20th International Conference on
Harmonisation within Atmospheric Dispersion Modelling for Regulatory Purposes
14-18 June 2021, Tartu, Estonia

TESTING OF AN ANALYTICAL FORMULA FOR ESTIMATING PLUME RISE AND TOUCHDOWN FOR A DENSE PLUME, USING THE JACK RABBIT II TRIAL 8 OBSERVATIONS

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Abstract: Trial 8 of the Jack Rabbit II chlorine release field experiment involved an upwards-directed jet of two-phase chlorine, where about 2400 kg were released from a pressurized tank in about 30 s. The mass release rate decreased monotonically over the time period. The observed dense jet rose initially to a maximum height of about 40 m and then descended to the ground, with touchdown at a distance of about 50 m from the source. As the release rate decreased over time, the plume rise was observed to slightly increase at first, reaching 60 m, and then steadily decrease, and the touchdown distance increased to 100 m. We test analytical formulas for maximum plume rise, distance to touchdown, and maximum concentration at touchdown suggested by Hoot Meroney and Peterka (HMP), derived using basic plume rise theory and wind tunnel observations. It is seen that the HMP formulas agree with the plume observations within a factor of two, and that they provide the correct variation of the plume rise and touchdown distance as the mass release rate decreased.

Key words: *dense gas plume rise, Jack Rabbit chlorine releases, plume touchdown*

INTRODUCTION

Upwards-directed dense plumes are interesting because they initially rise up due to their momentum, but then can settle down to the ground if the excess density flux is large enough. This scientific topic has been studied over the past 50 years or more using theoretical (analytical) analysis, laboratory experiments, and full scale experiments. Hoot, Meroney and Peterka (HMP, 1973) carried out laboratory experiments and used dimensional analysis principles applied to available plume rise theories (e.g., Briggs, 1969) to derive empirical formulas that best fit the observations. Ooms et al. (1974) also carried out theoretical analyses and fluid modeling studies to develop a set of plume rise equations for dense plumes. Also, Briggs pointed out that his formulas for the rise of plumes with densities different from that of air would work just as well for negatively buoyant plumes (dense plumes) as for positively buoyant plumes.

It should be pointed out that several of the current operational models for dense gas plumes can directly simulate the settling of the dense plume after it has risen to to its initial momentum. Here, we focus only on the analytical models.

There have been few observations of full-scale dense plumes that could be used to test these formulas. However, as part of the 2015-2016 Jack Rabbit II field program, one trial (8) involved a release of 2400 kg of chlorine from a pressurized tank with a hole at the top. Plume rise and touchdown distance could be determined from many available photos and videos and lidar scans, and maximum ground level concentration could be obtained from an extensive network of near-ground samplers (Fox et al 2021, Chang et al 2021). Source details (e.g., mass emission rate as a function of time) are obtained from the summary papers by Spicer and Miller (2018) and Spicer and Tickle (2020).

We have submitted a detailed manuscript on this analysis of the Trial 8 plume behavior to a journal and it is currently under review (Hanna et al., 2021). The current paper gives the highlights.

JACK RABBIT II TRIAL 8

The JR II experiments took place over an extensive flat salt playa at DPG (Nicholson et al., 2017). The chlorine release during Trial 8 was a two-phase momentum jet with duration about 30 s, from a 15.2 cm diameter hole at the top of the tank. The release began at 9:01:45 am Mountain Daylight Time. The initial chlorine mass in the tank was 9100 kg, and total mass released in the momentum jet was 2400 kg. The observed air temperature was 15.8 °C and estimated air density was 1.05 kg/m^3. Wind speed at a height of 2 m near the source location was 2.1 m/s, and wind direction was from 120 degrees. However, over the next 20 minutes, the surface wind direction near the source slowly veered to about 160 degrees and wind speed at 2 m decreased to about 1.5 m/s. A weak upward sensible heat flux was observed by sonic anemometers at 2 m, and a shallow (4 to 8 m) unstable layer had formed. Above the 8 m level, the boundary layer remained stable. As done for all nine JR II trials, concentrations and winds were measured at all directions around the circle for downwind distances of about 50 to 120 m, and on 90 degree arcs at distances of 0.2 to 11 km (see Chang et al., 2021). Still photos and videos were taken by cameras located at several near-ground sites to the side of the expected cloud direction and from the rear. Fox et al. (2021) and Mazzola et al. (2021) include tables of basic characteristics of the releases for all JR II trials.

The videos from the side during Trial 8 showed the plume initially rising to a height of about 40 m at a distance of about 30 m downwind. The plume's density caused it to slump to the ground, initially reaching the ground at a downwind distance of about 50 m (see Figure 1). After about 40 s, the plume rise increased to about 60 m (apparently the effects of the decrease in excess density with time overwhelmed the decrease in momentum). Over a period of about 40 s, the plume "touchdown" distance increased to about 100 m. After that, due the decrease in density of the plume, the plume centerline remained aloft.

