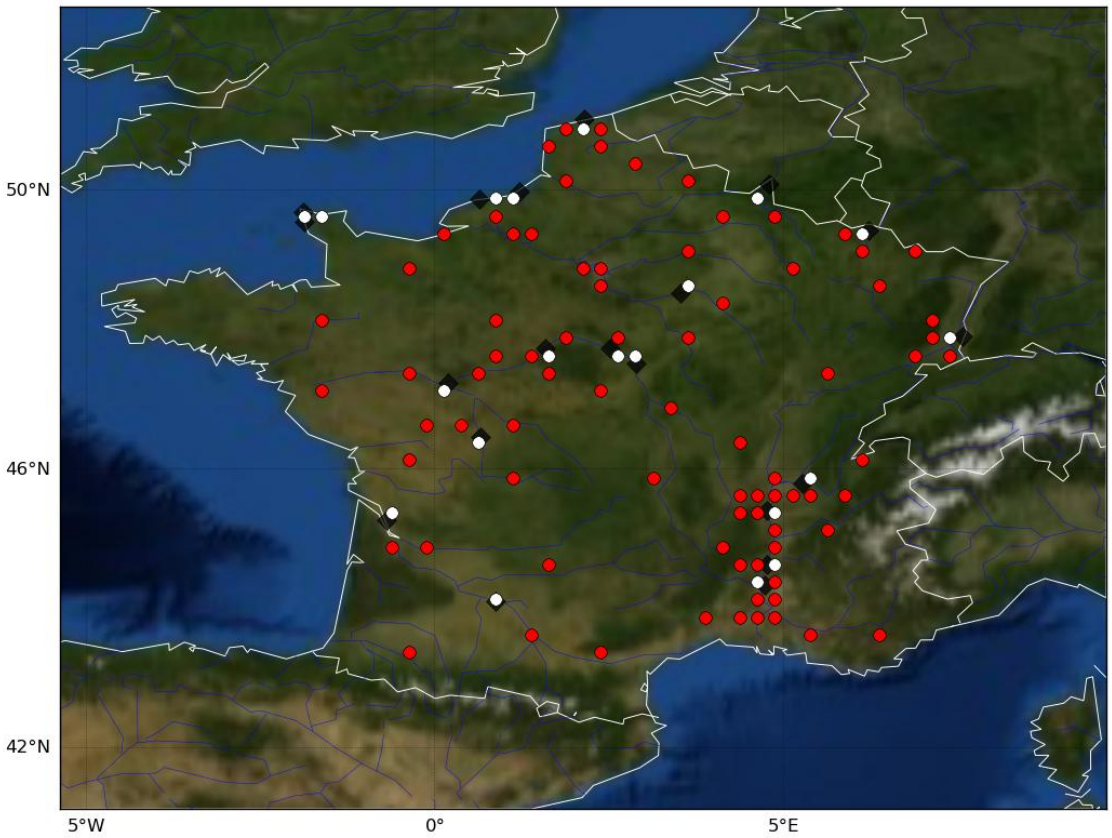


Optimal network of 100 stations taking into account 34 European nuclear power plants

