ACCOUNTING FOR EFFECTS OF WIND ROTATION IN THE PBL ON THE PLUME CHARACTERISTICS

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INTRODUCTION

Most of the regulatory models are based on the Gaussian plume solution:

$$c(x,y,z)=\frac{\tilde{c}_0}{\sqrt{2\pi}\sigma_y(x)}\exp\left[-\frac{y^2}{2\sigma_y^2}\right] \quad \tilde{c}_0=\frac{1}{\sqrt{2\pi}\sigma_z}\left(e^{-\frac{(z-h)^2}{2\sigma_z^2}}+e^{-\frac{(z+h)^2}{2\sigma_z^2}}\right) \tag{1}$$

The present work aims at constructing a more complicate modification of (1), by which the influence of a number of dynamic effects in the Planetary Boundary Layer (PBL) on the pollution concentration $c(x,y,z)$, unaccounted for by (1), can be studied.

FORMULATION OF THE PROBLEM

The diffusion plume model applied in the study is based on the momentum method (*Saffman, P.*, 1962). The following system of equations, written in a co-ordinate system with x axis oriented along the wind at the effective height h level:

$$\frac{\partial c_0}{\partial t}+u\frac{\partial c_0}{\partial x}+(w-w_0)\frac{\partial c_0}{\partial z}+\alpha c_0=\frac{\partial}{\partial x}k_x\frac{\partial c_0}{\partial x}+\frac{\partial}{\partial z}k_z\frac{\partial c_0}{\partial z}+\overline{Q} \tag{2}$$

$$\frac{\partial c_1}{\partial t}+u\frac{\partial c_1}{\partial x}+(w-w_0)\frac{\partial c_1}{\partial z}+\alpha c_1=\frac{\partial}{\partial x}k_x\frac{\partial c_1}{\partial x}+\frac{\partial}{\partial z}k_z\frac{\partial c_1}{\partial z}+vc_0 \tag{3}$$

$$\frac{\partial c_2}{\partial t}+u\frac{\partial c_2}{\partial x}+(w-w_0)\frac{\partial c_2}{\partial z}+\alpha c_2=\frac{\partial}{\partial x}k_x\frac{\partial c_2}{\partial x}+\frac{\partial}{\partial z}k_z\frac{\partial c_2}{\partial z}+2vc_1+2k_yc_0 \tag{4}$$

is solved under boundary conditions:

$$k_z\frac{\partial c_n}{\partial z}=(B-w_0)c_n \text{ at } z=z_0 \ , \ k_z\frac{\partial c_n}{\partial z}=0 \text{ at } z=H_T \ , \ n=0,1,2 \tag{5}$$

for a stationary point source:

$$\overline{Q}(x,y,z)=Q\delta(x)\delta(y)\delta(z-h) \tag{6}$$

The quantities used in (2)-(6) are as follows: $c_0(x,z,t)$, $c_1(x,z,t)$ and $c_2(x,z,t)$ are respectively the zero (cross-wind integrated concentration), the first and the second momentum; $u(z)$, $v(z)$ and $w(z)$ - the wind components: H_T - diffusion layer height; w_0 , B , α - gravity deposition velocity, deposition parameter, describing the admixture-soil interaction and the chemical transformation parameter; k_z and $k_x \sim k_y$ - the vertical and horizontal turbulent exchange coefficients; h - the effective stack height. The admixture concentration $c(x,y,z,t)$ can be obtained by the formula:

$$c(x,y,z,t)=\frac{c_0(x,z,t)}{\sqrt{2\pi}\sigma_y}\exp\left[-\frac{(y-\overline{Y})^2}{2\sigma_y^2}\right] \tag{7}$$

where $\overline{Y}$ - the mean displacement along y and the corresponding dispersion σ_y are calculated by the well known formula:

$$\overline{Y}(x,z,t)=c_1/c_0 \;\;,\;\; \sigma_y^2(x,z,t)=c_2/c_0-\overline{Y}^2 \tag{8}$$