Figure 1. Jack Rabbit II Trial 8 plume 12 s (left) and 28 s (right) after chlorine release begins. The plume center at maximum rise is about 40 m.

MASS EMISSION RATE AND OTHER INPUTS

The mass emission rate and other characteristics of the initial jet were determined by Spicer and Miller (2018) and Spicer and Tickle (2020). In addition, as explained in Hanna et al. (2021), two alternate expansion models were used to determine the flow characteristics in Table 1:

Expansion Model 1. Velocity increases above the exit value due to acceleration by excess pressure at the exit. This is also referred to as a momentum conservation model.

Expansion Model 2. Velocity unchanged from exit velocity.

Table 1. Initial plume conditions assumed for time periods after the release is initiated.

Release time ending	td obs Duration of release	Mass flux Qc	T obs cloud	Mass fract vapor	rhoc Model 1 cloud density	rhoc Model 2 cloud density	Qc/rhoc Model 1 Volume flux Vc	Qc/rhoc Model 2 Volume flux Vc	wo Model 1 Initial vert vel	wo Model 2 Initial vert vel	Ro Model 1 Initial Radius	Ro Model 2 Initial Radius
sec	(s)	kg/s	K		kg/m3	kg/m3	m3/s	m3/s	m/s	m/s	m	m
5	5	139.4	286.2	0.19	16.71	16.35	8.34	8.53	84.6	51.0	0.185	0.235
10	5	66.8	283.7	0.31	10.11	9.71	6.61	6.88	148.2	87.1	0.120	0.160
15	5	48.8	281	0.42	7.57	7.23	6.45	6.75	182.3	108.1	0.110	0.140
20	5	37.8	277.9	0.53	5.96	5.66	6.34	6.68	210.0	126.4	0.100	0.130
25	5	30.1	274.9	0.66	4.80	4.56	6.27	6.60	234.2	143.7	0.095	0.120
30	5	24.3	271.8	0.81	3.93	3.73	6.18	6.51	255.8	160.7	0.090	0.115
40	10	18.1	267.4	0.98	3.21	2.88	5.64	6.28	263.3	176.0	0.085	0.105
50	10	12.4	262	1	3.16	2.84	3.92	4.37	221.1	175.3	0.080	0.090
60	10	8.64	257.4	1	3.16	2.89	2.73	2.99	164.3	161.0	0.075	0.075
80	20	5.24	252.3	1	3.08	2.95	1.70	1.78	97.9	97.9	0.075	0.075
100	20	2.71	247.2	1	3.05	3.01	0.89	0.90	49.7	49.7	0.075	0.075

DENSE GAS PLUME MODELS TESTED

The Hoot-Meroney-Peterka (1973) model for upwards-directed dense plumes was developed by combining basic science concepts with wind tunnel observations. It consists of the following analytical formulas:

Maximum plume rise Δh above source: $\Delta h/2R_o = 1.32\,(w_o/u)^{1/3}(\rho_o/\rho_a)^{1/3}[w_o^2/(2R_o g')]^{1/3}$ (1)

where $g' = g(\rho_o-\rho_a)/\rho_o$; g is acceleration of gravity (9.8 m/s^2), ρ_a is ambient air density, u is wind speed, and ρ_o, R_o, and w_o are initial plume density, radius and vertical velocity after depressurization.

Distance to maximum plume rise: $x/2R_o = w_o u/(2R_o g')$ (2)

Plume touchdown distance x_g downwind:

$$x_g/2R_o = w_o u/(2R_o g') + 0.56\{(\Delta h/2R_o)^3\,[(2 + h_s/\Delta h)^3-1]u^3/(2R_o w_o g_a')\}^{1/2} \quad (3)$$

where $g_a' = g(\rho_o-\rho_a)/\rho_a$ and h_s is elevation of the stack or vent opening above the ground (2 m).
Air density ρ_a is assumed to be 1.055 kg/m^3. The HMP model also has a formula for the maximum concentration, C, at plume touchdown:

$$C/C_o = 2.43\,(w_o/u)\,[(h_s + 2\Delta h)/(2R_o)]^{-1.95} \quad (4)$$

An alternate basic dense gas plume model uses the Briggs (1969) fundamental equation for the trajectory of a buoyant (or negatively buoyant) plume as a function of distance, x:

$$\Delta h = [(19(\rho_o/\rho_a)(M_o/u^2)x - 4.2\,(B_c/u^3)x^2]^{1/3} \quad (5)$$

where $M_o = w_o^2R_o^2$ is proportional to the initial momentum flux and $B_c = g[(\rho_o-\rho_a)/\rho_a]w_oR_o^2$ is proportional to the initial buoyancy flux (here assumed positive for a dense cloud). The plume touchdown distance can be calculated as the distance where $\Delta h = 0$ (i.e., the first term in the equation equals the second term). That is, $x_g = 4.52(\rho_o/\rho_a)uM_o/B_o = 4.52\, w_o u/[g(\rho_o - \rho_a)/\rho_a]$. However, to be fair, Briggs intended eq 5 to be used only in the plume rise phase and not for determining plume touchdown.

