PARAMETERIZATION OF CONVECTIVE PBL USING SURFACE DATA FOR THE WIND AND STABILITY CLASSES

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INTRODUCTION

The aim of the present work is to determine the vertical profiles of wind velocity and turbulent exchange coefficient in the Convective Planetary Boundary Layer (CPBL) using data collected from the automatic meteorological stations. A simple two-layer model of convective PBL developed in accordance with the similarity theory (see Yordanov, D. and M. Kolarova, 1988), that consists of a Surface Layer and an Ekman layer over it, is used to obtain the vertical profiles of the temperature, wind velocity and the turbulent exchange coefficient in PBL from the surface wind measurements and atmospheric stability data. As input to this model the internal to CPBL parameters are needed. The internal parameters can be obtained from the experimental data applying two approaches: first one (called "up-down" approach) uses data for the geostrophic wind and the potential temperature and was described in several papers by (Yordanov, D., M. Kolarova, 1988, 1989, 1990, 1994). The second one (called here "down-up" approach) uses data from the surface meteorological observations. The description of this approach is the subject of the present paper.

WIND AND TEMPERATURE PROFILES IN THE SURFACE LAYER (SL)

In the present work the empirical curves of Golder (1972), which relate the atmospheric stability classes to the surface roughness and the Monin-Obukhov length scale, are used for obtaining the vertical distribution of the meteorological characteristics in the PBL. Following the approach presented in (Zanetti, 1990) the empirical curves of Golder are fitted using power low functions so that:

$$L^{-1} = a\, z_0^{\,b}\,, \tag{1}$$

where z_o is the roughness length, the constants *a* and *b* depend on the stability class.

Here the Monin-Obukhov length scale L is defined as: $L = -u_*^3 \big/ \kappa\beta(\overline{w'\theta'})_0$,

where u_* is the friction velocity, κ - the von Karman constant, $\beta = g/\overline{\theta}$ - buoyancy parameter, g – gravity acceleration, $\overline{\theta}$ - averaged for the whole layer potential temperature, $(\overline{w'\theta'})_0$ -the normalized by $c_p\rho$ vertical turbulent heat flux at the ground, c_p is the specific heat capacity at constant pressure, and ρ - the air density. The values of the constants ***a*** and ***b*** for unstable and neutral conditions are given in Table 1.

Table 1. The values of a and b for unstable and neutral conditions

Stability class	*A*	*B*	*C*	*D*
a	-0.0875	-0.03849	-0.00807	0.0
b	-0.1029	-0.1714	-0.3049	0.0

Having the stability classes experimentally determined from the measurements of the pulsation characteristics, the corresponding Monin-Obukhov length scale L can be calculated from equation (1) at given z_0 (representative roughness for the region).

According to the Monin-Obukhov similarity theory the universal profiles of the wind velocity and of the potential temperature in SL can be presented as:

$$\kappa \frac{u}{u_*} = f_u(\zeta, \zeta_0)\,; \qquad \frac{(\theta - \theta_0)}{\theta_*} = \frac{(\theta - \theta_0)\kappa^2 \beta L}{u_*^2} = f_\theta(\zeta, \zeta_0) \tag{2}$$

where u и θ are the wind velocity and potential temperature at height z, θ_0 is the potential temperature at height z_0, $\theta_* = \dfrac{u_*^2}{\kappa^2 \beta L}$ is the temperature scale, f_u and f_θ are universal functions of the dimensionless height $\zeta = z/L$ and dimensionless roughness $\zeta_0 = z_0/L$. The turbulent exchange coefficients for momentum k_m and for heat k_θ can be expressed as:

$$k_m = \frac{\kappa u_* z}{\eta_u(\zeta)}\,, \qquad k_\theta = \frac{\kappa u_* z}{\eta_\theta(\zeta)}, \tag{3}$$

where $\eta_u(\zeta) = \zeta \dfrac{\partial f_u(\zeta, \zeta_0)}{\partial \zeta}$, $\quad \eta_\theta(\zeta) = \zeta \dfrac{\partial f_\theta(\zeta, \zeta_0)}{\partial \zeta}$.

A number of expressions for the universal SL profiles f_u and f_θ (respectively for η_u and η_θ) can be found in the literature worked out mainly by fitting experimental data.

The turbulent friction velocity u_* enters explicitly in the expressions (2) and (3). If the universal SL profiles f_u and f_θ are known, u_* can be determined from equation (2) considering the wind velocity u_a and the anemometer height z_a as known parameters:

$$u_* = \frac{\kappa u_a}{f_u(\zeta_a, \zeta_0)}, \tag{4}$$

where $\zeta_a = z_a / L$, and L is determined from equation (1).