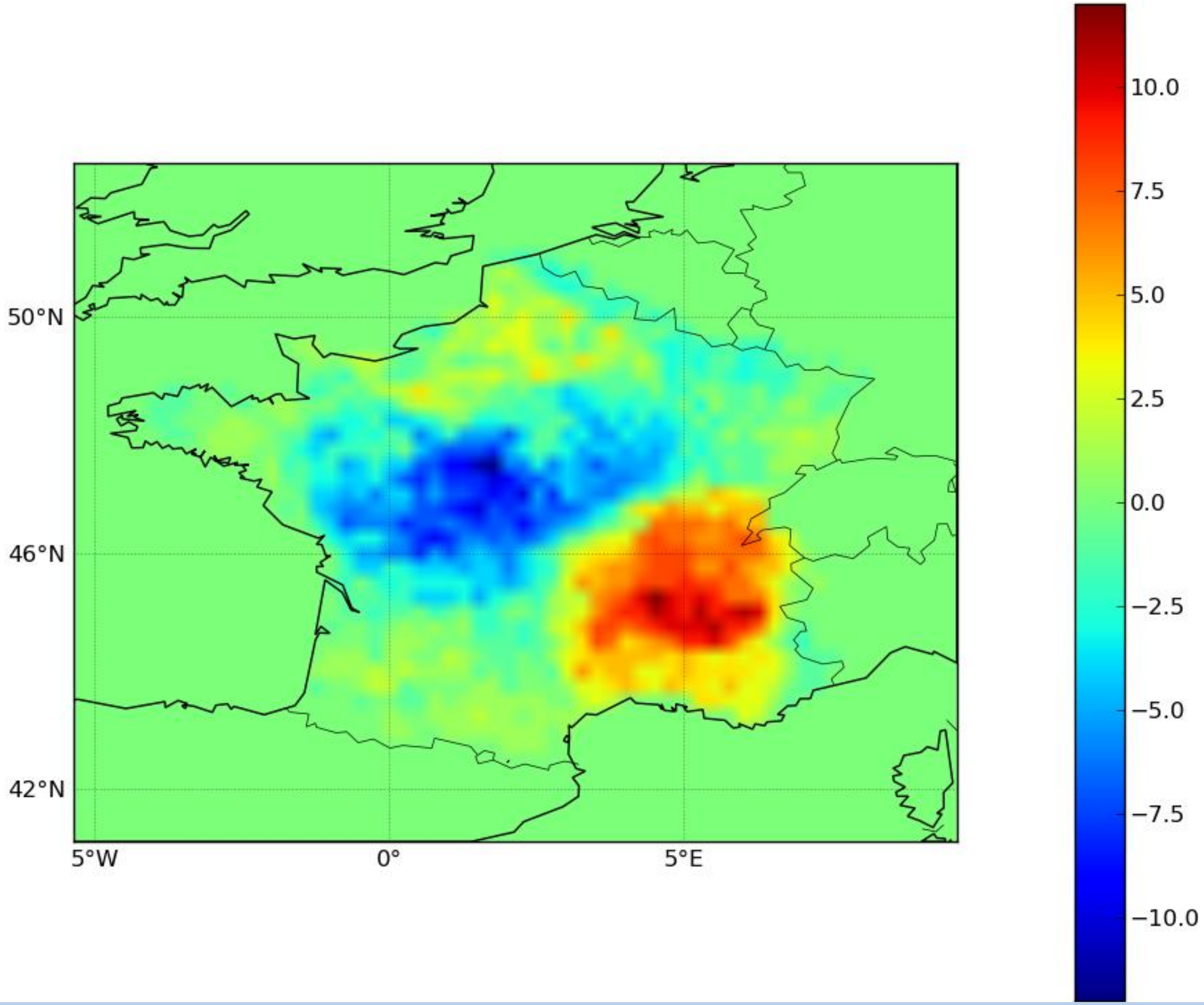


Smoothed discrepancy between reference network and the network optimised for 54 sites including 34 nuclear power plants.

New definition of cost function $J_1 = \frac{1}{N}\sum_{i=1}^{q}\sum_{k=1}^{n} W_k \left|\overline{c_k^i} - c_k^i\right|$



Optimal network of 100 stations taking into account the French density population



Smoothed discrepancy between reference network optimised for 20 sites in France and network optimised in taking into account French density population

How to deploy the measurements stations ?

100 stations to deploy in several years

Where to place the first stations ?

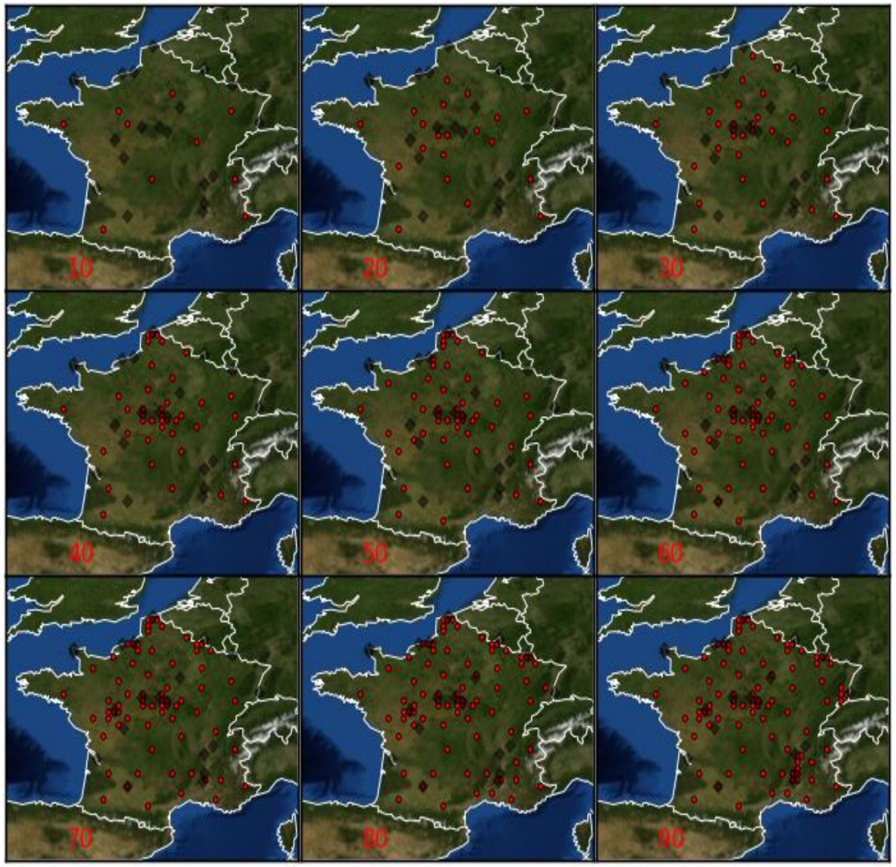
- Strategy 1a: The network is deployed by batch of 10 stations. Each batch is optimised on the full domain, given the stations already deployed. Theoretically, this strategy could lead to a sub-optimal final network, as compared to the reference optimal design.