It should be noted that in spite the classic plume form of (7), the whole formulation (2)-(8) makes it possible to take into account the effects of vertical heterogeneity of k_z and the wind speed, as well as the wind rotation. The classic Gaussian plume solution ($c_0 \rightarrow \widetilde{c}_0$) can be obtained from (7) after some simplifications: the assumptions $\frac{\partial c_0}{\partial t}=w=w_0=B=\alpha=k_x=0$, $u(z)=const=u(h)$, $v(z)=0$ (which means $\overline{Y}=0$), $H_T \rightarrow \infty, c(H_T)\rightarrow 0$ and after formal application of the statistical theory and Taylor's frozen turbulence hypothesis $k_z=\frac{1}{2}\frac{\sigma_z^2}{x}u(h)$, $\sigma_y^2(x,z,t)\rightarrow\sigma_y^2(x)$, in which case usually semi-empirical dispersion relations (for example Pasquil – Gifford) are used.

The dynamic parameters $u,v,\, k_z$, ($k_x \sim k_z$) in (2)-(6) are determined by a model of a none-stationary, stratified, baroclinic PBL over a sloping terrain, with x axis oriented along the slope (fig.1.):

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial z}k_z\frac{\partial u}{\partial z}+s_* f[v-v_g(z)]+\beta\vartheta'\psi \tag{9}$$

$$\frac{\partial v}{\partial t}=\frac{\partial}{\partial z}k_z\frac{\partial v}{\partial z}-s_* f[u-u_g(z)]+\beta\vartheta'\varphi \tag{10}$$

$$\frac{\partial \vartheta'}{\partial t}=\frac{\partial}{\partial z}k_z\frac{\partial \vartheta'}{\partial z}+\Gamma\{[u-u_g(z)]\psi+[v-v_g(z)]\varphi\} \tag{11}$$

The boundary conditions for (9)-(11) are the following:

$$u=v=0, \vartheta'=\vartheta_0' \quad \text{at } z=z_0 \tag{12}$$

$$u=u_g(H),\, v=v_g(h),\, \vartheta'=0 \quad \text{at } z=H \tag{13}$$

The initial conditions for (9)-(11) may be prescribed, or obtained by successive iterations at fixed parameters. The following parameters are used in (9)-(13): $s_*=1,-1$ for the North and South hemisphere respectively; β is the buoyancy parameter; Γ is the free-atmosphere potential temperature gradient; $\vartheta'=\vartheta-\vartheta(z=H)$, ϑ - potential temperature; $u_g=u_{go}+S_x z$, $v_g=v_{go}+S_y z$, S_x and S_y - baroclinicity parameters (they can be also defined in the form $S_x=M\cos\Phi$, $S_y=M\sin\Phi$, $M=(S_x^2+S_y^2)^{1/2}$); $u_{g0}=G_0\cos\chi$, $v_{g0}=G_0\sin\chi$; φ and ψ - angles which characterise the terrain slope; $H(t)$ is the PBL height; $\vartheta_0'(t)$ is the disturbance of the surface potential temperature. It may be prescribed, or determined (for example by the "force-restore" method) in which case additional information – short-wave solar radiation, soil characteristics, cloud cover, etc. should be implemented.

The geometry of the problem is demonstrated in Fig.1.

The problem (9)-(13) is closed by introducing mixing lengths of Blakadar type, modified for a stratified PBL. The model can work in two regimes – a none-stationary for studying the PBL evolution and a stationary for studying typical meteorological situations. The problem is generally enough parameterized by a set of external dimensionless parameters (*Syrakov E.,* 1990):

$$Ro(t), S(t), Ro_i(t), S_x, S_y, (or\, M, \Phi), \varphi, \psi \;, \tag{14}$$

where $Ro = G_0 / fz_0$, $Ro_i = G_0 / fH_i$, $S = -\beta\delta\vartheta / G_0 f$ are the geostrophyc and inversion Rosby number and the external stratification parameter. A large number of different meteorological regimes can be obtained by varying these parameters.

Figure 1. Scheme of the coordinate axes and angles configuration: $0x, 0x_g, 0x_T, 0x_\tau$ *are correspondingly oriented along the slope, the surface geostrophic wind, the surface termal wind, the surface wind (North hemisphere)*

RESULTS AND DISCUSSIONS

The simulations are carried out by following the procedure: the PBL dynamics problem (9)-(13) is numerically solved and $u(z), v(z)$, $k_z(z)$ are obtained for given parameters (14). Then the problem (2)-(6) is solved (it should be reminded that (2)-(6) is formulated in a coordinate system with x axis oriented along the wind at the effective height h level), the momentums $c_0(x,z,t)$, $c_1(x,z,t)$, $c_2(x,z,t)$ and the corresponding $\overline{Y}(x,z,t)$ and $\sigma_y^2(x,z,t)$ are obtained. Finally the concentration $c(x,y,z,t)$ is determined from (7).