For the surface layer the universal profiles of Businger et. al. (1971) are used as shown in (Yordanov, D. and M. Kolarova 1988, 1994, 1997). In this case the functions f_u and f_θ has the form:

$$f_u(\zeta, \zeta_0) = \Phi_u(\zeta) - \Phi_u(\zeta_0) \qquad \alpha_\theta(0) f_\theta(\zeta, \zeta_0) = \Phi_\theta(\zeta) - \Phi_\theta(\zeta_0) \tag{5}$$

where the functions $\Phi_u(\zeta)$ and $\Phi_\theta(\zeta)$ are the following:

$$\Phi_u(\zeta) = \ln(-\zeta) + 2arcctg(\eta_u) - \ln\left[(1+\eta_u^2)(1+\eta_u)^2 \eta_u^{-4}\right] \quad \text{and}$$

$$\Phi_\theta(\zeta) = \ln(-\zeta) - 2\ln\left[(1+\eta_\theta)\eta_\theta^{-1}\right].$$

Here η_u and η_θ are defined as:

$$\eta_u = (1 - \gamma_u \zeta)^{-1/4}\,, \qquad \eta_\theta = (1 - \gamma_\theta \zeta)^{-1/2} \tag{6}$$

and $\gamma_u = 15$; $\gamma_\theta = 9$; $\alpha_\theta(0) = 1.35$.

Replacing the expressions for the functions f_u and f_θ given by equation (5) in equation (4) we can determine the friction velocity u_* .

EXTERNAL TO CPBL PARAMETERS ($\overrightarrow{V_g}$, α, and $\delta\theta$)

The turbulent regime in a barotropic PBL capped by an inversion is parameterized on the basis of the similarity theory using the following non-dimensional external parameters:

$$Ri_B = \frac{\beta\,\delta\theta\, z_i}{\vec{V}_g^{\,2}}, \quad Z_0 = \frac{z_0}{z_i}, \quad \text{and} \quad Ro_i = \frac{\left|\vec{V}_g\right|}{f z_i}, \tag{7}$$

where Ri_B is the bulk Richardson number, $\overline{\theta}$ is mean potential temperature in the layer, $\delta\theta = (\theta_{z_i} - \theta_0)$ is the difference between the potential temperature at the inversion height and the ground (at z_0), $\vec{V}_g$ is the geostrophic wind vector, Z_0 is the non-dimensionless roughness, z_i is the inversion height, Ro_i is the bulk inversion Rossby number, f is the Coriolis parameter. The turbulent regime in the CPBL is uniquely determined by the following internal parameters:

$$c_g = \frac{u_*}{\left|\vec{V}_g\right|}, \quad \alpha, \quad \mu_c = \frac{z_i}{L} \quad \text{or} \quad \mu = \frac{\kappa\, u_*}{f\, L} \tag{8}$$

where c_g is the drag coefficient, α is surface wind deviation from the geostrophic wind or cross-isobaric angle, μ and μ_c are the internal stratification parameters.

The relationship between the external and internal CPBL parameters is given by the resistance laws:

$$\frac{\kappa}{c_g} = \left\{\left[\left(\ell n Z_0 + A(\mu_c\right)\right]^2 + B^2(\mu_c, \mu)\right\}^{1/2} \tag{9}$$

$$\sin|\alpha| = \frac{c_g}{\kappa} B(\mu_c, \mu) \tag{10}$$

$$\alpha_\theta(0) Ri_B = -\frac{c_g^2}{\kappa^2}\left[\ell n Z_0 + C(\mu_c)\right] \tag{11}$$

where $\alpha_\theta(0) = 1.35$ and $a_\theta(0) = K_\theta / K_m$ is the inverse Prandtl number (expressing the ratio between the turbulent exchange coefficients for heat and for momentum at the ground).

For the absolute value of the geostrophic wind the following expression can be obtained from equations (9):

$$\left|\overrightarrow{V_g}\right| = \frac{u_*}{\kappa}\left[\left(\ell n Z_0 - A(\mu_c)\right)^2 + B^2(\mu_c, \mu)\right]^{1/2} \tag{12}$$

and the absolute value of the cross isobaric angle α can be obtained from equation (10):

$$|\alpha| = \arcsin\left[\frac{c_g}{\kappa} B\left(\mu_c, \mu\right)\right], \tag{13}$$

α - being negative in the Northern Hemisphere.

From (11) we obtain the expression for the difference between the potential temperature at the top and at the bottom of the PBL $\delta\theta$ expressed as:

$$\delta\theta = \theta_{z_i} - \theta_0 = \frac{\theta_* \mu_c}{\alpha_\theta(0)}\left[-\ell n Z_0 - C(\mu_c)\right]. \tag{14}$$

The universal functions A, B , C, in the resistance lows are subject of many theoretical and experimental investigations. In this case we use the functions derived by Yordanov, D. and M. Kolarova (1988) for the CPBL model. The numerical solution of the system (9-11) is given in (Kolarova, M. et al., 1989).