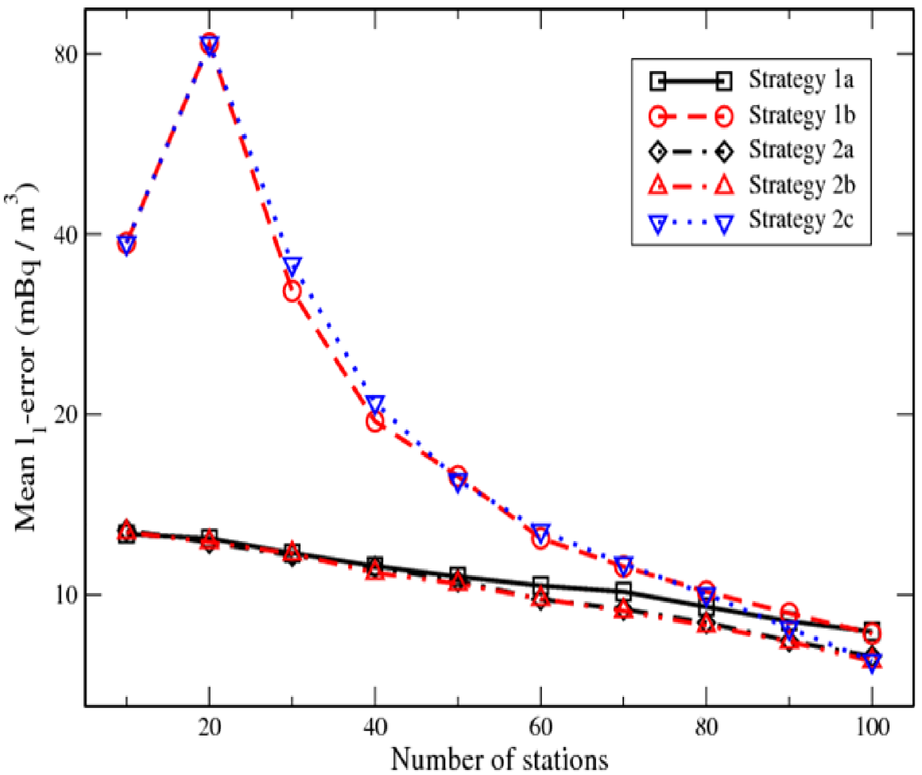
- Strategy 1b: The same as 1a except that the first 20 stations are placed at the nuclear site locations. This strategy could also lead to a sub-optimal final network. Note that the very first 10 stations are optimally chosen within the group of 20 fixed ones.

- Strategy 2a: We consider a pre-computed optimal network of 100 stations, without constraints. The network is deployed by batch of 10 whose locations are optimally chosen among the remaining sites of the pre-computed optimal network. This way, we guarantee that the final network is optimal.

- Strategy 2b: The same as 2a, except that the pre-computed optimal network of 100 stations has 20 fixed stations, one for each nuclear site.

- Strategy 2c: The same as 2b, except that the first 20 stations to be deployed are positioned at the nuclear sites. As for case 1b, the very first 10 stations are optimally chosen within the group of 20 fixed ones.





Sequential deployment of the network following optimal strategy : strategy 2b

Mean error for deployment strategies

- Strategies 1b, 2c : poor intermediary networks
- Strategies 2a, 2b : efficient intermediary networks

Conclusion

- Development of a reduction method : speed-up the optmisation of an atmospheric monitoring network (CPU time divided by 20)
- Flexibility of the reduction method
- To take into account foreign nuclear power plants, French density population in optimisation process and to define an optimal strategy to deploy sequentially a network over several years
- The retained strategy for the deployment leads to performant intermediary networks

Perspective

To use these optimal networks for data assimilation to improve the accidental plume forecast dispersion [Winiarek et al, 2010]

References

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