Due to the volume limitations a small number of typical stationary situations are considered in the present work. These are three cases of barotropic ($S_x = S_y = 0$) PBL without ($Ro_i \le 10^2$) and with inversion ($H_i = 200m$, $Ro_i = 500$), over a flat terrain ($\psi = \varphi = 0$) for strong stability ($S = 670,\ Ro = 5.10^5$) and instability ($S = -670,\ Ro = 5.10^5$) as well as for neutral case ($S = 0,\ Ro = 5.10^5$).

A characteristic, which explicitly demonstrates the differences between the present and the classic plume model is $\overline{Y}$, shown in Figure 2. It can be seen that for the case of low source $\overline{Y}$ tends to right deviation (according to the orientation of the wind at $z = 10m$), which is consistent with the Ekman's spiral. This effect is especially well displayed at the higher levels for the case of stable stratification. For the higher source the $\overline{Y}$ deviations are, in most of the cases, smaller, because the wind at level $z = 100m$ is closer to the geostrophic one. The inversions have a dual effect on the $\overline{Y}$ behaviour – the maximal wind rotation angle is significantly larger, but this does not necessarily cause larger deviations. The $\overline{Y}$ deviation is also subject to the vertical turbulent exchange, which in inversion cases is smaller. The joint influence of these two factors determines the dominating rotation trend.

The surface concentrations $c_{00} = c(x, y = \overline{Y}, 0)$, obtained by (7) are shown in Figure 3.a. Figure 3.b. shows the ratio $c_{00} - plume / c_{00}$, where $c_{00} - plume$ is the surface concentration along the plume axis obtained by (2)-(7), but in the case of classic plume approximation - $u(z) = const = \sqrt{u(h)^2 + v(h)^2}, v(z) = 0$. In this case not only $\overline{Y} = 0$, but the plume concentrations along the plume axis are much smaller at distances less then several km from the source.