THE CPBL MODEL

The convective planetary boundary layer (CPBL) model YORCON, developed on the basis of the similarity theory, consists of two layers - surface layer (SL) and Ekman layer over it, in which the turbulent exchange coefficient is assumed constant with height. In the SL the profiles defined by equations (2) and (3) are used. The turbulent exchange coefficient is determined from the turbulent kinetic energy balance equation under the assumptions about the type of the turbulent kinetic energy and its dissipation (see Yordanov, D. and M. Kolarova, 1988; BC-EMEP, 1994). The following asymptotes are found:

$$k_m = (2c)^{-1/3} w_* z_i \quad \text{at } \mu_c \le -11.5; \quad \text{and} \quad k_m = (3c)^{-1/4} u_* z_i \quad at\, \mu_c \ge -11.5 \tag{15}$$

where $w_* = \left[\beta\left(\overline{w'\theta'}\right)_0 z_i\right]^{1/3}$ is the convective velocity scale, $c = 2.10^6$.

The dimensionless height of the SL is determined slightly dependent on μ_c and equal to $h = h_*/z_i = 0.04$.

For the dimensional PBL wind profile we obtain:

$$u + iv = \left|\overrightarrow{V_g}\right|\cos\alpha + P\frac{u_*}{\kappa} + i\left[\left|\overrightarrow{V_g}\right|\sin\alpha + Q\frac{u_*}{\kappa}\right] \quad \text{at } h \le Z \le 1 \tag{16}$$

Where the non-dimensional velocity defects P and Q have the form:

$$P + iQ = \begin{cases} \left[\frac{1}{3} - (1-Z)^2\right]\frac{a}{2(-\mu_c)^{1/3}} + i\frac{\kappa u_*}{f z_i} & at\;\; h \le Z \le 0.4 \\ i\frac{\kappa u_*}{f z_i} & at\; 0.4 \le Z \le 1 \end{cases} \tag{17}$$

here $a = \kappa^{4/3}(2c)^{1/3}$.

In order to obtain the external to CPBL parameters we use the universal similarity functions *A, B, C*, obtained from the CPBL model:

$$A + iB = \ell n(-\mu_c) - \frac{m_1}{\mu_c^{1/3}} + m_2 + i\frac{\kappa u_*}{f z_i} \tag{18}$$

$$C = \ell n(-\mu_c) - \frac{m_3}{\mu_c^{1/3}} + m_4 \tag{19}$$

where

$$m_1 = a/3; \quad m_2 = -\ell n 8\kappa + \pi/2 - \Phi_u(\zeta_h); \quad m_3 = \frac{a\alpha_\theta(0)}{2\alpha_\theta(\zeta_h)}; \quad m_4 = -\ell n 4\kappa - \Phi_\theta(\zeta_h);$$

and $\zeta_h = h\mu_c$. Replacing the universal similarity functions given by equation (18) and (19) in the resistance lows (9-11) we obtain a system of non-linear algebraic equations for the internal parameters $c_g, \alpha,$ and μ_c. The numerical solution of this system of equations at different values of the external parameters Ri_B, Z_0, Ro_i are obtained by Kolarova, M. et al. (1989).

THE INVERSION HEIGHT

The evolution of the convective PBL height (mixing layer height) at conditions of horizontal homogeneity is calculated applying the equation (20) as shown in (Yordanov, D. et al., 1990):

$$\frac{dz_i}{dt} = \frac{(1+2c_1)(\overline{w'\theta'})_0}{\Gamma z_i} + c_2 \frac{u_*^3}{\Gamma \beta z_i^2} \tag{20}$$

where Γ is the potential temperature gradient above the inversion layer, $c_1 = 0.2$, $c_2 = 2.5$ are constants. The evolution of the inversion height depends on the surface turbulent heat and momentum fluxes by the following way:

$$(\overline{w'\theta'})_0 = -\frac{\mu f u_*^2}{\kappa^2 \beta} = -\frac{\mu_c u_*^3}{\kappa \beta z_i} \quad \text{and} \quad u_* = c_g \left|\vec{V}_g\right|, \tag{21}$$

i.e. on the internal CPBL parameters.

Applying the “down-up” approach the surface turbulent fluxes are estimated from the surface observations calculating u_* and $(\overline{w'\theta'})_0 = -\frac{u_*^3}{\kappa \beta L}$ from equation (4) and (1). The equation (21) is solved numerically at initial condition : $z_i = 0.1\kappa u_* / f$ at $t = t$sunrise .

CONCLUSIONS

Applying the similarity theory and resistance lows expressed through equations (9-11) we can generally determine the surface turbulent fluxes defined by L and u_* from the external to CPBL parameters $|V_g|$, $|\alpha|$ and $\delta\theta$, normally determined from the numerical weather prediction. The proposed approach solves the inverse problem that consists of determination of the external to CPBL parameters and the vertical profiles of the temperature, wind velocity and the turbulent exchange coefficient (given by the CPBL model) from the ground station meteorological measurements of the wind and atmospheric stability.

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