RESULTS

The analytical plume rise formulas listed above assume steady state conditions. To account for the large decrease in mass emission rate during the Trial 8 release, we solve the steady state equations piecewise for 5, 10 or 20 s blocks of time, up to 100 s. The points in the "time series" plots in Figure 2 are representing solutions for emissions for these blocks. The point represents the end time of the block. Solutions are given for the two initial jet expansion models (1: momentum and 2: velocity conservation). The red dots are the observations, from our scaling of the visible plume on the videos and still photos.

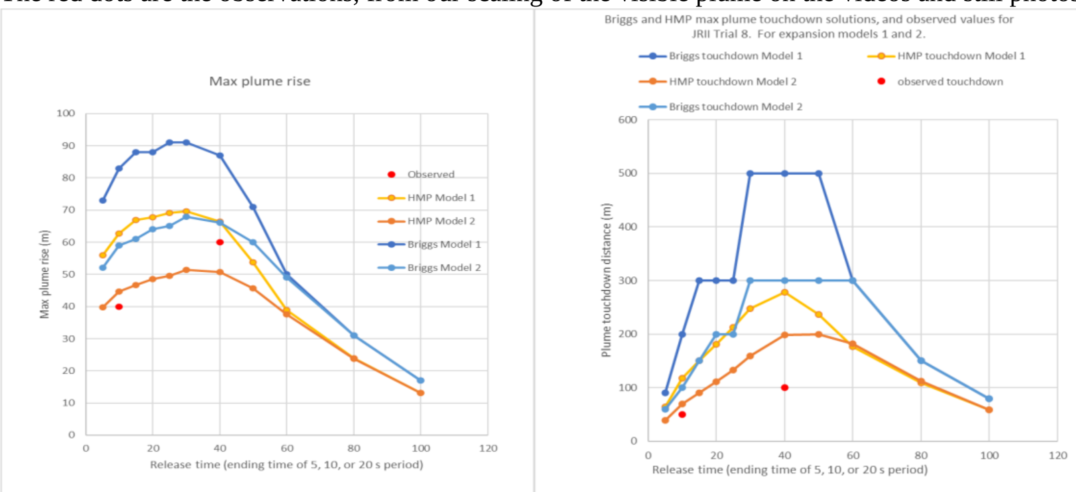

Figure 2. Observed and modeled maximum plume rise (left) and touchdown distance (right), calculated using emissions rates for 11 time segments after release was initiated. Red dots are observed.

The Briggs formula appears to always predict a maximum plume rise that is about 30 to 40 % larger than the HMP formula. Also, use of expansion model 1 always predicts maximum plume rise about 40 % larger at $t < 40$ s, but the solutions approach each other at $t > 40$ s. The HMP model prediction of maximum plume rise agrees better with the two observed values than the Briggs model. Perhaps HMP Model 2 performs the best, but the difference may not be statistically significant.

To be fair, Briggs was thinking primarily about positively buoyant plumes when he wrote his Plume Rise book in 1969. We are applying his model because it should be valid for negatively buoyant plumes, too, but perhaps not for the very large negative buoyancy in Trial 8.

The HMP model also produces a solution (eq 4) for the maximum concentration at plume touchdown. Table 2 lists the observation and the predictions of HMP Models 1 and 2. It is seen that the observed value is in between the two model predictions. Model 1 is about 21% low and model 2 is about 46% high

Table 2. Comparison of observed and modeled maximum concentration. Model predictions are at touchdown distance.

	Observed at 85 m	HMP Model 1	HMP Model 2
Max C (ppm)	12080	9570	17660

CONCLUSIONS

The HMP dense gas plume rise model (with either momentum or velocity conservation expansion models) is able to match the observations of maximum plume rise, touchdown distance, and maximum ground level concentration at JR II Trial 8 within a factor of about two. The Briggs plume rise model tends to predict larger maximum plume rise than the HMP model by about 40 %. Further details and results of inclusion of DRIFT model (Gant et al., 2020; Tickle and Carlisle 2008 and 2013) predictions are given in Hanna et al. (2021)

ACKNOWLEDGEMENTS

The research was supported by the U.S. Defense Threat Reduction Agency (DTRA). I appreciate the contributions of Graham Tickle of GT Science and Software, UK; Tom Mazzola of SAIC, Lorton, VA USA; and Simon Gant of Health and Safety Executive (HSE), UK.

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