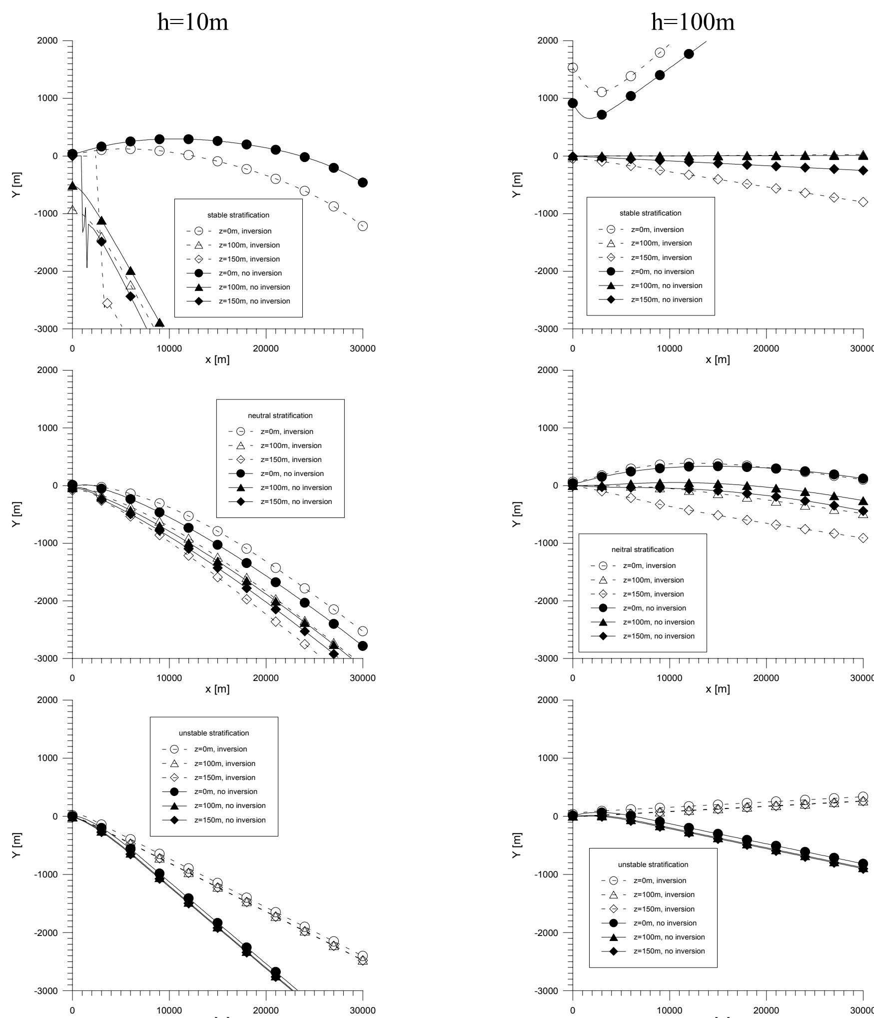

Figure 2. Plots of $\overline{Y}(x)$ at different levels (z=0, 100, 150m) and source heights (h= 10 and 100m) at stable, neutral and unstable stratification for cases with inversion (height 200m) and without inversion.

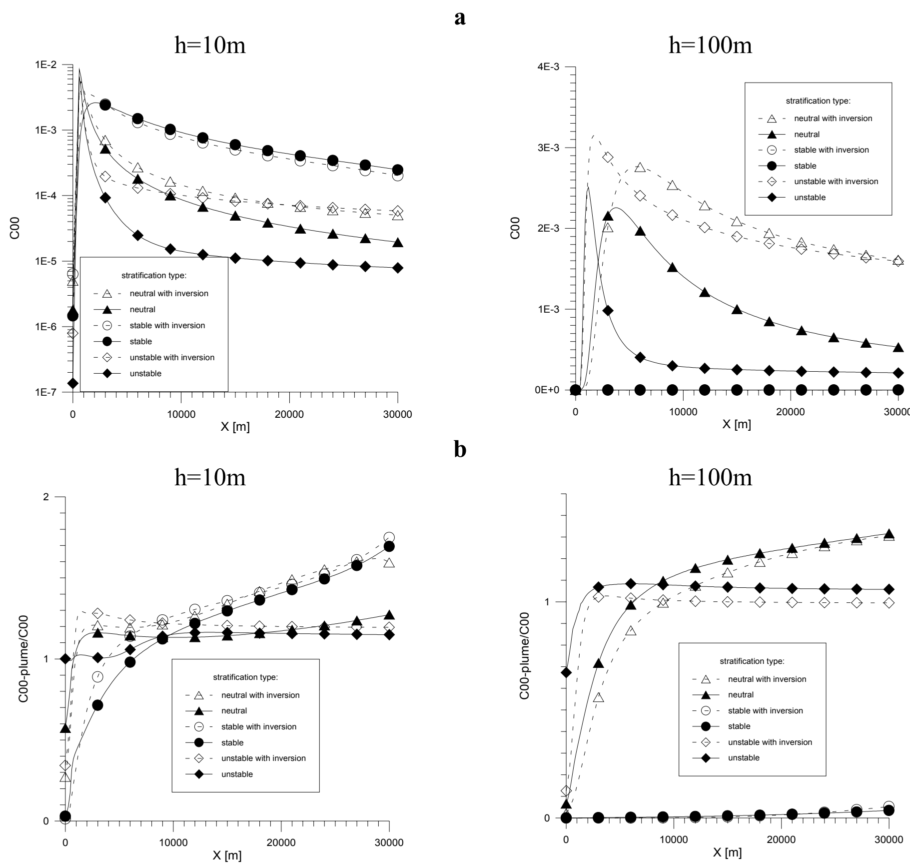

Figure 3. Plots of c_{00} *(a) and* $c_{00} - plume / c_{00}$ *(b) for source heights h= 10 and 100m at stable, neutral and unstable stratification for cases with inversion with height 200m and without inversion.*

CONCLUSION

Even these relatively simplified examples demonstrate the very complex influence of the PBL characteristics on the plume behaviour. More extensive studies also of the other factors and more detailed comparisons of (2)-(7) with the classic plume (1) can be an objective of some future work.

REFERENCES

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